When Did The Options Market in Enron Lose Its' Smirk?

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Abstract:

The Enron Corporation went from a $65 billion dollar market capitalization to bankruptcy in just 16 months. Using statistical techniques for extracting the implied probability distributions built into option prices, I examine the market's expectation of Enron's risk of collapse. I find that the options market remained far too optimistic about the stock until just weeks before their bankruptcy filing.

`Keywords`: volatility smile; options; Enron; bankruptcy.

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The Enron Corporation was widely praised by Wall Street analysts even after the bear market began in early 2000. In August of 2000, its' share price peaked at $90. With a market capitalization of $65 billion, it was the seventh largest publicly traded company in the U.S. Beginning in the August 2001 though, a series of questions about the company's financial statements emerged following the resignation of their CEO. Four months later, the company was in bankruptcy.

Enron's collapse was due to excessive debt, disguised from the public through off balance sheet entities. Wall Street buy side analysts were either deceived or dishonest. Many maintained strong buy ratings virtually up to their bankruptcy filing. Was everyone caught off guard by the unraveling of Enron's complex financial structure? This paper examines the implied probability distributions built into option prices to answer this question.

There are two general approaches to obtaining probabilistic information from options. The first group of methods generalize the stochastic processes in the Black-Scholes model which assume no discontinuous changes in the stock price and constant volatility. Merton (1976) and Bates (1991) have made an important extension by allowing for jumps. The stochastic volatility literature, with contributions from Wiggins (1987), Hull and White (1987), Stein and Stein (1991) and Heston (1993), allows for volatility to change over time. The GARCH model also allows for time dependent volatility and has been applied in this context by Duan (1995). A related literature, with papers by Dumas, Whaley and Fleming (1998) and Das and Sundaram (1999), has looked at deterministic variations in volatility with the level of the stock price or with time[1].

This paper utilizes a second approach that looks directly as the probability distribution. I parameterize the underlying as a mixture of log normals as in Ritchey (1990), and Melick and Thomas (1997). Alternative parameterizations include binomial trees, as in Rubinstein (1994) and Jackwerth and Rubinstein (1996), trinomial trees, developed by Derman, Kani, and Chriss (1996), polynomial expansions, as in Longstaff (1995) and Rubinstein (1998), and the finite difference methods used in Andersen and Brotherton-Ratcliffe (1998). Nonparametric approaches, using kernel density estimation, have been proposed by Ait-Sahalia and Lo (1998).

The mixture density is quite promising in explaining the departures from Black-Scholes observed in my Enron option sample. In Haas, Mittnik, and Mizrach (2005), this model provides some evidence of an early warning in the two ERM exchange rate crisis. With Enron, the market appears to have been too sanguine. In the last month before the company went bankrupt, the op-

[1] In the literature, these deterministic volatility functions are often called *local volatility.* They appeal to practitioners because they often fit better the observed market prices.

tions market saw no greater risk of a 50% decline in price than it did before the details of Enron's accounting irregularities came to light. The smart money in the options market appears to have been fooled as much as unwitting retail investors.

The paper begins with the implied density information contained within options. This enables options to be priced under very general assumptions. The variation of the call price with the strike motivates the presentation of the histogram estimators in Section 2. Section 3 develops the simulated moments estimation procedure. Section 4 discusses facts about Enron. Section 5 looks at data and estimation. Section 6 concludes.

1. Implied Probability Densities

1.1 Motivation

Departures from the Black-Scholes distributional assumptions may account for the observed variation of implied volatility with the strike price. Because this variation generally has a parabolic shape, it is often called the volatility "smile." The smile is often present on only one part of the distribution giving rise to a "smirk."

A small sampling of the literature indicates that these effects are present across a wide variety of markets and instruments. Haas, Mittnik, and Mizrach (2005) find a smile in European exchange rate options. Bates (1991) found negative skewness in U.S. stock index options consistent with a crash-risk premium. Tompkins (2001) found similar results for the Japanese, German and British markets stock index options. Tompkins also finds variation in implied volatilities across strikes in British, German, Japanese and U.S. bond futures options as well.

1.2 How volatility varies with the strike

In the Black-Scholes case, volatility does not vary with the striking price. As I have noted in the prior section, this assumption seems violated in practice. I make the first attempt at characterizing the relationship between the moneyness of the call price and the volatility.

Let $f(S_T)$ denote the terminal risk neutral probability that $S = x$ at time T, and let $F(S_T)$ denote the cumulative probability . A European call option at time t, expiring at T, with striking price K, is priced

$$C(K, \tau) = e^{-r\tau} \int_K^\infty (S_T - K) f(S_T) dS_T, \tag{1}$$

where $\tau = T - t$, and r is the annualized interest rate. In the case where $f(.)$ is the normal density and volatility σ is constant with respect to K, this yields the Black-Scholes formula,

$$BS(S_t, K, \tau, r, \sigma) = S_t N(d_1) - K e^{-r\tau} N(d_2), \tag{2}$$
$$\begin{aligned} d_1 &= \frac{\ln(S_t/K) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}} \\ d_2 &= d_1 - \sigma\sqrt{\tau}, \end{aligned}$$

where $N(.)$ is the cumulative normal distribution.

Since the risk neutral distribution is unobservable, a large empirical and theoretical literature has devised ways to extract the implied distribution from options prices. Breeden and Litzenberger (1978) provided a road map for a variety of approaches by showing the link the change in the option price with respect to moneyness. The first derivative is a function of the cumulative distribution,

$$\partial C/\partial K \mid_{K=S_T} = -\exp^{-r\tau}(1 - F(S_T)). \tag{3}$$

The second derivative then extracts the density,

$$\partial^2 C/\partial K^2 \mid_{K=S_T} = \exp^{-r\tau} f(S_T). \tag{4}$$

The principal problem in estimating f is that we don't observe a continuous function of option prices and strikes. Early attempts in the literature like Shimko (1994) interpolated between option prices. Later attempts turned to either specifying a density family for f or a more general stochastic process for the spot price. This paper follows Ritchey (1990) and Melick and Thomas (1997) by specifying f as a mixture of log normal distributions.

