# Collateral Constraints, Debt Management and Investment Incentives

Elettra Agliardi, Rainer Andergassen *

Department of Economics, University of Bologna

P.zza Scaravilli 2, 40126 Bologna (Italy)

April 2005

Abstract. This paper analyses the hedging decisions of an emerging economy which is exposed to market risks and whose debt contract is subject to collateral constraints. Within a sovereign debt model with default risk and endogenous collateral, we study the optimal choice of hedging instruments when both futures and non-linear derivatives are available. We examine in which way the hedging policy is affected by the cost of default and the financial constraints of the economy and provide some implications in terms of resource allocation.

Keywords: hedging strategies; financial constraints; default cost; endogenous collateral; emerging markets.

JEL: D84, E32, E40, F30, G15, G28, G32

*E-mail addresses: agliardi@economia.unibo.it (E. Agliardi), anderga@economia.unibo.it (R. Andergassen).

# 1 Introduction

Emerging markets have been exposed to remarkable market risks and it is by now folk-wisdom that, if given a choice, they should be endowed with instruments of hedging against downside risks (see Caballero (2003), Caballero and Panageas (2003), Shiller (2003)). Finding out which factors are the fundamental source of volatility for each country - for example, the prices of oil for Mexico, of coffee for Brasil, of semiconductors for Korea, of copper for Chile, and so on - is recognized as a crucial step in order to construct the appropriate hedging instruments, which will be contingent on observable variables (Caballero (2003)). Yet, it is still to be answered the question concerning the proper application of derivative securities that can be used to construct hedging strategies and the optimal hedging policy. The purpose of this paper is to examine the hedging decisions of an economy which is exposed to market risks and is subject to collateral constraints. The model we consider is a sovereign debt one, with default risk and endogenous collateral.

Collateral is typically used to secure loans. Since the paper by Kiyotaki and Moore (1997) it has been pointed out that if collateral is endogenous, then the debt capacity of firms is altered, causing fluctuations in output (Krishnamurthy, 2003). In this paper we discuss a model where the use of hedging instruments may affect collateral values and thus the debt capacity of the debtor.

In most literature relating to the 1980's debt crisis and following the Bulow and Rogoff models (1989, 1991) a given proportion of output or exports are assumed to be available for repayment of outstanding debt. This means that repayment is modelled as an "output tax" and actual repayment is the minimum of this amount and debt. Alternatively, in other models ((Eaton and Gersowitz (1981), Eichengreen (2003), Thomas (2004)) a fixed sanction is established in the case of default, which is not a direct claim on the country's current resources and is not received by the creditors, but may represent the future losses due to diminished reputation. In this paper we develop a model where the amount of repayment by the debtor country is determined endogenously by an optimizing choice of the debtor and where the two above-mentioned aspects of the repayment contract are present. Indeed, the debt contract is a collateralized one, where profits on internationally tradable goods can be used for repayment, constituting the endogenous collateral; additionally, in the case of default, a sanction is imposed which affects non-tradable goods, which represents the cost to the debtor of defaulting. Within this framework, hedging may be driven by the desirability to reduce expected default costs. As Smith and Stulz (1985) have shown, by hedging a debtor is able to reduce the likelihood of default by increasing the income it gets in the downside.

Our paper is most related to the literature on risk management. Recently, a few papers have studied the optimal choice of hedging instruments of a firm when either futures or options are available.

It has been shown that in the model of competitive firms with output price uncertainty, where all input decisions are made simultaneously prior to resolution of uncertainty, hedging with futures does provide a perfect hedge and there is no scope for non linear instruments such as options as pure hedging instruments. Albuquerque (2003) characterizes optimal currency hedging in three cases, namely in the presence of bankruptcy costs, with a convex tax schedule and in the case of a loss-averse manager. In all these cases, he shows that futures dominate options as hedging instruments against downside risk. Batterman, Braulke, Broll, Schimmelpfennig (2000) study the optimal choice of hedging instruments of an exporting firm exposed to exchange rate risk, when both currency futures and standard options are available. They show that the hedge effectiveness of futures is larger than that of options.

Wong (2003) studies the optimal hedging decision of an exporting firm which faces hedgeable exchange rate risk and non-hedgeable price risk, when price and exchange rate risk have a multiplicative nature. This source of non-linearity creates a hedging demand for non-linear payoff currency options distinct from that for linear payoff currency futures. Moschini and Lapan (1992) analyze the problem of hedging price risk under production flexibility, yielding nonlinearity of profits in output price, and show that there is a role for options even when the use of futures is allowed. In Froot, Scharfstein, Stein (1993) it is shown that firms may decide not to hedge fully, if there is correlation between investment opportunities and the availability of funds; moreover, options may be needed in addition to futures to implement the optimal hedge when there are state-dependent financing opportunities.

