The Term Structure of Credit Spreads: Theory and Evidence on Credit Default Swaps

Ren-raw Chen

Rutgers Business School , Rutgers University

Xiaolin Cheng

Rutgers Business School , Rutgers University

Liuren Wu

Zicklin School of Business, Baruch College

Ren-raw Chen and Xiaolin Cheng are at Rutgers Business School, New Brunswick, NJ 08903; tel: (732) 445-4236; fax: (732) 445-2333; rchen@rci.rutgers.edu (Chen) and linlycheng@yahoo.com (Cheng). Liuren Wu is at Zicklin School of Business, Baruch College, One Bernard Baruch Way, Box B10-225, New York, NY 10010; tel: (646) 312-3509; fax: (646) 312-3451; Liuren_Wu@baruch.cuny.edu; http://faculty.baruch.cuny.edu/lwu.

The Term Structure of Credit Spreads: Theory and Evidence on Credit Default Swaps

ABSTRACT

Using a large data set on credit default swaps, we perform a joint analysis of the term structure of interest rates, credit spreads, and liquidity premia. We select reference companies that fall into two broad industry sectors and two broad credit rating classes. Within each sector and credit rating class, we divide the companies into two liquidity groups based on the quote updating frequency. We then study how the term structures of credit default risk premia differ across industry sectors, credit rating classes, and liquidity groups.

We develop a class of dynamic term structure models that include two benchmark interest-rate factors, two credit risk factors for the high-liquidity groups, and an additional default risk factor and a liquidity risk factor that capture the difference between the two liquidity groups. We link these factors to the instantaneous benchmark interest rate and credit spread via both an affine function and a quadratic function, and compare their relative performance.

We estimate the models using a three-step procedure. First, we estimate the interest-rate factor dynamics and the instantaneous interest rate function using the libor and swap rates. Second, we take the interest-rate factors and estimate the default-risk dynamics and the instantaneous credit spread function using the average credit default spreads of the high-liquidity group for each industry sector and credit rating class. Third, we identify an additional credit risk factor and a liquidity risk factor using the credit default swap spreads in the low-liquidity group. At each step, we cast the models into a state-space form and estimate the model parameters using quasi-maximum likelihood method.

Estimation shows that the quadratic specifications generate better and more uniform performance across the term structure of interest rates and credit spreads. Furthermore, firms in different industry and credit rating classes have different default risk dynamics. Nevertheless, in all cases, default risks exhibit intricate dynamic interactions with the interest-rate factors. Interest-rate factors both predict the default risk and have a contemporaneous impact on it.

Within each industry and credit rating class, the average credit default swap spreads for the high-liquidity group are significantly higher than for the low-liquidity group. Estimation shows that the difference is driven by both default risk and liquidity difference. The low-liquidity group has a lower default arrival rate, and also a much heavier discounting due to low liquidity.

JEL Classification Codes: E43, G12, G13, C51.

Key Words: credit default swap; credit risk; credit premium; term structure; liquidity risk; liquidity premium; quasi-maximum likelihood estimation.

The Term Structure of Credit Spreads: Theory and Evidence on Credit Default Swaps

How to quantify, forecast, and price default risk, and how the default risk interacts with the interest-rate risk in determining the term structure of credit spreads are vitally important for decisions in corporate finance, asset pricing, investment, and risk management. However, data availability has severely hindered the study of default risk. Since defaults are rare events that often lead to termination of the underlying reference entity such as a corporation, researchers need to rely heavily on cross-sectional averages of different entities over a long history to obtain any reasonable estimates of statistical default probabilities. Although corporate bond prices contain useful information on the default probability and the price of default risk, the information is often mingled with the pricing of the underlying interest rate risk and other factors such as liquidity and tax.[1]

The recent development in credit derivative securities provides us with an excellent opportunity to better understand the pricing of default risk and the term structure of credit spreads. The most common credit derivative is in the form of a credit default swap (CDS) contract, written on a reference entity such as a sovereign country or a corporate company. According to surveys by the International Swaps and Derivatives Association, Inc., the outstanding notional amount of CDS contracts has reached $3.58 trillion by the end of 2003, surpassing the size of the total equity derivatives market, which stands at $3.44 trillion for the same time period.

A credit default swap is an over-the-counter contract that provides insurance against credit risk. The protection buyer pays a fixed fee or premium, often termed as the "spread," to the seller for a period of time. If a certain pre-specified credit event occurs, the protection seller pays compensation to the protection buyer. A credit event can be a bankruptcy of the reference entity, or a default of a bond or other debt issued by the reference entity. If no credit event occurs during the term of the swap, the protection buyer continues to pay

[1]Many researchers strive to identify and distinguish the different components of corporate bond yields. Prominent examples include Jones, Mason, and Rosenfeld (1984), Longstaff and Schwartz (1995), Duffie and Singleton (1997), Duffee (1999), Elton, Gruber, Agrawal, and Mann (2001), Collin-Dufresne, Goldstein, and Martin (2001), Delianedis and Geske (2001), Liu, Longstaff, and Mandell (2000), Eom, Helwege, and Huang (2003), Huang and Huang (2003), Collin-Dufresne, Goldstein, and Helwege (2003), and Longstaff, Mithal, and Neis (2004).

the premium until maturity. Should a credit event occur at some point before the contract's maturity, the protection seller owes a payment to the buyer of protection, thus insulating the buyer from a financial loss.

CDS contracts can be used as a way to gain exposure to credit risk. Although the risk profile of a CDS is similar to that of a corporate bond of the reference entity, there are several important differences. A CDS does not require an initial funding, which allows leveraged positions. A CDS transaction can be entered where a cash bond of the reference entity at a particular maturity is not available. Furthermore, by entering a CDS contract as a protection seller, an investor can easily create a short position in the reference credit. With all these attributes, CDS contracts can be a great tool for diversifying or hedging an investor's portfolio.

We obtain a large data set on credit default swap spreads. The data set includes daily CDS spread quotes from May 21, 2003 to May 12, 2004 on hundreds of corporate companies and across seven fixed maturities for each company from one to ten years. We select reference companies that fall into two broad industry sectors (financial and non-financial) and two broad credit rating classes (A and BBB). Within each sector and credit rating class, we divide the companies into two liquidity groups based on the quote updating frequency on credit default swap spreads.[2] We also download from Bloomberg the eurodollar libor and swap rates of matching maturities and sample periods.

Equipped with this large data set, we study the dynamics and pricing of default risk and their impacts of the term structure of credit default swap spreads across different industry sectors and credit rating classes. Specifically, through model development and estimation, we address the following fundamental questions regarding default risk and default risk premia:

- It is well known that the term structure of interest rates is governed by multiple factors. How many factors govern the term structure of credit spreads? How these factors interact with the interest-rate factors?
- Unlike the term structure of interest rates that is universal across the whole market, credit risk is individually unique and firm-specific. Credit default swaps have been written on thousands of reference

[2] There are a few AA companies and a very limited number of AAA companies but none of those companies satisfies our liquidity classification criterion.

entities. How the default risk dynamics and pricing differ across different industry sectors and credit rating classes?

- Even within the same industry sector and credit rating class, the activities and liquidities of the credit default swap contracts vary dramatically across different reference entities. What causes the liquidity difference and how does the different liquidity affects the pricing of credit default swaps?