Dupire (1994) clarifies the isomorphism between the approaches that specify the density and those that specify price process. He shows that for driftless diffusions, there is a unique stochastic process corresponding to a given implied probability density.

2. A Mixture of Lognormals Specification

I first parameterize the data generating mechanism for the stock price as a mixture of log normals. I then simulate from the distribution in the three baseline examples to illustrate the range of possible volatility smiles.

2.1 The data generating mechanism

I assume that the stock price process is a draw a mixture of three log normal distributions, $N(\mu_i, \sigma_i)$,

$i = 1, 2, 3$ with $\mu_3 \geq \mu_2 \geq \mu_1$. Three additional parameters p_1, p_2 and p_3 define the probabilities of drawing from each log normal. To nest the Black-Scholes, I restrict the central log normal to have the same mean as the risk free rate, $\mu_2 = r$. Risk neutral pricing then implies restrictions on either the other means or the probabilities. I chose to let μ_2, p_1 and p_3 vary, which implies

$$\mu_3 = \mu_1 p_1 / p_{3,} \tag{5}$$

and

$$p_2 = 1 - p_1 - p_3. \tag{6}$$

For estimation purposes, this leaves me six free parameters $\Theta = (m_1, s_1, s_2, m_3, s_3, \pi_1, \pi_3)$. I take exponentials of all the parameters because they are constrained to be positive. The left hand mixture is given by

$$N(\mu_1, \sigma_1) = N(r - \exp(m_1), 100 \times \exp(s_1)). \tag{7}$$

The only free parameter of the middle lognormal is the standard deviation,

$$N(\mu_2, \sigma_2) = N(r, 100 \times \exp(s_2)). \tag{8}$$

I parameterize the probabilities using the logistic function to bound them on $[0, 1]$,

$$p_1 = \exp(\pi_1)/(1 + \exp(\pi_1)), \tag{9}$$

$$p_3 = \exp(\pi_3)/(1 + \exp(\pi_3)). \tag{10}$$

The probability specification implies the following mean restrictions on the third log normal,

$$N(\mu_3, \sigma_3) = N\left((r - \exp(m_1)) \times \frac{\exp(\pi_1)/(1 + \exp(\pi_1))}{\exp(\pi_3)/(1 + \exp(\pi_3))}, 100 \times \exp(s_3)\right). \tag{11}$$

In the baseline simulations, I show that this data generating mechanism can match a wide range of shapes for the volatility smile.

2.2 Baseline examples

In all the following examples, I look at a set of 41 European calls with equally spaced strikes from 20 to 60 around a spot price of 40. I assume a risk free rate of 4% and no dividends. All three examples have a weighted average volatility of between 37 and 38%.

The model nests the Black-Scholes by making the transition probabilities zero by setting $\pi_1 = \pi_3 = -\infty$, or making all the means and standard deviations equal, $m_1 = 0$, and $s_1 = s_2 = s_3$.Either parameterization gives the flat Black-Scholes profile with respect to the strike.

A smile can be generated by a fat-tailed distribution. I set the standard deviations of the middle and right tail mixtures quite high, and $\sigma_2 = 49.5\%$ and $\sigma_3 = 50.2\%$ per annum, with $s_2 = -3.0$ and $s_3 = -0.69$. I then lower the left tail standard deviation to $\sigma_1 = 23.3\%$, $s_1 = -1.46$. I assume you draw more frequently from the less volatile tail, $p_1 = 45.3\%$, $\pi_1 = -0.19$. than the right, $p_3 = 32.6\%$, $\pi_3 = -0.73$. The lower tail and upper tail have means $\mu_1 = 2.87\%$ and $\mu_3 = 5.56\%$, by setting $m_1 = 0.12$.

The smirks require risks of sizable jumps. I generate the right smirk by assuming a large a large jump in the right tail, $\mu_3 = 7.95\%$ and a smaller jump down, $\mu_1 = 1.90\%$, with $m_1 = 0.74$. The probability that the stock price will move down is just slightly higher, $p_1 = 43.2\%$, and $p_3 = 23.1\%$, setting $\pi_1 = -0.27$ and $\pi_3 = -1.21$. The standard deviations here increase as you move left to right, $\sigma_1 = 32.2$, $\sigma_2 = 39.3$, $\sigma_3 = 22.31$, with $s_1 = -1.13$, $s_2 = -0.93$, and $s_3 = -0.79$.

The left smirk is generated with a larger jump down than up, $\mu_1 = 1.85\%$, $\mu_1 = 6.73\%$, with $m_1 = 0.76$. The jump probabilities are approximately equal, $p_1 = 44.9\%$, and $p_3 = 35.3\%$, with $\pi_1 = -0.20$ and $\pi_3 = -0.60$. The standard deviations here are smaller in the upper mixture. $\sigma_1 = 37.6\%$, $\sigma_2 = 55.9\%$, and $\sigma_3 = 26.0\%$. The corresponding model parameters are $s_1 = -0.99$, $s_2 = -0.58$, and $s_3 = -1.35$. All three examples are charted in Figure 1.

[INSERT Figure 1 Here]

I consider next the probability distributions implicit in these volatility patterns. I graph the probability distributions of spot price outcomes 90 days into the future in Figure 2. The left (right) smirk has a higher (lower) mean and longer right (left) tail.

[INSERT Figure 2 Here]

These distributions are more leptokurtic and positively skewed than a lognormal distribution with the same average volatility.

[INSERT Table 1 Here]

Later, we will see if these shapes change in response to important events in the Enron case to which I now turn.[2]

[2] Bates (2000) examines the case where investors preferences may include an explicit skewness premium.

3. The Enron Case

The Enron case is the second largest bankruptcy in U.S. history.[3] It involved the meltdown of the seventh largest corporation in the U.S. in a matter of a few months. It is a case of fraud, greed, and regulatory failure. This section reviews the key events in the history of the company.