In this paper, we characterize optimal investment and hedging decisions. We show that the decision to use non-linear hedging strategies in addition to futures contracts can be optimal in relation to market conditions and financial constraint of the economy. In particular, we show in which way the optimal hedging decision is affected by the cost of default. In addition to a short position in futures, either concave or convex hedging with options is optimal, depending on the size of default costs. In particular, we find that if default costs are sufficiently large, options are used for financing purposes, that is to increase financial resources when these are needed for investment purposes. If default costs are sufficiently low, options are employed for speculative motives, i.e. financial resources are reduced when they are needed for investment purposes. Our results are thus closely related to those of Adam (2002, 2004) who shows how firms employ non-linear hedging strategies to match financial resources against financial needs at different time periods.

The remainder of the paper is organized as follows. Section 2 describes the model and the hedging problem of the economy. Section 3 contains the optimal hedging choices of a futures and straddles. Section 4 concludes. All proofs are in the Appendix.

# 2 The model

The model is a two-period model of sovereign debt with default risk[1] . Let us consider an economy having access to a technology producing an internationally tradable and a non-tradable good, denoted by $y_T$ and $y_{NT}$ respectively. In the production quasi-fixed inputs (e.g. capital goods) and variable inputs (e.g. labor) are used. The economy has no initial endowments. Thus, in order to produce, firms have to borrow capital from abroad. Borrowing is done with collateralized one-period-ahead debt contract in order to purchase and use in the production functions $k + z$ units of capital, where $k$ and $z$ are the units of capital employed in the production of $y_{NT}$ and $y_T$, respectively. Only the internationally tradable good can be used as a collateral.

At time 1 the price of the internationally tradable good $p$ is not known with certainty and the economy must commit to production plans by choosing the level of investment $z$ and $k$ in capital

[1] For a survey of the literature about sovereign debt see Eaton and Fernandez (1995),in Grossman and Rogoff (eds),Handbook of International Economics, Elsevier, Amsterdam.

goods. The price of the non-tradable good is known, constant over time.

In what follows, we assume that at time 1 producers can take positions in the futures market and in the option market to hedge their exposure. At time 2 uncertainty is resolved and the economy chooses the level $y_T$ ($y_{NT}$) conditional on $z$ ($k$) and on the open futures and options positions determined at time 1. We normalize the risk free interest rate to zero.

## 2.1 Time 2

At time 2, when price uncertainty is resolved, the usual profit maximization yields:

$$g(z,p) = \max_{y_T} \{ p y_T - c_1(y_T, z) \}$$

where $c_1(y_T, z)$ is the variable cost function which is conditional on the level of $z$. In what follows we assume that the production function is $y_T = \widetilde{A} z^{\frac{\beta}{2}} L^{\frac{1}{2}}$, where $L$ is labor and $0 < \beta < 1$. Therefore, $g(z,p) = p^2 A z^{\beta}$.

We assume that in the case of default, a sanction is imposed exogenously which leads to a reduction of $(1-\widetilde{\alpha})\%$ of non-tradable goods, with $1 \geq \widetilde{\alpha} > 0$. Let $q$ be the constant price of the non-tradable good. The production problem of the non-tradable good $y_{NT}$ at time 2 is given as follows

$$\phi^1(k) = \max\nolimits_{y_{NT}} \{ q y_{NT} - c_2(y_{NT}, k) \} \qquad \text{in case of no default}$$

$$\phi^2(k,\alpha) = \max\nolimits_{y_{NT}} \{ \widetilde{\alpha} q y_{NT} - c_2(y_{NT}, k) \} \qquad \text{in case of default}$$

where $c_2(y_{NT}, k)$ is a twice continuously differentiable function with positive first and second derivative in $y_{NT}$ and $c_2(0,k) = 0$. To simplify the exposition, we consider the following production function $y_{NT} = \widetilde{B} k^{1-\eta} L^{\eta}$, where $1 > \eta > 0$, and consequently $\phi^1(k) = Bk$ and $\phi^2(k,\alpha) = \alpha B k$, with $\alpha = (q\widetilde{\alpha})^{\frac{1}{1-\eta}}$, $1 \geq \alpha > 0$.

Consumption occurs in period 2. Consumers are risk-neutral and gain utility just from the consumption of the non-tradable good. Thus maximizing aggregate utility corresponds to maximizing $k$.