To address these questions, we develop a class of dynamic term structure models of interest rate risk, credit risk, and liquidity risk. First, we model the term structure of the benchmark libor and swap rates using two interest-rate factors. Second, we assume that the default arrival intensities at each industry sector and credit rating class are governed by either one or two additional dynamic factors. Furthermore, we allow the interest-rate factors to impact the default risk both dynamically and contemporaneously. We link these factors to the instantaneous benchmark interest rate and credit spread via both an affine and a quadratic specification, and compare their relative performances via estimation. Finally, we use an additional default risk factor and a liquidity risk factor to capture the difference between the credit spreads of the two liquidity groups with each industry sector and credit rating class.

We estimate the models using a three-step procedure. In the first step, we estimate the interest-rate factor dynamics and the instantaneous interest rate function using the benchmark libor and swap rates. In the second step, we take the interest-rate factors extracted from the first step as given, and estimate the default-risk dynamics and the instantaneous credit spread function for each industry sector and credit rating class using the average credit default spreads of the high-liquidity group for that sector and rating class. In the third step, we identify the additional credit risk factor and the liquidity risk factor using the average credit default swap spreads in the low-liquidity group. At each step, we cast the models into a state-space form, obtain efficient forecasts on the conditional mean and variance of observed interest rates and credit default swap spreads using an efficient filtering technique, and build the likelihood function on the forecasting errors of the observed series, assuming that the forecasting errors are normally distributed. We estimate the model parameters by maximizing the likelihood functions.

Comparing the affine and quadratic specifications, we find that the quadratic specifications generate better and more uniform performance across the term structure of interest rates and credit spreads. The interest-rate and credit-risk dynamics are also estimated with more precision under the quadratic specification, an indication of less model mis-specification.

Our estimation shows that one affine default-risk factor can price the moderate-maturity credit default swap spread well, but the performance deteriorates on both ends of the credit spread curve. Two affine default-risk factors can price the whole term structure of credit spreads well. In contrast, under the quadratic specification, one default-risk factor is sufficient to explain over 90 percent of the variation on each of the seven credit spread series for each industry sector and credit rating class. Adding an additional quadratic credit risk factor does not dramatically improve the performance. Hence, with a nonlinear, richer dynamic specification, one default-risk factor can explain the majority of the credit spread variation in the high liquidity group.

Our estimation also shows that firms in different industry sectors and credit rating classes have different default risk dynamics. Nevertheless, in all cases the default risks show intricate dynamic interactions with the interest-rate factors. Interest-rate factors both predict the default risk and have a contemporaneous impact on it.

Within each industry sector and credit rating class, we find that the average credit default swap spreads for the high-liquidity group are significantly higher than for the low-liquidity group. The mean term structure of credit spreads is also more upward sloping for the high-liquidity group. Estimation shows that the different spreads between the two groups are driven by both credit risk differences and liquidity differences. On average, the low-liquidity group has lower default arrival rates, and hence a lower instantaneous credit spread. For each industry sector and credit rating class, we are able to identify a significant default-risk factor for the low-liquidity group. This risk factor shows strong risk-neutral persistence, indicating that it affects the term structure of credit spreads across both short and long maturities. We also identify a highly volatile but less persistent liquidity risk factor for the credit spreads on the low-liquidity group. This liquidity factor induces a strongly positive instantaneous spread on the discount factor, showing that lower liquidity induces a heavier discounting on the pricing as a compensation for liquidity premium. The lower credit risk and heavier liquidity discounting jointly determine the lower spread on the credit default swap contracts for the low-liquidity group in each industry sector and credit rating class.

The remainder of this paper is organized as follows. The next section provides some background information on the credit default swap contract and the related literature. Section 2 develops the dynamic term structure models of interest rates, credit risks, and liquidity premia. Section 3 describes the data set and our processing of the data. Section 4 elaborates on the estimation strategy. Section 5 discusses the estimation results. Section 6 concludes.

1. Background Information on Credit Default Swap Spread

The International Swaps and Derivatives Association, Inc. estimates the outstanding notional CDS amount at $0.92 trillion dollars by the end of 2001. Since then, the notional amount has more tnan tripled to $3.58 trillion by the end of 2003. This explosive development can be attributed to fours sets of players. The largest players in the CDS market are commercial banks. Traditionally, a bank's business involves credit risk since the bank originates loans to corporations. The CDS market offers a bank an attractive way to transfer the credit risk without removing assets from its balance sheet and without involving borrowers. Furthermore, a bank may use CDS contracts to diversify its portfolio, which often is concentrated in certain industries or geographic areas. Banks are the net buyers of credit derivatives. According to Fitch's 2003 survey, global banks held net bought positions of $229 billion in credit derivatives, with gross sold positions of $1,324 billion.

On the other hand, insurance companies are increasingly becoming dominant participants in the CDS market, primarily as sellers of protection, to enhance investment yields. Globally, insurance companies had net sold positions of $137 billion in 2003. Other players include financial guarantors, who are also big sellers of protection, with net sold positions of $166 billion. Global hedge funds are also rumored to be active players in the CDS market, but their activities are notoriously opaque and are not detected on any survey's radar screen.

Sovereign names were prevalent as reference entities in the early days of the CDS market, but the share of sovereigns as reference entities had declined from over 50 percent in 1997 to less than 10 percent in 2003. In contrast, corporate reference entities have become more common, accounting for over 70 percent of all reference entities in 2003. This shift in reference entities reflects the rapid growth of the corporate bond market after the mid-1990s.

The premium paid by the protection buyer to the seller, often called the“spread,” is quoted in basis points per annum of the contract’s notional value and is usually paid quarterly. These spreads are not the same type of concept as “yield spread” of a corporate bond to a government bond. Rather, CDS spreads are the annual price of protection quoted in basis points of the notional value, and not based on any risk-free bond or any benchmark interest rates. Periodic premium payments allow the protection buyer to deliver the defaulted bond at par or to receive the difference of par and the bond’s recovery value. For example, the five-year credit default swap for Ford was quoted around 160 basis points on April 27, 2004. This quote means that if someone wants to buy the five-year protection for a $10 million exposure to Ford credit, the buyer will pay 40 basis points of the notational exposure, or $40,000, every quarter as an insurance premium for the protection that the buyer receives. There are no limits on the size or maturity of CDS contracts. However, most contracts fall between $10 million to $20 million in notional amount. Maturity usually ranges from one to ten years, with the five-year maturity being the most common maturity.

Given the nascent nature of the CDS contracts, researches using CDS data are relatively few. Skinner and Diaz (2003) look at early CDS prices from September 1997 to February 1999 for 31 CDS contracts. They compare the pricing results of the Duffie and Singleton (1999) and Jarrow and Turnbull (1995) models. Blanco, Brennan, and Marsh (2004) compare the CDS spreads with credit spread derived from corporate bond yields and find that they match each other well. When the two sources of spreads deviation from each other, they find that CDS spreads have a clear lead in price discovery. In another related study, Longstaff, Mithal, and Neis (2004) regard the spread from the CDS prices as purely due to credit risk and use it as a benchmark to identify the liquidity component of the corporate yield spread. Similar to Blanco, Brennan, and Marsh, they find that the majority of the corporate spread is due to default spread. In addition to comparing bond spreads and CDS spreads, Hull, Predescu, and White (2004) examine the relation between the CDS spreads and announcements by rating agencies. Similar to these studies, we choose to use credit default swap data to estimate the default risk dynamics and default risk premium. Different from them, our work constitutes the first comprehensive study on the joint term structure of interest rates, credit spreads, and liquidity premia using the CDS data.