3.1 Rapid rise

Enron was founded in 1985 from the merger of two natural gas companies, Houston Natural Gas Omaha based InterNorth. In 1989, it began a global trading business in natural gas that grew rapidly once government price regulations were lifted in the 1990s. It first traded electricity in North America in June of 1994 and expanded into Europe in 1995. Enron also pioneered the market in weather derivatives, trading its first products in August of 1997. Eventually they would expand their trading business into a wide array of products ranging from pulp and paper to broadband telecommunications. In 1985, the company had revenues of $10.25 billion, and a net income of $125 million. By the year 2000, the company exceeded $100 billion in revenue and nearly $1 billion in net profits, with the energy trading business[4] responsible for 72%. Nearly all of this growth, as can be seen in Table 2, took place after 1995.

[INSERT Table 2 Here]

CFO magazine praised the company for its rapid transformation "from a heavily regulated domestic natural-gas pipeline business to a fully integrated global energy company with thriving activities in natural gas, electricity, infrastructure development, marketing and trading, energy financing, and risk management." They cited the pioneering efforts of chief financial officer Jeffrey Fastow's "unique financing techniques" and awarded him the CFO of the Year award in 1999.[5] On February 6, 2001, Fortune named the company the "most innovative in the U.S." for the sixth consecutive year. Fortune's award was based on a survey of 10,000 executives, directors, and analysts.[6]

The company was also a favorite of Wall Street analysts, and they helped propel Enron shares to an all-time high of $90.75 on August 23, 2000, well after the Nasdaq bubble had burst. Salomon

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[3] WorldCom, Inc. is now the largest.
[4] In the annual report, this is called the Wholesale Energy Division.
[5] *CFO* Magazine, October 1, 1999.
[6] *Fortune*, February 6, 2001.

Smith Barney's report of January 22, 2001 was typical of Wall Street's admiration for this company at the top. The placed a price target of $100 on the company based on "$60 in implied value for Enron's energy merchant platform and $40 for bandwidth trading and other extensions of their risk merchant franchise." They projected that bandwidth trading, a business which lost $32 million on only 232 transactions in the fourth quarter of 2000, would "within 5 years, exceed...the entire value of energy marketing..." Like many other firms on Wall Street, they also praised the company for disposing of nearly all of its physical assets.

3.2 The beginning of the end

Hoping perhaps to go out at the peak or avoid a storm he could see forming, Kenneth Lay, CEO since 1986, stepped down and was replaced by president and chief operating officer Jeffrey Skilling on December 12, 2000. Skilling's eight month tenure was to be marked by a series of negative news. Enron was criticized for profiteering during the California energy crisis that begin in the summer of 2000 and extended into 2001. Enron's remaining physical assets began to report disappointing results. On December 15, 2000, Enron agreed to buy back shares of a failing water subsidiary, Azurix Inc., that it had spun off xx months earlier. A deal with Blockbuster Inc. to deliver movies via the Internet also collapsed in April of 2001. Dabhol Power Co., a wholly owned subsidiary of the company, lost its contract with the Indian government at the end of May. An Enron spin-off, New Power Co. fell dramatically in August 2001.

The stock price slid steadily during Skilling's tenure. It fell from $83.13 per share at the end of 2000 to $44.07 per share on June 25, 2001. This 47% decline exceeded the 7.7% decline in the S&P500 over the same period. Skilling began to face criticism from the public its role in the California crisis and even from a handful of Wall Street analysts over the company's leverage. Skilling insult here.

The stock had a short lived rally at the end of June into July on the basis of strong reported second quarter results. In mid-July though, the stock price continued slowly downward, declining 28.7% from July 16 to August 13, 2001 compared to a 6.2% decline in the S&P 500. The next day, Skilling shocked the market by resigning as CEO, and Kenneth Lay returned to the position. The stock price fell another 12.7%, reaching $36.85 on August 16th but then stabilized in that region for the rest of the month. As Enron's troubles began to emerge, analysts stayed with the company. As late as September 2001, 15 of the 16 covering analysts on Enron had either buy or strong buy ratings on the stock.

To show confidence in the company, Lay exercised options on 68,000 shares at prices of $20.78 to $21.56 on August 20-21, 2001. Lay, however, disposed of some the shares shortly afterwards to repay a loan. It is unclear if Lay realized how much trouble the company was in. On August 15, 2001, finance executive Sherron Watkins sent an anonymous letter to Lay warning that the company "will implode in a wave of accounting scandals." She met him face to face on Aug. 22, 2001.

Enron's accounting firm, Arthur Andersen, appears to have been complicit in this fraud. Oct. 12, 2001, David Duncan, the chief auditor for Andersen's Enron account, organized a two-week document destruction effort to discard many records, according to auditor Arthur Andersen. On October 23, 2001, Andersen began to shred Enron documents.

In its' earnings releases the company slowly began to reveal the extent of their problems. Oct. 16, 2001, Enron reported its first quarterly loss in over four years after taking charges of $1 billion on poorly performing businesses. Nov. 8, 2001, Enron says it overstated earnings dating back to 1997 by almost $600 million. Until late November, it appeared that Enron would be bought by a much smaller rival, Dynegy, despite a Securities and Exchange Commission (SEC) investigation that Enron acknowledged on October 22, 2001. The SEC was investigating the private partnerships created by CFO Andrew Fastow that were at the core of the Enron fraud. Dynegy backed out of its deal with Enron on November 28, 2001 after Enron's credit rating is downgraded to "junk" bond status.

3.3 Chapter 11

Without a buyer in sight and bankruptcy unavoidable, Enron shares plunged below $1 on November 28 amid the heaviest single-day trading volume ever for a NYSE or Nasdaq-listed stock. On Dec. 2, 2001, Enron filed for Chapter 11 bankruptcy. On Dec. 12, 2001, Congressional hearings began on Enron's collapse.