## 2.2 Time 1

At time 1 the country borrows from foreign creditors funds to purchase and use $k + z$ units of capital. Since there are only two periods, the loan has to be paid back at time 2. All debt contract has to be collateralized. Let $r$ be the repayment price per unit of capital. Let $x$ represent the futures position ($x > 0$ is short) and $s$ the straddle[2] position ($s > 0$ is short) that firms take to hedge the risk associated with price uncertainty. Denote the random profit of the economy at time 1 by:

$$\pi(p) = p^2 A z^\beta - rz - rk + (f - p)x + (t - v)s \tag{1}$$

where $f = E(p)$, $t = E(|p - p^*|)$ and $v = |p - p^*|$, where $p^*$ is the strike price. Then, the collateral constraint requires $\pi(p) \geq 0$. Notice that for $s > 0$, i.e. a short position in straddles, the economy increases its financial resources available for investment in the first period at the cost of reducing them in the second period, while for $s < 0$, i.e. a long position in straddles, the opposite occurs. Since in the present model the economy has no initial endowments, for $s > 0$ straddles are used for financing purposes since shortening straddles reduces financial constraints in the first period where investment decisions have to be taken. For $s < 0$ straddles are used for speculative purposes since financial resources are reduced when these are needed for investment purposes, while financial constraints are alleviated in the second period when repayments are due to. The same argument holds true for short and long positions in futures.

Given the collateral constraint, at time 1 when the price uncertainty has not been solved yet, the problem is specified as follows

$$\max_{k,z,x,s} \Omega(k, \alpha, \chi) \equiv Bk\left[1 - (1 - \alpha)(1 - \chi)\right] \tag{2}$$

where $\chi = \int_P I_{\pi(p) \geq 0} \psi^*(p)\, dp$, $I_{\pi(p) \geq 0}$ is an indicator function, $\psi^*(p)$ is the probability density function of the price of $y_T$, defined over the set $P$. For simplicity[3], we define $p = \overline{p} + \varepsilon$, where $E(\varepsilon) = 0$ and assume that $\varepsilon \in [-\overline{p}, \overline{p}]$ and is symmetrically and uniformly distributed, with probability density function $\psi(\varepsilon) = \frac{1}{2\overline{p}}$. We assume that $p^* = \overline{p}$. Thus, $f = \overline{p}$, $t = \frac{\overline{p}}{2}$ and $v = |\varepsilon|$.

[2] A long/short straddle is a portfolio which consists of a long/short put and a long/short call on the same asset with the same strike price and exercise time

[3] The assunptions of a symmetric distribution of prices and of a profit function quadratic in price is also in Moschini and Laplan (1992), (1995), where they show that futures and options have a role in hedging price risk.

### 2.3 Benchmark

Consider the case where the price of the collateral is known with certainty, and equal to its average value, i.e. $p = \overline{p}$, where $\overline{p} = E(p)$. The problem reduces to

$$\max_z \left\{ \overline{p}^2 A z^\beta - rz \right\}$$

From the first order condition we obtain $z^\circ = \left( \frac{\beta \overline{p}^2 A}{r} \right)^{\frac{1}{1-\beta}}$ and thus optimal $k$ is obtained from condition $\pi(\overline{p}) = 0$ which yields $k^\circ = \frac{1-\beta}{\beta} z^\circ$.

# 3 Optimal hedging

Since $g(z,p)$ is quadratic in $p$, we get:

$$\pi(\varepsilon) = \overline{p}^2 A z^\beta - r(z+k) + \left[ 2\overline{p} A z^\beta - x \right] \varepsilon + A z^\beta \varepsilon^2 + \left( \frac{\overline{p}}{2} - |\varepsilon| \right) s$$

Since $\varepsilon$ is symmetrically distributed over the set $[-\overline{p}, \overline{p}]$ we can rewrite $\pi$ considering only positive values of $\varepsilon$. Thus, for $\varepsilon \geq 0$, we have

$$\pi(\varepsilon) = \overline{p}^2 A z^\beta - rz - rk + \frac{\overline{p}}{2} s + \left[ 2\overline{p} A z^\beta - x - s \right] \varepsilon + A z^\beta \varepsilon^2$$

$$\pi(-\varepsilon) = \overline{p}^2 A z^\beta - rz - rk + \frac{\overline{p}}{2} s - \left[ 2\overline{p} A z^\beta - x + s \right] \varepsilon + A z^\beta \varepsilon^2$$

The following result can be obtained.

**Proposition 1**. *A short futures position $x = g_p(z, \overline{p}) = 2\overline{p} A z^\beta$ is optimal.*

Optimality requires a short position in futures equal to $2\overline{p} A z^\beta$. Thus, a short futures position increases the funds available at time 1 for investment purposes. Moreover, the future position does not depend on the cost of default $\alpha$.