2. A Dynamic Term Structure Model of Interest, Default, and Liquidity

We value the credit default swap spread using the reduced-form framework of Duffie (1998), Lando (1998), Duffie and Singleton (1997), Duffie and Singleton (1999), and Duffie, Pedersen, and Singleton (2003). First, we use r_t to denote the instantaneous benchmark interest rate. Historically, researchers often use the instantaneous riskfree rate as the benchmark. Houweling and Vorst (2003) perform daily calibration of reduced-form models using credit default swap spreads and find that swap rates are better suited than the Treasury yields in defining the benchmark yield curve. Here, we define the benchmark instantaneous interest rate based on the eurodollar libor and swap rates. Libor and swap rates contain a default risk component. Using them as benchmarks, the estimated default risk can be regarded as relative default risk.

Second, we use $\{\lambda_t^i\}_{i=1}^n$ to denote the intensity of the Poisson process governing the default of each industry sector and credit rating class i. We divide the underlying firms for which we have credit default swaps data into two industry sectors: financial and non-financial (corporate). Within each industry sector, we consider two credit rating classes: A and BBB, where the most actively quoted credit default swaps reside. Hence, we have $n = 4$ industry sector and credit rating classes. By modeling the dynamics of the benchmark interest rate r, we determine the term structure of the benchmark libor and swap rate curve. By modeling the dynamics of the Poisson intensities λ_i and their interaction with the benchmark interest rates, we determine the term structure of credit default swap spreads for the ith industry and credit rating class.

Third, within each credit rating and industry class, the credit default swap spread quotes have different levels of liquidity in terms of quote updating frequency across different reference firms. We divide them into two groups as high- and low-liquidity groups. To capture the liquidity difference between these two groups within each industry sector and credit rating class, we use $\{q_t^i\}_{i=1}^n$ to denote an instantaneous liquidity spread. By modeling the dynamics of q_t^i, we analyze the term structure of liquidity premium within each credit rating and industry class. To study whether the two groups also differ in credit risk, we also specify an additional credit risk component m_t^i for the low-liquidity group.

Formally, let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{Q})$ be a complete stochastic basis and $\mathbb{Q}$ be a risk-neutral probability measure. Under this measure $\mathbb{Q}$, the fair value of a benchmark zero-coupon bond with maturity τ relates to the instantaneous benchmark interest rate dynamics by,

$$P(\tau) = \mathbb{E}\left[\exp\left(-\int_0^\tau r_u du\right)\right], \tag{1}$$

where $\mathbb{E}[\cdot]$ denotes the expectation operator under the risk-neutral measure $\mathbb{Q}$. Our notation implicitly states our focus on time-homogeneous specifications.

Following Duffie (1998), Lando (1998), and Duffie and Singleton (1999), we can represent the value of a defaultable coupon-bond in terms of the benchmark instantaneous interest rate r and the Poisson intensity λ of the default arrival:

$$\begin{aligned} CB(c,w,\tau) &= \mathbb{E}\left[c\int_0^\tau \exp\left(-\int_0^t (r_u+\lambda_u)du\right)dt\right] \\ &\quad +\mathbb{E}\left[\exp\left(-\int_0^\tau (r_u+\lambda_u)du\right)\right] \\ &\quad +\mathbb{E}\left[(1-w)\int_0^\tau \lambda_t \exp\left(-\int_0^t (r_u+\lambda_u)du\right)dt\right], \end{aligned} \tag{2}$$

where c denotes the coupon rate and $(1-w)$ denotes the recovery rate. For expositional clarity, we assume continuous coupon payments.

For a credit default swap contract, we use S to denote the premium paid by the buyer of default protection. Assuming continuous payment, we can write the present value of the premium leg of the contract,

$$\text{Premium}(\tau) = \mathbb{E}\left[S\int_0^\tau \exp\left(-\int_0^t (r_u+\lambda_u)du\right)dt\right]. \tag{3}$$

Similarly, the present value of the protection leg of the contract is

$$\text{Protection}(\tau) = \mathbb{E}\left[w\int_0^\tau \lambda_t \exp\left(-\int_0^t (r_u+\lambda_u)du\right)dt\right]. \tag{4}$$

Hence, by setting the present values of the two legs equal, we can solve for the credit default swap spread as

$$S = \frac{\mathbb{E}\left[w \int_0^\tau \lambda_t \exp\left(-\int_0^t (r_u + \lambda_u) du\right) dt\right]}{\mathbb{E}\left[\int_0^\tau \exp\left(-\int_0^t (r_u + \lambda_u) du\right) dt\right]}, \tag{5}$$

which can be thought of as the weighted average of the expected default loss. In model estimation, we discretize the above equation according to quarterly premium payment intervals. For the credit default swap contracts that we have, w is fixed at 40 percent, which is the average recovery rate for senior unsecured debts.

For an inactively traded credit default swap contract, the premium could potentially include a liquidity component. Duffie, Pedersen, and Singleton (2003) show that this liquidity component can also be modeled via an instantaneous liquidity premium spread, q.

$$S = \frac{\mathbb{E}\left[w \int_0^\tau \lambda_t \exp\left(-\int_0^t (r_u + \lambda_u + q_u) du\right) dt\right]}{\mathbb{E}\left[\int_0^\tau \exp\left(-\int_0^t (r_u + \lambda_u + q_u) du\right) dt\right]}. \tag{6}$$

Under this framework, the benchmark libor and swap rate curve is determined by the instantaneous benchmark interest rate (r) dynamics. The credit default swap spreads are determined by the joint dynamics of instantaneous benchmark interest rate r and the Poisson arrival rate λ. Furthermore, when the credit default swap contract is illiquid, the spreads may also include a liquidity premium that is controlled by the dynamics of the instantaneous liquidity premium spread q. We specify the three sets of dynamics in the following subsections.

2.1. Benchmark interest rate dynamics and the term structure

We use $X \in \mathbb{R}^2$ to denote a two-dimensional vector Markov process that represents the systematic state of the benchmark yield curve. We assume that under the risk-neutral measure $\mathbb{Q}$, the state vector is governed by a pure-diffusion Ornstein-Uhlenbeck (OU) process,

$$dX_t = (\theta_x - \kappa_x X_t)\, dt + dW_{xt}, \tag{7}$$

where $\kappa \in \mathbb{R}^{2\times 2}$ controls the mean reversion of the vector process and $\kappa_x^{-1}\theta_x \in \mathbb{R}^2$ controls the long-run mean. For the OU process to be stationary, the real part of the eigenvalues of κ must be positive. For identification reasons, we normalize the state vector to have identity diffusion matrix. We also constrain κ to be a lower triangular matrix. Then, the diagonal values of the κ matrix correspond to its eigenvalues. To maintain stationarity, we constrain the diagonal values of κ_x to be positive in our estimation.

We further assume that the instantaneous benchmark interest rate r is affine in the state vector X,

$$r_t = a_r + b_r^\top X_t, \tag{8}$$

where the parameter $a_r \in \mathbb{R}$ is a scalar and $b_r \in \mathbb{R}^{2+}$ is a vector.