On Jan. 17, 2002, Enron decided to fire Andersen, blaming the auditor for destroying Enron documents government investigators were seeking for a probe into the energy trader's murky bookkeeping. The entire Arthur Andersen firm was indicted by the Justice Department in March 2002, a very unusual move.

The first indictments for Enron activity came on August 2002 when Michael Koppers, who worked with Fastow on the partnership deals, pleaded guilty to charges. As the one year anniversary of the bankruptcy now looms, the legacy of Enron still reverberates on Wall Street. The case has brought about important reforms in corporate governance and accounting oversight. Parenthetically, the energy trading market has nearly collapsed. Morgan Stanley had bought Enron's trading operation but shut down most of it in October 2002.

I now turn to the questions of whether the options market provided any indication that such a collapse was in the offing.

4. Data and Estimation

4.1 Sample

I have American style options for all strikes and expirations for the period July 16 to November 15, 2001. I filter the data in the following fashion: volume more than 5 contracts, more than 5 days to expiration, and implied volatility no more than twice the volatility of an at the money call. It leaves me with more than 1,500 daily observations of a range of strikes and maturities.

[INSERT Table 3 Here]

Table 3 shows that the stock price was still at almost $50 per share at the start of the sample period and fell under $10 at the end. The strikes range from 50% above and below the spot price at the beginning of the sample to more than four times the spot in November. Puts and calls were traded at roughly the same ratios until the last four weeks of the sample.

4.2 Estimation

There are two key issues in estimating the model to this data. The first is handling the early exercise provision of the American options. The second is choosing the metric for estimation.

4.2.1 Early exercise

Enron paid an annual dividend of $0.50 per share of common stock from[7] October 13, 1998 through the end of my sample period. The company only suspended dividend payments on December 11, 2001. Nonetheless, the dividend makes early exercise a possiblity on American calls as well as puts. Because the dividend was constant at $0.50 per share over the sample, I assumed a continuous dividend yield based on the current spot price.

[7] The dividend was increased from $0.45 to $0.50 a share on that date, adjusted for the split on yyyy, dd, mm.

Melick and Thomas (1997) use arbitrage bounds for determining the range of possible American options prices. They note that the bounds are remarkably close for reasonable discount factors. Bates (2000) notes that the proportional markup is between $[1, e^{r\tau}]$ which is very small for options of 6 months or less.

I chose to approximate the value of the early exercise feature using the Bjerksund and Streslund (1993) analytical approximation. Hoffman (2000) shows that the Bjerksund-Streslund algorithm is as accurate as the Barone Adesi and Whaley (1987) quadratic approximation and computationally much more efficient. I also found it more stable in my estimation as well. Very similar results were obtained using a mixture of binomial trees and simply ignoring the early exercise provision.

4.2.2 Metric

$f(S_T)$ is the object I am trying to estimate, and I have assumed that it is a mixture of log normals. It might be tempting to proceed by matching the moments of the density to time series data on the stock price. This approach is not suitable because $f(S_T)$ is the risk neutral density and is not directly observable.[8]

The only sample "moments" I observe are the option prices. Let $\{c(\tau_i, K_i), \ldots, c(\tau_{n_1}, K_{n_1}), p(\tau_{n_1+1}, K_{n_1+1}), \ldots, p(\tau_n, K_n)\} \equiv \{d_{i,t}\}_{i=1}^n$denote the n dimensional sample of data at time t on the American calls c and puts p struck at K_i and expiring in τ_i years. Denote the pricing estimates from the model as $\{d_{i,t}(\theta)\}_{i=1}^n$.

In matching model to data, Christofferson and Jacobs (2001) emphasize that the choice of loss function is important. Bakshi, Cao and Chen (1997), for example, match the model to data using the squared pricing errors. While using the percentage pricing errors minimizes the impact of deeply in the money options, this can contribute to estimation problems for low-priced options. I obtained the best fit on deeply in and out of the money options using the implied Bjerksund and Streslund volatility,

$$\sigma_{i,t} = BJST^{-1}(d_{i,t}, S_t, r). \tag{12}$$

Let the estimated volatility be denoted

$$\sigma_{i,t}(\theta) = BJST^{-1}(d_{i,t}(\theta), S_t, r). \tag{13}$$

I then minimize, in estimation,

$$\min_{\theta} \sum_{i=1}^n (\sigma_{i,t}(\theta) - \sigma_{i,t})^2 \tag{14}$$

[8] Grundy (1991) does note that the risk neutral distribution does imply bounds for the true one.

for each day in my sample. As Christoffersen and Jacobs note, this is just a weighted least squares problem that, with the monotonicity of the option price in θ satisfies the standard regularity conditions in White (1981).

5. Results

I estimate the six parameter model day-by-day for the 77 day sample. I report R^2 and other summary statistics in Table 4. The overall fit is quite good with an average goodness of fit of 38%. 30 of the 77 days are higher than 50%. I next want to test whether the data are strong enough to reject the Black-Scholes.

[INSERT Table 4 Here]

5.1 Tests of the adequacy of Black-Scholes

I would like a formal test of whether the model's mixture parameters are providing much additional explanatory power. In the standard case, I could construct a likelihood ratio test of the model, restricting all the parameters but the Black-Scholes volatility s_2 to zero. The problem with that approach in the mixture case is that under the Black-Scholes alternative, the parameters in the two tail lognormals are nuisance parameters, giving the likelihood ratio statistic, as Hansen (1997) notes, a non-standard distribution. Computing proper p-values requires numerical techniques.

I report $\sup LR$ tests and p-values from 1,000 bootstrap replications in the second and third columns of Table 4. On 39 days, I can reject the Black-Scholes at the 99% level and on 44 days at the 95% level. Most of the rejections occur after September 1. On 30 of the 45 days in September thru November, the model rejects the Black-Scholes at the 99% level.

5.2 Implied probabilities of an Enron collapse

The mixture model provides a complete characterization of the risk neutral probability density at any time horizon. I chose to examine the possibility of a 50% decrease in the stock price in a 3-month time frame. To provide a reference for comparison, I express this as a ratio of the implied probability based on a log normal distribution with a standard deviation equal to an at-the-money call. This helps to focus attention on the added forecasting contribution of the model.