For $x = 2\overline{p} A z^\beta$ we obtain $\pi(-\varepsilon) = \pi(\varepsilon)$, where

$$\pi(\varepsilon) = \overline{p}^2 A z^\beta - r(z+k) + \left( \frac{\overline{p}}{2} - \varepsilon \right) s + A z^\beta \varepsilon^2$$

$\pi(\varepsilon) \geq 0$ for values external to the two roots:

$$\varepsilon_{1,2} = \frac{s \pm \sqrt{s^2 - 4 \left[ \overline{p}^2 A z^\beta - r(z+k) + \frac{\overline{p}}{2} s \right] A z^\beta}}{2 A z^\beta} \tag{3}$$

We define $\delta = \frac{s}{s^*}$, where $s^* \equiv \overline{p} A z^{\beta}$. We assume that only a finite amount of straddles are available on the market. This corresponds to imposing upper and lower bounds on $\delta$, i.e. $|\delta| \leq \overline{\delta}$. In order to find a solution to problem (2) we proceed in two steps. First, using the first order condition for $z$, we find the optimal level of capital $k$ which yields a given probability of default $c$, where $c \in [0, 1]$. In this way we obtain $k$ as a function of $c$ and $\delta$. The payoff function in (2) can be rewritten as

$$\Omega\left(c, \delta\right) = k\left(c, \delta\right)\left[1 - \left(1 - \alpha\right) c\right] \tag{4}$$

In the second step we find the optimal position in straddles and the optimal probability of default $c \in [0, 1]$. From (4) we observe that maximizing the payoff function with respect to $\delta$ reduces to maximizing $k\left(c, \delta\right)$ over appropriate values of $\delta$, for each given $c$. Subsequently, it can be shown (see the Appendix) that $k\left(c, \delta^*\right)$, where $\delta^*$ is the optimal value of $\delta$, is an increasing function of $c$. Thus, in maximizing the payoff function with respect to $c$, the economy has to trade-off a larger expected punishment due to default against larger values of $k$. The size of the expected punishment depends on the value of $\alpha$. The larger is this value, the lower is the punishment in the case of default. Consequently, the solution to this trade-off depends on the size of $\alpha$.

The following result can be obtained.

**Proposition 2** *There exists a critical level $\alpha^*\left(\beta, \overline{\delta}\right)$ such that for $0 \leq \alpha < \alpha^*\left(\beta, \overline{\delta}\right)$ the optimal choice is $\delta = 1$ and $c = 0$, while for $\alpha^*\left(\beta, \overline{\delta}\right) < \alpha \leq 1$ the optimal choice is $\delta = -\overline{\delta}$ and $c \in \left(\frac{1}{2}, 1\right]$, where $\alpha^*\left(\beta, \overline{\delta}\right)$ is a decreasing function of $\beta$ and $\overline{\delta}$ and is strictly positive for $\beta < \beta\left(\overline{\delta}\right)$ and 0 otherwise, where $\beta'\left(\overline{\delta}\right) < 0$.*

Proposition 2 states that optimality requires non-linear hedging. For sufficiently low values of $\alpha$, i.e. sufficiently large costs of default, optimality requires a short position of $s^* \equiv \overline{p} A z^{\beta}$ straddles. Moreover, in this regime, the economy is induced never to default. The intuition for this result is as follows. Short selling straddles increases financial resources available for investment in the first period while it increases financial constraints in the second period. Thus, if default costs are sufficiently large, borrowing constraints are tighter, and thus the economy uses straddles to reduce these constraints in the first period and chooses not to default. Thus, in this regime straddles are used for financing purposes. For sufficiently large values of $\alpha$, i.e. sufficiently low costs of default, optimality requires a long position of $s = -\overline{\delta}\overline{p}Az^{\beta}$. Moreover, in this regime, the economy is induced to default with a probability larger than $\frac{1}{2}$. In this regime default costs are low and consequently financial constraints in the first period and borrowing constraints are loose. Thus, in this regime straddles are employed for speculative motives and furthermore the country will default with a probability larger than $\frac{1}{2}$.

Thus, the event of default can be avoided for $\beta < \beta\left(\overline{\delta}\right)$, chosing an $\alpha$ lower than $\alpha^{*}\left(\beta, \overline{\delta}\right)$.

**Corollary 1** *The optimal investment in $k$ is an increasing function of $\alpha$.*

The above mentioned optimal hedging strategies have direct implication in terms of resource allocation for the economy. It is straightforward to prove the following.