Our specifications in (7) and (8) belong to the affine class of term structure models of Duffie and Kan (1996) and Duffie, Pan, and Singleton (2000). The model-implied fair value of the zero-coupon bond with maturity τ is exponential affine in the current level of the state vector, X_0,

$$P(X_0,\tau) = \exp\left(-a(\tau) - b(\tau)^\top X_0\right), \tag{9}$$

where the coefficients $a(\tau)$ and $b(\tau)$ are determined by the following ordinary differential equations:

$$\begin{aligned} a'(\tau) &= a_r + b(\tau)^\top \theta_x - b(\tau)^\top b(\tau)/2, \\ b'(\tau) &= b_r - \kappa_x^\top b(\tau), \end{aligned} \tag{10}$$

subject to the boundary conditions $b(0) = 0$ and $c(0) = 0$.

2.2. Default risk dynamics and the term structure of CDS spreads

We assume that the Poisson arrival rate of default λ^i underlying each industry sector and credit rating class i is governed by a vector of interest-rate factors X and default-risk factors $Y \in \mathbb{R}^k$:

$$\lambda_t^i = a_i + b_i^\top X_t + c_i^\top Y_t, \tag{11}$$

where $b_i \in \mathbb{R}^2$ denotes the instantaneous response of the arrival rate λ to the two benchmark interest-rate factors X, and $c_i \in \mathbb{R}^{k+}$ denotes the instantaneous response to the default-risk factors Y. By allowing the default arrival intensity to be an explicit function of the benchmark interest rate factors, our model specification captures the empirical evidence that credit spreads are related to interest rate levels (e.g., Collin-Dufresne, Goldstein, and Martin (2001)). For model estimation, we consider both a one-factor and a two factor structure of the default risk factors $k = 1, 2$ and compare their relative performances.

The dynamics of the default risk factors under the risk-neutral measure $\mathbb{Q}$ follow,

$$dY_t = (\theta_y - \kappa_{xy} X_t - \kappa_y Y_t)\, dt + dW_{yt}, \tag{12}$$

where the benchmark interest-rate factors X_t are also allowed to have a direct feedback effect on the default risk factor through $\kappa_{xy} \in \mathbb{R}^{2\times k}$. Thus, interest rate factors both have an contemporaneous effect on default risk and predict default risk. For identification, we again normalize the instantaneous covariance to an identity matrix. In the two-factor specification, we further constrain κ_y to be a lower-triangular matrix with positive diagonal values.

The joint $\mathbb{Q}$-dynamics of $Z = [X^\top, Y^\top] \in \mathbb{R}^{2+k}$ is, in matrix form,

$$dZ_t = (\theta - \kappa Z_t)\, dt + dW_t, \quad \texttt{with} \quad \theta = \begin{bmatrix} \theta_x \\ \theta_y \end{bmatrix}, \quad \kappa = \begin{bmatrix} \kappa_x & 0 \\ \kappa_{xy} & \kappa_y \end{bmatrix}. \tag{13}$$

Given this compact specification, the present value of the premium leg of the credit default swap contract becomes,

$$\mathsf{Premium}(Z_0, \tau) = \mathbb{E}\left[S \int_0^\tau \exp\left(-\int_0^t (r_u + \lambda_u) du\right)\right] = \mathbb{E}\left[S \int_0^\tau \exp\left(-\int_0^t (a_Z + b_Z^\top Z_u) du\right)\right] \tag{14}$$

with $a_Z = a_r + a_i$ and $b_Z = [(b_r + b_i)^\top, c_i^\top]^\top$. The solution is exponential affine in the state vector Z_0,

$$\mathsf{Premium}(Z_0, \tau) = S \int_0^\tau \exp\left(-a(t) - b(t)^\top Z_0\right), \tag{15}$$

where the coefficients $a(t)$ and $b(t)$ are determined by the following ordinary differential equations:

$$\begin{aligned} a'(t) &= a_Z + b(t)^\top \theta - b(t)^\top b(t)/2, \\ b'(t) &= b_Z - \kappa^\top b(t), \end{aligned} \tag{16}$$

subject to the boundary conditions $b(0) = 0$ and $c(0) = 0$.

The value of the protection leg becomes,

$$\begin{aligned} \text{Protection}(Z_0, \tau) &= \mathbb{E}\left[w \int_0^\tau \lambda_t \exp\left(- \int_0^t (r_u + \lambda_u) du \right) \right] \\ &= \mathbb{E}\left[w \int_0^\tau \left(c_Z + d_Z^\top Z_t \right) \exp\left(- \int_0^t (a_Z + b_Z^\top Z_u) du \right) \right], \end{aligned} \tag{17}$$

with $c_Z = a_i$ and $d_Z = [b_i^\top, c_i^\top]^\top$. The solution is

$$\text{Protection}(Z_0, \tau) = w \int_0^\tau \left(c(t) + d(t)^\top Z_0 \right) \exp\left(-a(t) - b(t)^\top Z_0 \right), \tag{18}$$

where the coefficients $[a(t), b(t)]$ are determined by the ordinary differential equations in (16) and the coefficients $[c(t), d(t)]$ are determined by the following ordinary differential equations:

$$c'(t) = d(t)^\top \theta - b(t)^\top d(t), \quad d'(t) = -\kappa^\top d(t), \tag{19}$$

with $c(0) = c_Z$ and $d(0) = d_Z$. The credit default swap spread can then be solved as,

$$S(Z_0, \tau) = \frac{w \int_0^\tau \left(c(t) + d(t)^\top Z_0 \right) \exp\left(-a(t) - b(t)^\top Z_0 \right)}{\int_0^\tau \exp\left(-a(t) - b(t)^\top Z_0 \right)}. \tag{20}$$

2.3. Liquidity risk and the term structure of liquidity risk premium

For each credit rating and industry group i, we divide the firms into two subgroups based on the frequency of quote updates. We first estimate the above default risk factors to the credit spreads of the high-liquidity group, and then ask whether the difference in credit spreads for the low-liquidity group is due to different credit risk, liquidity risk, or both.

To answer this question, we introduce an idiosyncratic credit risk spread (m_t^i) and an idiosyncratic liquidity risk premium (q_t^i), with the following risk-neutral dynamics,

$$m_t^i = a_m + c_m \xi_t^i, \quad d\xi_t^i = \left(\theta_m - \kappa_m \xi_t^i\right) dt + dW_{mt}, \tag{21}$$

$$q_t^i = a_q + b_q \zeta_t^i, \quad d\zeta_t^i = \left(\theta_q - \kappa_q \zeta_t^i\right) dt + dW_{qt}. \tag{22}$$

Then, we can expand the definition of the state vector $Z = [X^\top, Y^\top, \xi, \zeta] \in \mathbb{R}^{4+k}$, with

$$\theta = \begin{bmatrix} \theta_x \\ \theta_y \\ \theta_m \\ \theta_q \end{bmatrix}, \quad \kappa = \begin{bmatrix} \kappa_x & 0 & 0 & 0 \\ \kappa_{xy} & \kappa_y & 0 & 0 \\ 0 & 0 & \kappa_m & 0 \\ 0 & 0 & 0 & \kappa_q \end{bmatrix}.$$

The solution to the premium on this credit default swap spread has the same form as in (20), with the following redefinitions: $a_Z = a_r + a_i + a_m + a_q$, $b_Z = [(b_r + b_i)^\top, c_i^\top, c_m, b_q]^\top$, $c_Z = a_i + a_m$, and $d_Z = [b_i^\top, c_i^\top, c_m, 0]^\top$.