[INSERT Figure 3 Here]

A 10-day moving average of the ratio remains above one until November 1, 2001. It is tempting to try to match up that date with events in the Enron chronology. On that day, Citibank and JP Morgan Chase provided a $1 billion cash infusion. The probability of this 50% collapse does peak on that day, but the risk falls quickly back to the levels of mid-September. The options market appears to have turned relatively more optimistic very late in the process, barely a month before the company declared bankruptcy.

5.3 Hypothesis tests on the implied densities.

To conduct a proper statistical comparison, I adapt the framework of Christoffersen (1998). I will examine three hypotheses: (1) are the model probabilities fat-tailed relative to the benchmark forecast; (2) are the model probabilities asymmetric; (3) does the model imply a higher risk of a crash overall. In each of the three cases, I construct the test of the null hypothesis from a seqeuence of Bernoulli trials,

$$I_t^{\alpha_L} = \left\{ \begin{array}{ll} 1, & \Pr\left[S_T < S_t/1.5\right] > \alpha_L \\ 0 & \Pr\left[S_T < S_t/1.5\right] < \alpha_L \end{array} \right\}, \tag{15}$$

where

$$\alpha_L = F[(S_t/1.5 - \mu)/\sigma],$$

with

$$\mu = \ln(S_t) + \tau \times (r - q) - \sigma^2/2,$$

where $F[.]$ is the cumulative normal distribution, and σ is the standard deviation of the at-the-money call. Define symmetrically

$$I_t^{\alpha_U} = \left\{ \begin{array}{ll} 1, & \Pr\left[S_T > 1.5 \times S_t\right] > \alpha_U \\ 0 & \Pr\left[S_T > 1.5 \times S_t\right] < \alpha_U \end{array} \right\}, \tag{16}$$

where

$$\alpha_U = 1 - F[(1.5 \times S_t - \mu)/\sigma].$$

Under what Christoffersen calls unconditional coverage, I test

$$H_1 : E[I_t^{\alpha_L} \times I_t^{\alpha_U}] = 0.25, \tag{17}$$

using the likelihood ratio

$$LR_1 = 0.75^{n_0} 0.25^{n_1} / [(1 - \widehat{\pi}_1)^{n_0} \widehat{\pi}_1^{n_1}. \tag{18}$$

Under H_1, the likelihood ratio is distributed $\chi^2(1)$.

The second hypothesis is whether the implied probability of a crash is greater than a similar size jump up. Define the Bernoulli random variable,

$$I_t^{\alpha_{LU}} = \left\{ \begin{array}{ll} 1, & \Pr\left[S_T < S_t/1.5\right] > \Pr[S_T > 1.5 \times S_t] \\ 0 & \Pr\left[S_T < S_t/1.5\right] < \Pr[S_T > 1.5 \times S_t] \end{array} \right\}. \tag{19}$$

I then test

$$H_2 : E[I_t^{\alpha_{LU}}] = 0.5. \tag{20}$$

The third hypothesis tests whether the lower tail probabilities are, on average, in the direction of greater crash risk,

$$H_3 : E[I_t^{\alpha_L}] = 0.5. \tag{21}$$

Both H_2 and H_3 can be tested using the likelihood ratio (18) with a probability of 0.5.

5.4 LR tests

The implied distributions are fat tailed compared to the lognormal benchmark. For H_1, $n_0 = 46, n_1 = 32$, and $\widehat{\pi}_1 = 0.41$. The likelihood ratio statistic is 9.59 which enables us to reject the null at the 99% level. The majority of this power comes in the earlier part of the sample. For the last month of the sample, only 2 of 23 observations are fat-tailed compared to the lognormal.

[INSERT Table 5 Here]

There are only 6 observations in which the probability of the 50% appreciation exceeds the probability of an equally sized move down. The likelihood ratio statistic for H_2 is 65.82, which overwhelmingly rejects the null.

The asymmetry of the large move risks should not imply that the options market fully understood the risks. For $H_3 : n_0 = 36, n_1 = 42$, and $\widehat{\pi}_1 = 0.54$. This is very close to the purely random sequence implied under the null. The likelihood ratio statistic is only 0.462 which cannot reject the null. The market information in the deeply out of the money (in the money) puts (calls) was apparently less reliable than the at the money volatility.

6. Conclusion

Options can provide a great deal of insight to economists because they incorporate information about the entire probability distribution of future events. This paper has utilized the mixture of lognormals to help draw inferences from the implied volatility surface.

The Enron case was certainly an epochal event on Wall Street. The options market as a whole priced in a great deal of risk. Nonetheless, once the stock had fallen substantially, the probability of a complete collapse into bankruptcy may not have been fully anticipated. Whether the excessive optimism of analysts or the overconfidence of dip buyers may have played a role is beyond the scope of this paper though.

Policy makers may find these tools and inference worthwhile in a variety of contexts. Their subjective weights between type I and type II errors should not only be tested ex-post but incorporated directly in the estimation. Both Skouras (2001) and Christoffersen and Jacobs (2001) have made progress along these lines. Loss aversion on the part of investors and traders may give them similar preferences.

In future work, I hope to examine whether the large bankruptcies that followed Enron like Worldcom and United Airlines did not result in a similar shrinkage in the crash risk.

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Table 1
Moments of Volatility Surfaces[9]

	Skewness	**Kurtosis**
Log Normal	0.54	3.46
Left Smirk	0.73	5.15
Smile	0.82	5.29
Right Smirk	0.62	3.91

[9] These are averages across 100 replications of each parameterization of the model.

Table 2
Reported Revenues and Profits at Enron (mn$): 1995-2000 [10]

Year	Revenue	Profit
1995	9,189.00	519.69
1996	13,289.00	584.00
1997	20,273.00	105.00
1998	31,260.00	703.00
1999	40,112.00	1,024.00
2000	100,789.00	979.00

[10] Source: Compustat.