**Corollary 2** *There is overinvestment in $k$, $z$ with respect to the benchmark case.*

# 4 Conclusion

This paper shows how financially constrained economies should hedge. It thus extends the literature on risk management that show why firms hedge and which are the optimal hedging instruments, and the contributions on emerging markets, which point out that if collateral is endogenous, then the debt capacity of an economy is altered.

Within a sovereign debt model with default risk and endogenous collateral, we study the optimal choice of hedging instruments when both futures and non-linear derivatives are available. We show that in addition to futures, optimality requires either concave or convex hedging, depending on the size of the default cost. If this latter is sufficiently large, then optimality requires a short position in straddles and furthermore the economy is induced never to default. If the default cost is sufficiently low, then optimility requires a long position in straddles and the economy is induced to default with a probability larger than $\frac{1}{2}$.

# Appendix

**Proof of Proposition 1.** $\pi(\varepsilon) \geq 0$ for values external to the two roots

$$\varepsilon_{1,2}^{+} = \frac{-\left(2\overline{p}Az^{\beta} - x - s\right) \pm \sqrt{\left(2\overline{p}Az^{\beta} - x - s\right)^{2} - 4\left[\overline{p}^{2}Az^{\beta} - r\left(z+k\right) + \frac{\overline{p}}{2}s\right]Az^{\beta}}}{2Az^{\beta}}$$

while $\pi(-\varepsilon) \geq 0$ for values external to the two roots

$$\varepsilon_{1,2}^{-} = \frac{2\overline{p}Az^{\beta} - x + s \pm \sqrt{\left(2\overline{p}Az^{\beta} - x + s\right)^{2} - 4\left[\overline{p}^{2}Az^{\beta} - r\left(z+k\right) + \frac{\overline{p}}{2}s\right]Az^{\beta}}}{2Az^{\beta}}$$

Maximizing[4] (2) with respect to $x$ yields:

$$\frac{\partial \varepsilon_1^{+}}{\partial x} - \frac{\partial \varepsilon_2^{+}}{\partial x} + \frac{\partial \varepsilon_1^{-}}{\partial x} - \frac{\partial \varepsilon_2^{-}}{\partial x} = 0 \tag{5}$$

Expression (5) is satisfied if $x = 2\overline{p}Az^{\beta}$. ■

**Proof of Proposition 2.** Three cases arise. Case 1: $\overline{p} \geq \varepsilon_{1,2} \geq 0$; case 2: $\overline{p} \geq \varepsilon_1 \geq 0$ and $\varepsilon_2 < 0$; case 3: $\overline{p} \geq \varepsilon_2 \geq 0$ and $\varepsilon_1 > \overline{p}$. Using the definition of $\delta$, (3) and the probability of default $c$, these conditions can be redefined as: case 1: $c \leq \delta \leq 2 - c$; case 2: $-\overline{\delta} \leq \delta < c$; and case 3: $\overline{\delta} \geq \delta > 2 - c$.

Case 1

**Result A1** *Given the probability of default $c \in [0,1]$, for each $c \leq \delta \leq 2 - c$, the optimal strategy is $\delta = 1$, $k = \frac{1-\beta}{\beta}z$ and*

$$k\left(c,1\right) = \frac{1-\beta}{\beta}\left(\frac{\beta A}{r}\overline{p}^{2}\frac{5+c^{2}}{4}\right)^{\frac{1}{1-\beta}} \tag{6}$$

Using the definition of $\delta$, the first order condition for $z$ requires

$$k\left(\delta\right) = \frac{1-\beta}{\beta}z - \frac{\delta\left(\delta-1\right)}{2r}\overline{p}^{2}Az^{\beta} \tag{7}$$

Now we the hold the probability of default constant, and find the optimal strategy $\delta$. Using (3) and (7), the probability of default $c = \frac{\varepsilon_1 - \varepsilon_2}{\overline{p}}$ yields $z\left(c,\delta\right) = \left(\frac{\beta A}{r}\frac{4+\delta^{2}+c^{2}}{4}\overline{p}^{2}\right)^{\frac{1}{1-\beta}}$. Thus, for $z\left(\delta\right)$

[4] For simplicity of exposition we consider here the case where all roots exists and are included in the interval $[-\overline{p}, \overline{p}]$. The result remains the same also in the other cases.

and the corresponding value of $k$ (7) the probability of default is $c$. The maximum payoff, subject to the condition of a constant probability of default, is obtained maximizing $k$ as in (7) over values of $\delta$, i.e.

$$\max_{\delta} k(c,\delta) = \left(\frac{\beta A}{r}\overline{p}^2 \frac{4+\delta^2+c^2}{4}\right)^{\frac{1}{1-\beta}} \left[\frac{1-\beta}{\beta} - \frac{\delta(\delta-1)}{\beta}\frac{2}{4+\delta^2+c^2}\right]$$

which yields $\delta = 1$.