2.4. Market prices of risks

Our estimation identifies both the risk-neutral and the statistical dynamics of the interest-rate, credit-risk, and liquidity risk factors. To derive the statistical dynamics, we assume an affine market price of risk on all the risk factors,

$$\gamma(Z_t) = \gamma_0 + \langle \gamma_1 \rangle Z_t \tag{23}$$

with γ_0 and γ_1 are both vectors of the relevant dimension and $\langle \cdot \rangle$ denotes a diagonal matrix, with the diagonal elements given by the vector inside. The affine market price of risk specification dictates that the state vector Z_t remains Ornstein-Uhlenbeck under the statistical measure $\mathbb{P}$, but with an adjustment to the drift term,

$$dZ_t = \left(\theta + \gamma_0 - \kappa^{\mathbb{P}} Z_t\right) dt + dW_t, \quad \kappa^{\mathbb{P}} = \kappa - \gamma_1. \tag{24}$$

For stationarity, we also constrain the diagonal elements of $\kappa^{\mathbb{P}}$ to be positive. For identification, we normalize the long-run mean of the state vector Z to zero under the statistical measure $\mathbb{P}$ so that $\theta = -\gamma_0$.

2.5. Positivity of interest rates and credit spreads: An alternative quadratic specification

The above specification assumes a Gaussian-affine structure for both the benchmark interest rates and the credit spreads. Therefore, both interest rates and credit spreads can become negative with positive probability. To guarantee positivity of interest rates and credit spreads, we also consider an alternative quadratic functional form for the instantaneous interest rate and credit spread, whiling maintaining the same number of parameters:

$$r_t = a_r + X_t^\top \langle b_r \rangle X_t, \quad \lambda_t^i = a_i + X_t^\top \langle b_i \rangle X_t + Y_t^\top \langle c_i \rangle Y_t. \tag{25}$$

According to Leippold and Wu (2002), the benchmark zero-coupon bond price becomes exponential quadratic in the state vector,

$$P(X_0, \tau) = \exp(-a(\tau) - b(\tau)^\top X_0 - X_0^\top B(\tau) X_0), \tag{26}$$

with the coefficients solving the following ordinary differential equations,

$$\begin{aligned}
a'(\tau) &= a_r + b(\tau)^\top \theta_x + \mathtt{tr} B(\tau) - b(\tau)^\top b(\tau)/2, \\
b'(\tau) &= 2B(\tau)\theta_x - \kappa_x^\top b(\tau) - 2B(\tau) b(\tau), \\
B'(\tau) &= \langle b_r \rangle - B(\tau)\kappa_x - \kappa_x^\top B(\tau) - 2B(\tau)^2,
\end{aligned} \tag{27}$$

starting at $B(0) = 0$, $b(0) = 0$ and $a(0) = 0$.

Analogously, we can derive the credit default swap premium as

$$S(Z_0, \tau) = \frac{w \int_0^\tau \left(c(t) + d(t)^\top Z_0 + Z_0^\top D(t) Z_0\right) \exp\left(-a(t) - Z_0^\top b(t) Z_0\right)}{\int_0^\tau \exp\left(-a(t) - b(t)^\top Z_0 - Z_0^\top B(t) Z_0\right)}, \tag{28}$$

with the coefficients solving the following ordinary differential equations:

$$\begin{aligned}
a'(\tau) &= a_Z + b(\tau)^\top\theta + \mathtt{tr}B(\tau) - b(\tau)^\top b(\tau)/2, \\
b'(\tau) &= l_Z + 2B(\tau)\theta - \kappa^\top b(\tau) - 2B(\tau)b(\tau), \\
B'(\tau) &= \langle b_Z\rangle - B(\tau)\kappa - \kappa^\top B(\tau) - 2B(\tau)^2, \\
c'(t) &= d(\tau)^\top\theta + \mathtt{tr}D(\tau) - d(\tau)^\top b(\tau), \\
d'(t) &= 2D(t)\theta - \kappa^\top d(t) - 2D(t)b(t) - 2B(t)d(t), \\
D'(t) &= -D(t)\kappa - \kappa^\top D(t) - 4B(t)D(t),
\end{aligned} \tag{29}$$

starting at $a(0)=0$, $b(0)=0$, $B(0)=0$, $c(0)=c_Z$, $d(0)=0$, and $D(0)=\langle d_Z\rangle$. In equation (29), l_Z is a vector of zeros, which will become nonzero in the presence of linear liquidity or credit risk factors. The details of the derivation are available upon request.

Since the signs of the idiosyncratic credit risk premium (m_t) and the idiosyncratic liquidity premium (q_t) can be either negative or positive, it is appropriate to maintain the original Gaussian affine assumption on both. In the presence of these two risk factors, the pricing formula for the credit default swap retains the same form as in (28), only with a corresponding expansion on the state vector $Z=[X^\top, Y^\top, \xi, \zeta]^\top$ and the following redefinitions on the coefficients: $a_Z = a_r + a_i + a_m + a_q$, $l_Z = [0,0,c_m,b_q]^\top$, $b_Z = [(b_r + b_i)^\top, c_i^\top, 0, 0]^\top$, $c_Z = a_i + a_m$, and $d_Z = [b_i^\top, c_i^\top, 0, 0]^\top$. Furthermore, the initial condition on $d(0)$ adjusts from zero to $d(0) = [0,0,c_m,0]^\top$.

3. Data and Evidence

The CDS data are from JP Morgan Chase. They are daily CDS spread quotes on seven fixed maturities at one, two, three, four, five, seven, and ten years from May 2003 to May 2004 (256 business days) on each reference company. We obtain the credit rating information on each reference company from Standard & Poors, and its sector information from Reuters, publicly available on Yahoo.

We apply several filters to the data. First, we exclude the reference entities that we cannot find credit rating information on. Second, we exclude companies whose credit rating has migrated during our sample period. Third, we include only investment grade companies. Furthermore, we exclude AA and AAA companies because they fail to meet our liquidity standards as described below. These filters leave us with 157 reference companies. We classify the remaining reference companies based on (i) two broad industry classifications: financial and corporate and (ii) two broad credit rating groups: A (including A+ and A-) and BBB (including BBB+ and BBB-).

The credit default swap spreads differ dramatically in liquidity across different reference entities. Within each sector and credit rating class, active quote updates are concentrated on only a few firms. To compare the quoting activity on each firm, we first expand the quote series into daily frequency by filling missing data points with previously available quotes. Then, we take daily differences. If the quotes are not updated between two consecutive days, the daily differences would be zero. Thus, we use the number of days that have non-zero daily quote differences to capture the quote updating frequency. Table 1 provides the summary statistics on the quote updating frequency at different industry sectors and credit rating classes. Each reference firm underlies seven credit default swap series at the seven fixed maturities. We compute the total number of active quote updates on all seven series for each firm. Our sample period includes 256 business days. The average of the median updating frequency across all seven series is 278 times, roughly about one update on the seven series per day, or one update on each series per seven business days. The most active firm has 938 updates, about one update on each series every two days. The least active only have 11 updates across the whole sample period.