Table 3
Characteristics of Enron Options Sample[11]

		Stock		Strikes		Maturity		# of:	
Start	**End**	**Low**	**High**	**Min**	**Max**	**Min**	**Max**	**Calls**	**Puts**
16-Jul-2001	27-Jul-2001	43.24	49.85	35	90	21	186	84	72
30-Jul-2001	10-Aug-2001	42.78	45.73	35	90	7	172	88	64
13-Aug-2001	24-Aug-2001	36.25	42.93	30	85	28	241	104	85
27-Aug-2001	07-Sep-2001	30.49	38.16	22.5	90	14	235	112	77
17-Sep-2001	21-Sep-2001	26.41	30.67	22.5	85	28	214	72	65
24-Sep-2001	05-Oct-2001	25.15	33.49	22.5	100	14	207	154	130
08-Oct-2001	19-Oct-2001	26.05	36.79	22.5	90	7	193	140	117
22-Oct-2001	02-Nov-2001	11.16	20.65	22.5	90	16	179	163	63
05-Nov-2001	16-Nov-2001	8.41	11.3	22.5	50	38	168	70	11

[11] Source: Daily closes from TBSP, Inc.

Table 4.1
Summary Statistics for Lognormal Options Mixture Model[12]

Date	R^2	$\sup LR$	p-value
16-Jul-2001	0.036	0.308	0.896
17-Jul-2001	0.040	0.053	0.984
18-Jul-2001	0.182	2.370	0.506
19-Jul-2001	0.008	0.017	0.996
20-Jul-2001	0.004	0.000	0.000
23-Jul-2001	0.009	0.023	0.988
25-Jul-2001	0.439	15.103	0.014
26-Jul-2001	0.195	4.346	0.202
27-Jul-2001	0.660	24.576	0.002
30-Jul-2001	0.015	0.194	0.930
31-Jul-2001	0.285	0.001	0.994
01-Aug-2001	0.354	7.648	0.144
02-Aug-2001	0.006	0.018	0.990
03-Aug-2001	0.019	0.186	0.938
06-Aug-2001	0.264	6.451	0.134
07-Aug-2001	0.891	130.487	0.000
09-Aug-2001	0.668	26.100	0.004
10-Aug-2001	0.555	16.397	0.008
13-Aug-2001	0.006	0.003	0.998
14-Aug-2001	0.005	0.000	0.990
15-Aug-2001	0.308	10.652	0.036
16-Aug-2001	0.149	3.308	0.294
17-Aug-2001	0.003	0.000	0.868
20-Aug-2001	0.073	1.550	0.530
21-Aug-2001	0.003	0.000	0.802
22-Aug-2001	0.001	0.003	0.998
23-Aug-2001	0.547	21.674	0.004
24-Aug-2001	0.002	0.000	0.722
27-Aug-2001	0.001	0.000	0.072
28-Aug-2001	0.032	0.900	0.696
29-Aug-2001	0.770	73.339	0.000
04-Sep-2001	0.938	0.062	0.878
05-Sep-2001	0.471	21.635	0.002
06-Sep-2001	0.550	41.535	0.000
07-Sep-2001	0.540	28.139	0.000
17-Sep-2001	0.478	22.007	0.000
18-Sep-2001	0.475	23.494	0.000

[12] The R^2 is the goodness of fit for a single day's estimation of the model. The $\sup LR$ stat is against the alternative that all the parameters but the Black-Scholes standard deviation s_2 are zero. The p-values are from 1,000 bootstrap replications of the test as described in Hansen (1997). The market was closed from September 11 to September 16, 2001 due to the terror attacks in New York and Washington, D.C.

Table 4.2
Summary Statistics for Lognormal Options Mixture Model[13]

Date	R^2	$\sup LR$	p-value
19-Sep-2001	0.532	25.560	0.000
20-Sep-2001	0.595	43.989	0.000
21-Sep-2001	0.559	30.362	0.000
24-Sep-2001	0.494	27.157	0.000
25-Sep-2001	0.441	17.322	0.010
26-Sep-2001	0.522	43.949	0.000
27-Sep-2001	0.477	18.654	0.002
28-Sep-2001	0.478	51.441	0.000
01-Oct-2001	0.505	30.215	0.000
02-Oct-2001	0.540	25.821	0.000
03-Oct-2001	0.675	49.619	0.000
04-Oct-2001	0.560	36.762	0.000
05-Oct-2001	0.089	2.154	0.422
08-Oct-2001	0.501	25.060	0.000
09-Oct-2001	0.601	34.638	0.000
10-Oct-2001	0.592	46.330	0.000
11-Oct-2001	0.621	56.699	0.000
12-Oct-2001	0.536	28.297	0.000
15-Oct-2001	0.420	16.644	0.000
16-Oct-2001	0.704	42.793	0.002
17-Oct-2001	0.325	7.217	0.122
18-Oct-2001	0.235	7.059	0.084
19-Oct-2001	0.319	10.718	0.028
22-Oct-2001	0.289	8.940	0.046
23-Oct-2001	0.513	40.396	0.000
24-Oct-2001	0.422	29.236	0.000
25-Oct-2001	0.576	44.895	0.000
26-Oct-2001	0.518	29.434	0.000
29-Oct-2001	0.498	16.880	0.010
30-Oct-2001	0.172	4.784	0.188
31-Oct-2001	0.185	4.321	0.176
01-Nov-2001	0.295	3.750	0.426
02-Nov-2001	0.676	27.044	0.000
05-Nov-2001	0.346	5.816	0.048
06-Nov-2001	0.521	16.297	0.010
07-Nov-2001	0.229	2.972	0.196
09-Nov-2001	0.459	4.588	0.216
13-Nov-2001	0.479	25.995	0.420
14-Nov-2001	0.839	3.044	0.000
15-Nov-2001	0.604	2.846	0.238

[13] The R^2 is the goodness of fit for a single day's estimation of the model. The $\sup LR$ stat is against the alternative that all the parameters but the Black-Scholes standard deviation s_2 are zero. The p-values are from 1,000 bootstrap replications of the test as described in Hansen (1997).