Thus, the problem reduces to find the optimal level of $c$,

$$\max_{c\in\left[0,\frac{1}{2}\right]} \Omega(c,1) \equiv B\frac{1-\beta}{\beta}\left(\frac{\beta A}{r}\overline{p}^2\frac{5+c^2}{4}\right)^{\frac{1}{1-\beta}} \left[1-(1-\alpha)c\right] \tag{8}$$

Case 2

**Result A2** *For each given $c \leq \frac{1}{2}$, $-\overline{\delta} \leq \delta < c$ is never optimal, while for $c > \frac{1}{2}$ it is optimal to choose $\delta = -\overline{\delta}$ and the corresponding capital level is*

$$k\left(c,-\overline{\delta}\right) = \left[\frac{\beta A}{r}\overline{p}^2\left(c^2+1\right)\right]^{\frac{1}{1-\beta}} \left(\frac{1-\beta}{\beta} + \frac{\overline{\delta}}{\beta}\frac{c-\frac{1}{2}}{c^2+1}\right) \tag{9}$$

From the first order conditions of $z$ we obtain

$$k_{1,2} = z\frac{1-\beta}{\beta} + \frac{s}{r}\left(\frac{\overline{p}}{2} \pm \sqrt{\frac{r}{\beta A}z^{1-\beta} - \overline{p}^2}\right) \tag{10}$$

For a given probability of default $c$, simple algebra shows that

$$k_1(c,\delta) = \left\{\frac{\beta A}{r}\overline{p}^2\left[(c-\delta)^2+1\right]\right\}^{\frac{1}{1-\beta}} \left[\frac{1-\beta}{\beta} + \frac{\delta}{\beta}\frac{\frac{1}{2}+c-\delta}{(c-\delta)^2+1}\right]$$

$$k_2(c,\delta) = \left[\frac{\beta A}{r}\overline{p}^2\left(c^2+1\right)\right]^{\frac{1}{1-\beta}} \left[\frac{1-\beta}{\beta} + \frac{\delta}{\beta}\frac{\frac{1}{2}-c}{1+c^2}\right]$$

For $c \leq \frac{1}{2}$, inspection shows that $k_1(c,\delta) < k_2(c,\delta)$ and further $k_2(c,\delta)$ is increasing in $\delta$ and thus the maximum is achieved in $\delta = c$. Furthermore $k_2(c,c)$ is increasing in $c$, and thus $k\left(\frac{1}{2},\frac{1}{2}\right) = \frac{1-\beta}{\beta}\left(\frac{\beta A}{r}\overline{p}^2\frac{5}{4}\right)^{\frac{1}{1-\beta}} < \frac{1-\beta}{\beta}\left(\frac{\beta A}{r}\overline{p}^2\frac{5+c^2}{4}\right)^{\frac{1}{1-\beta}} = k(c,1)$. Consequently, if $c \leq \frac{1}{2}$ is optimal, then $\delta = 1$ is optimal.

For $c > \frac{1}{2}$ we observe that $\frac{\partial}{\partial\delta}k_2(c,\delta) < 0$ and further that $k_2\left(c,-\overline{\delta}\right) > k_1(c,\delta)$ for each $\delta \in \left[-\overline{\delta},c\right]$.

Case 3

**Result A3** *For each given* $0 \le c \le \frac{1}{2}$, $\overline{\delta} \ge \delta > 2-c$ *is never optimal, while for* $c > \frac{1}{2}$ *it is optimal to choose* $\delta = \overline{\delta}$ *and the corresponding capital level is*

$$k\left(c,\overline{\delta}\right) = \left\{\frac{\beta A}{r}\overline{p}^2\left[1+(1-c)^2\right]\right\}^{\frac{1}{1-\beta}}\left[\frac{1-\beta}{\beta}+\frac{\overline{\delta}}{\beta}\frac{c-\frac{1}{2}}{1+(1-c)^2}\right] \tag{11}$$