Within each industry sector and credit rating class, we divide the data into two groups, with high and low liquidity, respectively. We classify a firm into the high-liquidity group if the spread quotes on the firm have no fewer than 364 total updates, corresponding an average of one update per week. Then, at each date and maturity, we average the spread quotes across the high-liquidity firms within each industry sector and credit rating class. We estimate the credit risk dynamics using the time series of these average high-liquidity credit default swap spreads on the seven maturities.

The rest firms are classified into the low liquidity group. To investigate the impact of liquidity on the term structure of credit spreads, we also average the quotes among the low-liquidity firms at each date, maturity, industry sector, and credit rating class. Nevertheless, quotes with very few updates are unlikely to be informative. Hence, for the low-liquidity group, we average across firms that have at least 182 quote updates, corresponding to an average updating frequency of at least once every two weeks.

Figure 1 plots the time series of the average credit default swap spreads at each industry sector and credit rating class, with left panels for high-liquidity firms and right panels for low-liquidity firms. The seven lines in each panel denote the seven fixed maturities from one to ten years. The spreads were high during the start of our sample following the high default year of 2002. The spreads declined and reached the bottom around January 2004 and started to pick up again since then.

[Figure 1 about here.]

Within each industry sector, spreads on the BBB rating class are much higher than the corresponding A group, due to the higher default probabilities. The seven maturity series under the BBB rating class are also closer to one another than the corresponding series under the A rating class, implying a flatter term structure for the higher credit rating class. Within the same industry sector and credit rating class, high-liquidity firms have markedly higher CDS spreads on low-liquidity firms.

Figure 2 plots the term structure of the credit default swap spreads at different days. To reduce clustering, we plot one term structure every seven business days for each panel. During our sample period, the CDS spreads on high-liquidity firms mostly show upward sloping term structures. The spreads on the corresponding low-liquidity firms are lower, and their term structures are also flatter, sometimes showing hump shapes. Within the same industry sector, CDS spreads on the A rating class have steeper term structures during our sample period than spreads on the BBB rating class.

[Figure 2 about here.]

Table 2 reports the summary statistics of the average credit swap spread series at the seven fixed maturities under each industry sector, credit rating class, and liquidity group. The mean spreads are higher at longer maturities and hence show upward sloping term structures in all groups. Within each sector and rating class, the high-liquidity groups generates much higher mean spreads than the low-liquidity group. The differences are especially large in the financial sector, where the mean spreads on the high-liquidity groups approximately double the mean spreads on the corresponding low-liquidity groups. Across the two credit rating classes, the mean spreads are larger for the BBB class than for the A class. The differences are again larger for the financial sector than for the corporate sector.

The term structure of the standard deviation is upward sloping for the financial sector and A rating class, but shows a hump-shape for all other groups. The skewness estimates are mostly positive, but the excess kurtosis estimates are mostly small. The daily autocorrelation estimates are between 0.96 to 0.99, showing that the spreads are persistent, albeit less so than the benchmark interest rates.

To obtain the benchmark libor interest rate dynamics, we also download from Bloomberg the U.S. dollar libor and swap rates that match the maturity and sample period of the credit default swap spreads data.

4. Estimation Strategy

We estimate the dynamics of benchmark interest-rate risk, credit risk, and liquidity risk in three consecutive steps, all using quasi-maximum likelihood method. At each step, we cast the models into a state-space form, obtain efficient forecasts on the conditional mean and variance of observed interest rates and credit default swap spreads using an efficient filtering technique, and build the likelihood function on the forecasting errors of the observed series, assuming that the forecasting errors are normally distributed. The model parameters are estimated by maximizing the likelihood function.

In the first step, we estimate the interest-rate factor dynamics using libor and swap rates. In the state-space form, we regard the two interest-rate factors (X) as the unobservable states and specify the state-propagation equation using an Euler approximation of statistical dynamics of the interest-rate factors embedded in equation (24):

$$X_t = \Phi_x X_{t-1} + \sqrt{\mathcal{Q}_x}\varepsilon_{xt}, \tag{30}$$

where $\Phi_x = \exp(-\kappa_x^{\mathbb{P}}\Delta t)$ denotes the autocorrelation matrix of X, $\mathcal{Q}_x = I\Delta t$ denotes the instantaneous covariance matrix of X, with I denoting an identity matrix of the relevant dimension and $\Delta t = 1/252$ denoting the daily frequency, and ε_{xt} denotes a two-dimensional i.i.d. standard normal innovation vector. The measurement equations are constructed based on the observed libor and swap rates, assuming additive, normally-distributed measurement errors,

$$y_t = \begin{bmatrix} LIBOR(X_t, i) \\ SWAP(X_t, j) \end{bmatrix} + e_t, \quad cov(e_t) = \mathcal{R}, \qquad \begin{array}{l} i = 12 \text{ months}, \\ j = 2,3,4,5,7,10 \text{ years}. \end{array} \tag{31}$$

In the second step, we take the estimated interest-rate factor dynamics in the first step as given, and estimate the credit-risk factor dynamics (Y) at each industry sector and credit rating class using the seven average credit default swap spread series for the high-liquidity groups. In the state-space form, we specify the state-propagation equation using an Euler approximation of statistical dynamics of the credit-risk factors embedded in equation (24):

$$Y_t = \Phi_y Y_{t-1} + \sqrt{\mathcal{Q}_y}\varepsilon_{yt}, \tag{32}$$

with $\Phi_y = \exp(-\kappa_y^{\mathbb{P}}\Delta t)$, $\mathcal{Q}_y = I\Delta t$, and ε_{yt} being a k-dimensional i.i.d. standard normal innovation vector. We estimate models with both $k = 1$ and $k = 2$. The measurement equations are defined on the seven credit default swap spreads,

$$y_t = S(X_t, Y_t, \tau, i) + e_t, \quad cov(e_t) = \mathcal{R}, \tau = 1, 2, 3, 4, 5, 7, 10 \text{ years}, \tag{33}$$

where $i = 1, 2, 3, 4$ denotes the ith industry sector and credit rating class. We repeat this step of each of four industry and credit rating classes.

In the third step, we estimate the idiosyncratic credit risk and liquidity risk dynamics for each industry sector and credit rating class using the average credit default spreads on the low-liquidity firms. The state-propagation equation is defined based on an Euler approximation of the idiosyncratic credit and liquidity risk dynamics in (21) and (22):

$$\begin{bmatrix} \xi_t \\ \zeta_t \end{bmatrix} = \Phi_q \begin{bmatrix} \xi_{t-1} \\ \zeta_{t-1} \end{bmatrix} + \sqrt{\mathcal{Q}_q}\varepsilon_{qt}, \tag{34}$$

with $\Phi_q = \langle \exp(-\kappa_m^{\mathbb{P}}\Delta t), \exp(-\kappa_q^{\mathbb{P}}\Delta t) \rangle$, $\mathcal{Q}_q = I\Delta t$, and ε_{qt} being a two-dimensional i.i.d. standard normal innovation vector. The measurement equations are defined on the seven average credit default swap spreads on the low-liquidity firms,

$$y_t = S(X_t, Y_t, \xi_t, \zeta_t, \tau, i) + e_t, \quad cov(e_t) = \mathcal{R}, \quad \tau = 1, 2, 3, 4, 5, 7, 10 \text{ years}. \tag{35}$$

We repeat this step on each of four industry sector and credit rating classes.