Table 5
Likelihood Ratio Tests of the Implied Probability Intervals[14]

	H_1	H_2	H_3
$\widehat{\pi}_1$	0.41	0.92	0.54
n1	32	72	42
n0	46	6	36
LR Stat	9.59	65.83	0.46
p-value	0.002	0.000	0.497

[14] The null hypotheses are given in (17), (20) and (21).

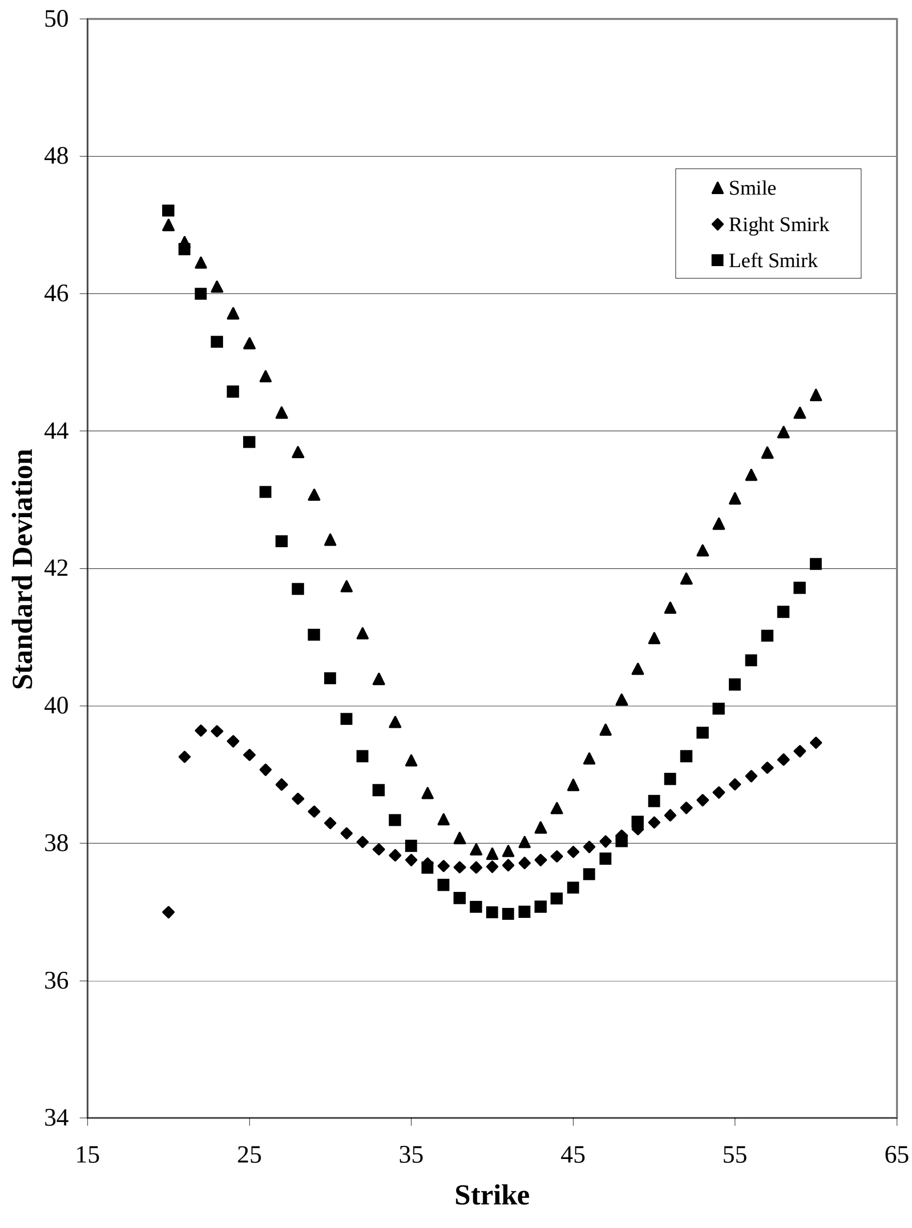