From the first order conditions of $z$ we obtain (10) and consequently, for a given probability of default $c$, simple algebra shows that

$$k_1 = \left\{\frac{\beta A}{r}\overline{p}^2\left[1+[\delta-(1-c)]^2\right]\right\}^{\frac{1}{1-\beta}}\left[\frac{1-\beta}{\beta}+\frac{\delta}{\beta}\frac{\frac{3}{2}-\delta-c}{1+[\delta-(1-c)]^2}\right]$$

$$k_2 = \left\{\frac{\beta A}{r}\overline{p}^2\left[1+(1-c)^2\right]\right\}^{\frac{1}{1-\beta}}\left[\frac{1-\beta}{\beta}+\frac{\delta}{\beta}\frac{c-\frac{1}{2}}{1+(1-c)^2}\right]$$

For each given $c \le \frac{1}{2}$, $\frac{\partial}{\partial\delta}k_{1,2} \le 0$ and consequently the maximum value of $k_{1,2}$ is obtained in $\delta = 2-c$. Simple inspection shows that for each $c \le \frac{1}{2}$, $k_2(c,2-c) \ge k_1(c,2-c)$. Furthermore, $k(c,2-c)$ is increasing in $c$ and $k\left(\frac{1}{2},2-\frac{1}{2}\right) = \frac{1-\beta}{\beta}\left(\frac{\beta A}{r}\overline{p}^2\frac{5}{4}\right)^{\frac{1}{1-\beta}} < \frac{1-\beta}{\beta}\left(\frac{\beta A}{r}\overline{p}^2\frac{5+c^2}{4}\right)^{\frac{1}{1-\beta}} = k(c,1)$.

For $c > \frac{1}{2}$ we observe that $\frac{\partial}{\partial\delta}k_2 > 0$ and further that $k_2\left(c,\overline{\delta}\right) > k_1(c,\delta)$, for each $\delta \in \left[2-c,\overline{\delta}\right]$.

We are now able to prove Proposition 2. First notice that as for each $\overline{\delta} \ge 1$, $k\left(c,-\overline{\delta}\right) > k\left(c,\overline{\delta}\right)$. Consequently the country preferes to buy straddles instead of shortening them, i.e. $\Omega\left(-\overline{\delta},1\right) > \Omega(1,1)$. Furthermore observe that, applying the envelope theorem, $\frac{\partial}{\partial\overline{\delta}}\Omega\left(-\overline{\delta},c\right) > 0$, $\frac{\partial}{\partial\alpha}\Omega\left(-\overline{\delta},c\right) > 0$ and $\frac{\partial}{\partial\alpha}\Omega(1,c) > 0$.

Consider the case of $\alpha = 1$ where no punishment occurs in the case of default. Since the optimal amount of capital $k(c,\delta)$ is increasing in $c$, it is always optimal to choose $c = 1$. Since $\Omega\left(-\overline{\delta},1\right) > \Omega(1,1)$ for each $\overline{\delta} \ge 0$, a long position in straddles is optimal.

Consider the case of $\alpha = 0$. Since $\Omega\left(-\overline{\delta},1\right) = 0$ and $\frac{\partial}{\partial c}\Omega\left(-\overline{\delta},\frac{1}{2}\right) > 0$, the optimal value of $c$ is obtained in $c \in \left(\frac{1}{2},1\right)$. Let us call $c_L = \arg\max_c \Omega\left(-\overline{\delta},c\right)$ and $c_S = \arg\max_c \Omega(1,c)$, then for $\beta \to 0$ and $\overline{\delta} = 2$, $\Omega\left(-\overline{\delta},c_L\right) < \Omega(1,0)$. Furthermore, computing $\Omega\left(-\overline{\delta},c_L\right)$ and $\Omega(1,c_S)$ for all possible values of $\alpha$, we observe that there exists a critical level of $\alpha$ such that for all values below this level it is optimal to short straddles ($\delta = 1$), while for values of $\alpha$ above this level it is optimal to buy straddles ($\delta = -\overline{\delta}$). Notice that $\Omega\left(-\overline{\delta},c_L\right)$ is increasing in $\overline{\delta}$ and thus the larger is $\overline{\delta}$, the lower is this critical level.