Given the definition of the state-propagation equation and measurement equations at each step, we use an extended version of the Kalman filter to filter out the mean and covariance matrix of the state variables conditional on the observed series, and construct predictive mean and covariance matrix of the observed series based on the filtered state variables. Then, we define the daily log likelihood function assuming normal forecasting errors on the observed series:

$$l_{t+1}(\Theta) = -\frac{1}{2}\log\left|\overline{V}_{t+1}\right| - \frac{1}{2}\left((y_{t+1}-\overline{y}_{t+1})^{\top}\left(\overline{V}_{t+1}\right)^{-1}(y_{t+1}-\overline{y}_{t+1})\right), \tag{36}$$

where $\overline{y}$ and $\overline{V}$ denote the conditional mean and variance forecasts on the observed series, respectively. The model parameters, Θ, are then estimated by maximizing log likelihood of the data series, which is a summation of the daily log likelihood values,

$$\Theta \equiv \arg\max_{\Theta} \mathcal{L}(\Theta, \{y_t\}_{t=1}^{N}), \quad \texttt{with} \quad \mathcal{L}(\Theta, \{y_t\}_{t=1}^{N}) = \sum_{t=0}^{N-1} l_{t+1}(\Theta), \tag{37}$$

where $N = 256$ denotes the number of observations for each series. For each step, we further assume that the measurement errors on each series are independent but with distinct variance.

We estimate both affine and quadratic models for the term structure of benchmark interest rates and credit spreads. The state dynamics are the same under both specifications. Hence, the state-propagation equations are the same for both model estimation. What differs is in the measurement equations since the two types of specifications generate different function forms for libor, swap rates, and credit default swap spreads as functions of the state variables.

5. The Pricing of Interest-Rate, Credit, and Liquidity Risks

First, we first summarize the performance of the different dynamic term structure models in pricing interest rates and credit default swap spreads. Then, from the estimated model parameters we discuss the dynamics and pricing of benchmark interest-rate risk, credit risk, and liquidity risk.

5.1. Model performance comparisons

Table 3 reports the summary statistics on the pricing errors of libor and swap rates under the two-factor affine and quadratic model specifications. The affine model explains the swap rates well, but fails miserably in explaining the 12-month libor. The discrepancy between libor and swap rates is well known in the industry. Nevertheless, the very poor performance reveals some deficiency of the two-factor affine specification. In contrast, the quadratic model performs much better on the libor series. Its performance across the six swap rates is also more uniform. Thus, the richer, nonlinear dynamic specification of the quadratic model captures the joint term structure of the libor and swap rates better.

The maximized log likelihood values ($\mathcal{L}$) are 5067.1 for the affine model and 5229.7 for the quadratic model, also indicating superior performance from the quadratic model. Since these two models are not nested, we cannot employ the standard likelihood ratio tests to gauge the significance of the likelihood difference. Nevertheless, we follow Vuong (1989) in constructing a statistic based on the difference between the daily log likelihood values from the two non-nested models:

$$lr_t = l_t^Q - l_t^A \tag{38}$$

where l_t^Q and l_t^A denote the time-t log likelihood value of the quadratic and affine models, respectively. Vuong constructs a statistic based on the likelihood ratio:

$$\mathcal{M} = \sqrt{T}\mu_{lr}/\sigma_{lr}, \tag{39}$$

where μ_{lr} and σ_{lr} denote the sample mean and standard deviation of the log likelihood ratio. Under the null hypothesis that the two models are equivalent, Vuong proves that $\mathcal{M}$ has an asymptotic normal distribution with zero mean and unit variance. We construct the log likelihood ratio, and estimate the sample mean at 0.6352, and sample standard deviation at 3.0054. The standard deviation calculation adjusts for serial dependence according to Newey and West (1987), with the number of lags chosen optimally according to Andrews (1991) based on an AR(1) specification. The $\mathcal{M}$-statistic is estimated at 3.38, indicating that the quadratic model performs significantly better than the affine model in explaining the benchmark libor term structure.

Table 4 reports the summary statistics of the pricing errors of the credit default swap spreads on the high-liquidity firms using one credit risk factor for both the affine and the quadratic specifications. The affine model provides an almost perfect fit for the four-year CDS spread, but the performance deteriorates at both short (one year) and long (ten year) maturities. In contrast, the performance of the quadratic specification is more uniform across different maturities. Under the quadratic specification, one credit risk factor, together with the previously identified two benchmark interest-rate factors, can explain all CDS spread series by over 90 percent. The one-factor quadratic model also generates higher likelihood values than the corresponding affine model for each of the four industry sector and credit rating classes, but the differences are not statistically significant in terms of the Vuong (1989) statistic.

For comparison, we also estimate models with two credit-risk factors. Table 5 reports the summary statistics of the pricing errors. Adding one additional credit risk factor significantly improves the performance of the affine model at the two ends of the CDS term structure. Two affine factors explain over 98 percent of the credit spread variations except for one series. With the quadratic specification, since one credit risk factor performs reasonably well, adding another credit risk factor does not generate as much improvement. Furthermore, with two credit-risk factors, the maximized likelihood values from the affine and quadratic specifications are close to one another. The quadratic specification no longer dominates the affine specification. In a sense, an extra factor in the affine model can play the richer dynamics of the quadratic model.

In the last step, to account for the idiosyncratic movements of the credit spreads for the low-liquidity firms, we introduce an idiosyncratic credit risk factor and a liquidity factor in addition to the two benchmark interest rate factors and the two credit risk factors identified from the spreads on high-liquidity firms. Table 6 reports the summary statistics of the pricing errors. These two additional factors can explain most of the idiosyncratic variation in the low-liquidity groups. Most series can be explained over 95 percent.

Overall, two interest-rate factors, especially in the quadratic forms, can explain the term structure of the benchmark interest rates well. Two additional credit-risk factors are more than enough to explain the term structure of credit spreads for high-liquidity firms under each industry sector and rating class. Finally, by incorporating an additional credit risk factor and a liquidity risk factor, the model also performs well in explaining the term structure of credit spreads on low-liquidity firms

5.2. Dynamics and term structure of the benchmark libor interest rates

Table 7 reports the first-stage parameter estimates and the absolute magnitudes of the t-statistics (in parentheses) that define the dynamics and term structure of the benchmark libor interest rates. Under both affine and the quadratic specifications, κ_x determines the mean-reversion of the interest-rate factor X under the risk-neutral measure $\mathbb{Q}$. The small estimates on the diagonal elements of κ_x capture the persistence of interest rates. The negative estimates on the off-diagonal element suggest that positive shocks on the first factor predicts larger values on the second factor.

The estimates for the constant part of the market price of risk γ_{x0} are negative for both factors under the affine model. Under the quadratic model, the market price is positive on the first factor and negative on the second factor. The proportional coefficients estimates, γ_{x1}, are small and not statistically different from zero for both factors under the affine specification, indicating that the market price of risk does not vary significantly with the factor level. The estimates under the quadratic model are also small and only statistically significant for the first factor.