Figure 1: Implied Volatility Surfaces for Alternative Parameterizations

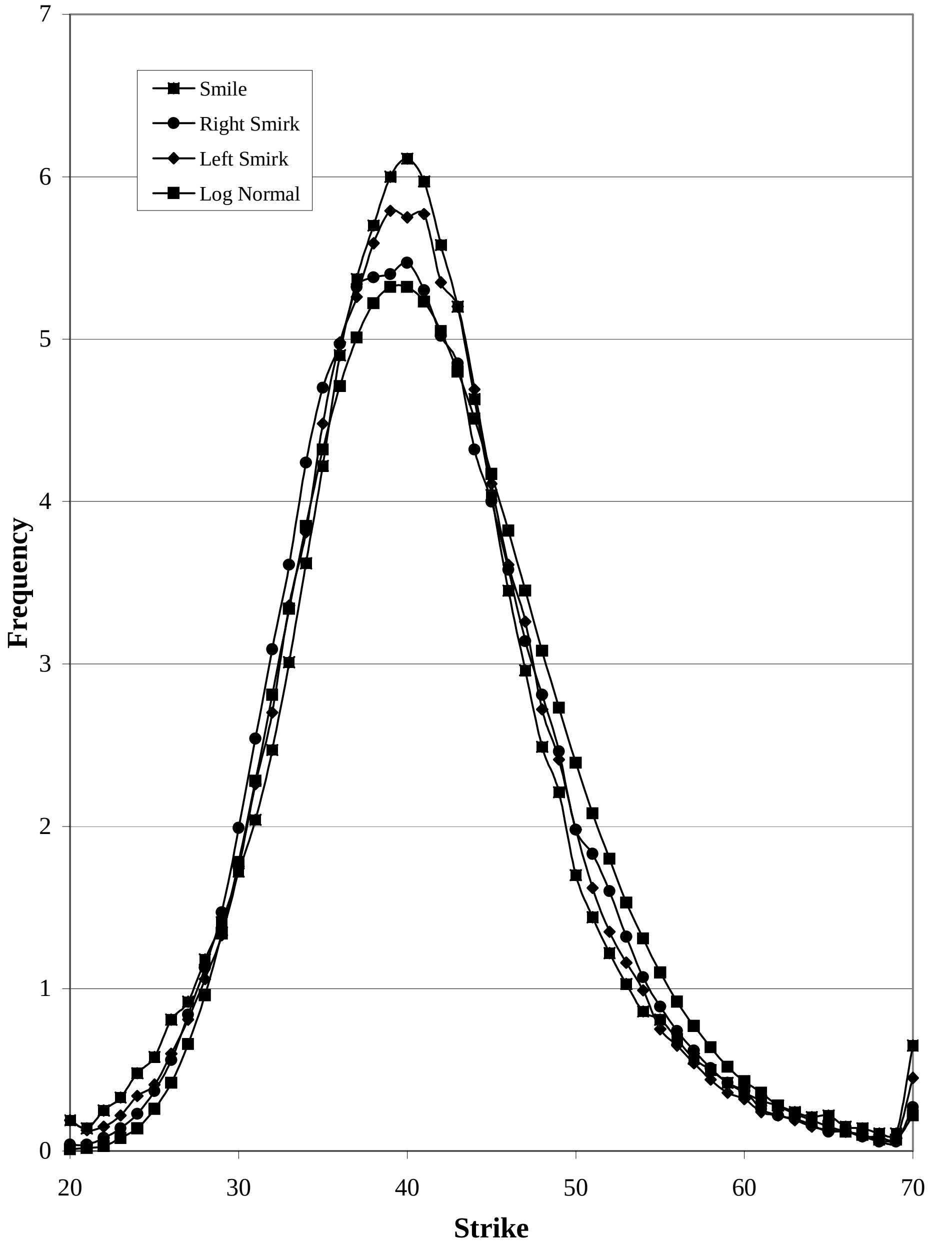

Figure 2: Implied Probability Densities for Alternative Models

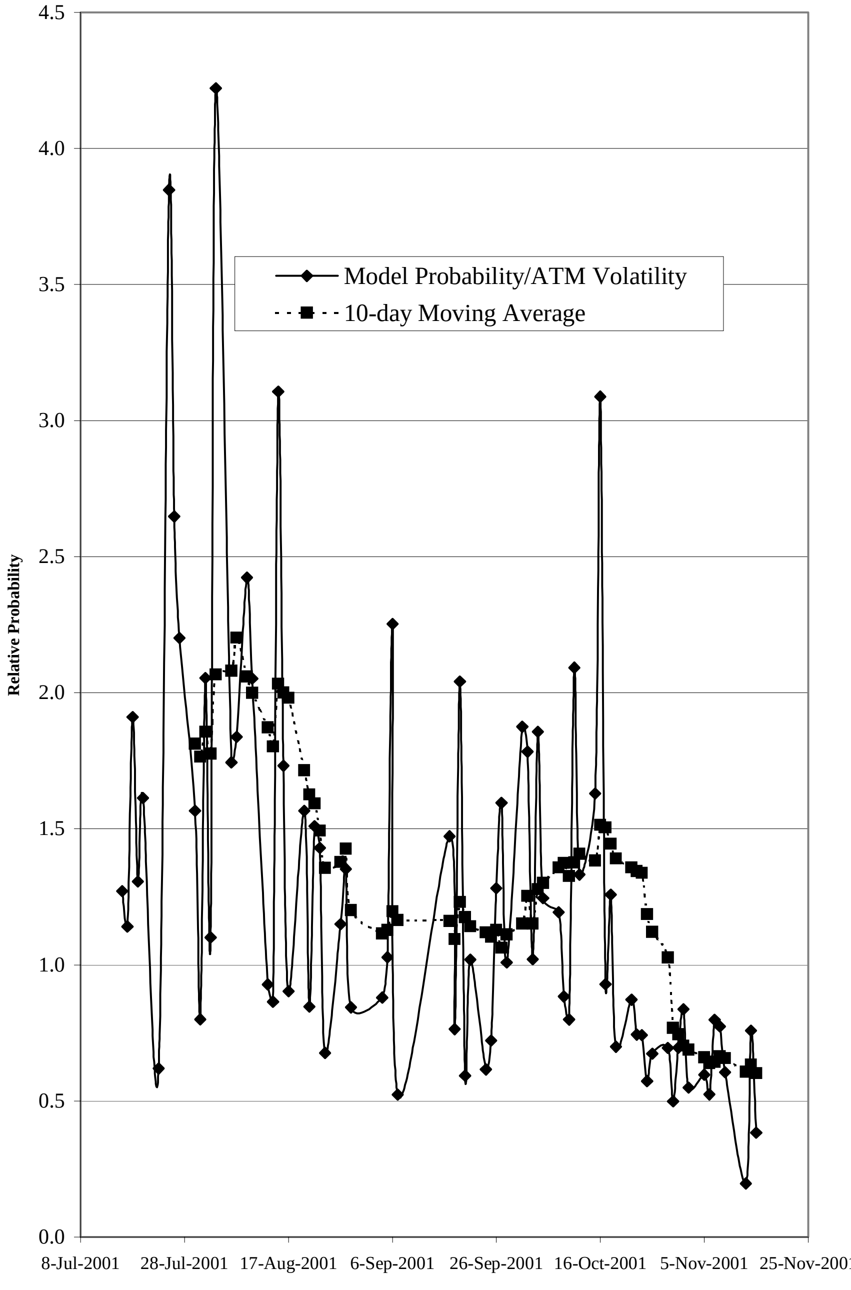

Figure 3: Relative Probability of a 50% Decline from the Model and an ATM Call