For $\alpha = 0$, $\frac{\partial}{\partial \beta}\Omega\left(-\overline{\delta}, c_L\right) > \frac{\partial}{\partial \beta}\Omega\left(1, c_S\right)$, for each value of $\beta$, and since for $\beta \rightarrow 1$, $c_L$, $c_S \rightarrow 1$ and $\frac{\Omega\left(-\overline{\delta}, c_L\right)}{\Omega(1, c_S)} \rightarrow \infty$ there exists a critical level of $\beta\left(\overline{\delta}\right)$ below which $\frac{\Omega\left(-\overline{\delta}, c_L\right)}{\Omega(1, c_S)} < 1$ and above which $\frac{\Omega\left(-\overline{\delta}, c_L\right)}{\Omega(1, c_S)} > 1$. Since $\Omega\left(-\overline{\delta}, c_L\right)$ is increasing in $\overline{\delta}$, this critical level is decreasing in $\beta$. We know that for $\alpha = 1$ $\frac{\Omega\left(-\overline{\delta}, c_L\right)}{\Omega(1, c_S)} > 1$ and thus computing for each $\beta < \beta\left(\overline{\delta}\right)$ and each value of $\alpha$ the payoffs $\Omega\left(-\overline{\delta}, c_L\right)$ and $\Omega\left(1, c_S\right)$ we observe that there exists a criticl value of $\alpha$ where $\frac{\Omega\left(-\overline{\delta}, c_L\right)}{\Omega(1, c_S)} = 1$. Since $\Omega\left(-\overline{\delta}, c_L\right)$ is increasing in $\overline{\delta}$ this critical value is decreasing in $\overline{\delta}$. ■

**Proof of Corollary 1.** The result follows from Proposition 2, (9) and from the fact that $c_L$ is increasing in $\alpha$. ■

**Proof of Corollary 2.** From Proposition 2 it follows that for $\alpha < \alpha^*$ the equilibrium is $\delta = 1$ and $c = 0$ and thus optimal investment in $z$ is $z = \left(\frac{\beta A}{r}\overline{p}^2 \frac{5}{4}\right)^{\frac{1}{1-\beta}} > z^{\circ}$. Furthermore, since $k = \frac{1-\beta}{\beta} z$, it follows from 6 that $k\left(0, 1\right) > k^{\circ}$. For $\alpha > \alpha^*$ the equilibrium is $\delta = -\overline{\delta}$ and $c \in \left(\frac{1}{2}, 1\right]$ and thus optimal investment in $z$ is $z = \left[\frac{\beta A}{r}\overline{p}^2\left(1 + c^2\right)\right]^{\frac{1}{1-\beta}} > z^{\circ}$. Furthermore, from 9 it follows that $k\left(c, -\overline{\delta}\right) > k^{\circ}$. ■

# References


Adam, T.R. (2002), Risk management and the credit risk premium, *Journal of Banking & Finance*, **26**, 243-269.

Adam, T.R. (2004), Why firms use non-linear hedging strategies, Mimeo, Hong Kong University of Science & Technology.

Albuquerque, R. (2003), Optimal Currency Hedging, Mimeo, University of Rochester.

Batterman, H.L., Braulke,M., Broll, U.,and Schimmelpfennig, J. (2000), The preferred Hedge Instrument, *Economics Letters*, **66** , 85-91.

Bulow,J. and K. Rogoff (1989), A Constant Recontracting Model of Sovereign Debt, *Journal of Political Economy*, **96**, 155-178.

Bulow,J. and K. Rogoff (1991), Sovereign Debt repurchases: no Cure for Overhang, *Quarterly Journal of Economics*, **106**, 1219-1235.

Caballero, R. (2003), The future of the IMF, *American Economic Review, Papers and Proceedings*, **93**, 31-38.

Caballero, R and S. Panageas (2003), Hedging Sudden Stops and Precautionary Recessions: a Quantitative Approach, Mimeo, MIT.

Eaton, J. and M. Gersovitz (1981), Debt with Potential Repudiation, *Review of Economic Studies*, **48**, 289-309.

Eichengreen,B.(2003), Restructuring Sovereign Debt, *Journal of Economic Perspectives*, **17**, 75-98.

Froot, K.A., Scharfstein, D.S. and J.C. Stein (1993), Risk Management: Coordinating Corporate Investment and Financing Policies, *The Journal of Finance*, **48**, 1629-1658.

Kiyotaki,N. and J. Moore (1997), Credit Cycles, *Journal of political Economy*, **105**, 211-248.

Krishnamurthy, A (2003), Collateral constraints and the amplification mechanism, *Journal of Economic Theory*, **11**, 277-292.

Moschini, G. and H. Lapan (1992), Hedging Price Risk with Options and Futures for the Competitive Firm with Production Flexibility, *International Economic Review*, **33**, 607-618.

Shiller, R.J. (2003), *The New Financial Order*, Princeton University Press, Princeton.

Smith, C.W. and R.M. Stulz (1985), The Determinants of Firm's Hedging Policies, *Journal of Financial and Quantitative Analysis*, **4**, 391-405.

Thomas, J.P. (2004), Default Costs, Willingness to Pay, and Sovereign Debt Buybacks, *Journal of Restructuring Finance*, **1**, 35-47.

Wong, K.P (2003), Currency Hedging with Options and Futures, *European Economic Review*, **47**, 833-839.