Under both models, the b_r estimates show that the second factor loads more heavily on the instantaneous benchmark interest rate. These instantaneous loading coefficients interact with the risk-neutral factor dynamics (κ) to determine the response of the whole yield curve to unit shocks from the interest-rate factors. Under the affine model, the contemporaneous responses of the continuously compounded spot rate to the two dynamic factors are linear, with $b(\tau)/\tau$ measuring the response coefficients. Equation (9) shows how $b(\tau)$ measures the loading coefficient to the dynamic factors and equation (10) shows how b_r and κ interact to determine $b(\tau)$. Figure 3 plots $b(\tau)/\tau$ as a function of maturity τ. The solid line represents the first element, which is the loading of the first interest-rate factor. This factor loads more heavily at longer maturities than at shorter maturities. The dashed line plots the impacts of the second factor, which loads more heavily at the short end of the yield curve. The different loading patterns relate not only to the different magnitudes of the two elements of the b_r estimates, but also to the difference in persistence between the two factors. Under the affine model, the first factor is estimated to be more persistent than the second factor. Hence, its impact extends to longer maturities.

[Figure 3 about here.]

5.3. Default arrival dynamics and the term structure of credit spreads

Tables 8 and 9 report the parameter estimates and t-statistics on the default arrival dynamics, market pricing, and their impacts on the term structure of credit spreads at each industry sector and credit rating class. Tables 8 reports estimates on the one-factor credit risk specifications. Tables 9 reports estimates on the two-factor credit risk specifications. From the two tables, we draw the follow common observations.

First, the default risk dynamics and pricing differ across different industry sectors and credit rating classes. Nevertheless, in all cases, the default risk dynamics show intricate dynamic interactions with the interest-rate factors. The κ_{xy} matrix captures the predictive power of interest-rate factors on the default risk factors, whereas the b_i vector captures the contemporaneous loading of the interest-rate factors on the default arrival λ^i. Estimates on both sets of parameters are significant in most cases, indicating that the interest-rate factors both predict default arrivals via the drift dynamics κ_{xy} and impact the default arrivals contemporaneously via the loading coefficients b_i.

Second, the estimates on κ_y under the one-factor affine model are very small and not significantly different from zero. Under the two-factor affine specification, the estimates for one of the diagonal elements of κ_y are close to zero. The small estimates indicate a near unit root behavior for the credit risk dynamics. However, the corresponding estimates are larger and also with better precision (larger t-values) under the quadratic specifications. Thus, with a nonlinear structure under the quadratic model, we can more accurately identify a more stationary credit-risk dynamics, while delivering a uniformly well pricing performance on CDS spreads across all maturities.

Although the quadratic models deliver better and more uniform performance, the affine models provide a linear and hence easier-to-interpret relation between factors and credit spreads. In Figure 4, we plot the contemporaneous loading of the two default risk factors on the continuously compounded spot rate at each industry sector and credit rating group, as captured by the third and four elements of $b^i(\tau)/\tau$ for the ith sector and rating class. Since the default risk factors do not enter the benchmark libor curve, the loading also directly measures the impact on the credit spread between the corporate spot rate and the libor spot rate. The solid line denotes the first credit risk factor and the dashed line the second credit risk factor.

[Figure 4 about here.]

Overall, both factor loadings are downward sloping under all four industry/rating classifications. And yet, a noticeable difference shows up between the financial and corporate sectors. First, the factor loadings in the financial sector present an exponential decay while in the corporate sector they are more or less linear. Second, the loading differences between the two rating classes are much larger for the financial firms than for non-financial firms, suggesting that financial firms are more sensitive to rating changes between A and BBB classes. A lower rating generates dramatically larger spreads for the financial firms.

5.4. Liquidity and liquidity premiums

The liquidity of the CDS contracts, as revealed by the quote updating frequency, varies greatly across different reference firms. Within each industry sector and credit rating class, the liquidity is concentrated on a few firms. An important question is what makes investors to concentrate the trading on one firm versus another. Also important is to understand whether and how liquidity impacts the pricing of the CDS contracts.

Table 10 reports the parameter estimates and t-statistics (in parentheses) on the idiosyncratic credit risk and liquidity factors that account for the idiosyncratic movements of the CDS spreads on the low-liquidity firms. First, the loading parameter estimates on the idiosyncratic credit risk factor (c_m) are strongly significant, showing that the default arrival rates for firms in the low-liquidity groups do have idiosyncratic movements that are independent of the default arrival dynamics identified from the high-liquidity group within the same industry sector and credit rating class.

Second, the estimates on the intercept a_m are negative under all four industry sector and credit rating classes. These negative estimates suggest that firms in the low-liquidity group on average have lower default risk and hence experience lower instantaneous credit spreads than firms in the corresponding high-liquidity group. This observation is intriguing and implies that, within the same industry and credit rating class, firms with active CDS trading activities are associated with higher perceived credit risk than firms with less active CDS trading activities. Either investors *choose* to trade CDS contracts on firms that they perceive to have higher chances of downward rating migrations, or high-profile firms generate more awareness of its potential default risk.

The instantaneous loading estimates on the liquidity risk factor (b_q) are large in magnitudes and also highly significant, showing that liquidity plays a key role in the credit spread differences between the two liquidity groups. The intercept estimates on a_q are all positive, suggesting a higher discounting for the low-liquidity contracts. Thus, the lower average CDS spreads on low-liquidity firms can be attributed to a combination of low credit risk and high liquidity discounting.

The estimates on κ_m, which measures the risk-neutral mean-reverting behavior of the idiosyncratic default risk factor, are very small, suggesting that the idiosyncratic credit risk factor has highly persistent risk-neutral dynamics and hence impacts the term structure of credit spreads almost linearly across maturities. Furthermore, the estimates for γ_{m0} are all negative, and in several cases significantly so, implying that this idiosyncratic credit risk has a negative market price of risk. Given the normalization of the statistical dynamics, the negative market price of risk implies a positive risk-neutral drift (θ) for the credit risk factor. As suggested by the ordinary differential equations in (16), a positive θ for the credit risk factor helps generate an upward sloping mean term structure of credit spreads.

The dynamics and pricing of the liquidity risk factor are dramatically different. The estimates on the risk-neutral mean-reversion parameter κ_q for the liquidity factor are larger than the estimates for κ_m and are all strongly significant, suggesting that the liquidity risk factor has a more stationary impact on the term structure of CDS spreads. Furthermore, the estimates on the pricing of the liquidity risk (γ_{q0}) are mostly positive and significant, especially for non-financial firms. Contrary to the negative market price on the credit risk factor, positive market prices on the liquidity risk factor make the term structure less upward sloping, and hence explains the flatter term structure of CDS spreads for low-liquidity firms.

Figure 5 shows the difference between the two factors in terms of their impacts on the spot rate curve of the low-liquidity firms. The impacts of the default risk factor are in the left panels. They are flat across maturities. The loadings are larger for BBB rating than for A rating firms, and the difference is especially large for financial firms, suggesting again that the financial firms are more sensitive to rating changes between A to BBB rating classes.

[Figure 5 about here.]

The loadings of the liquidity factor show quite different patterns. The loading are very high at short maturities, but the impacts decline quickly as maturity increases, generating a strongly downward sloping pattern. Also interestingly, the liquidity factor loadings on financial firms are also more sensitive to rating changes than non-financial firms.

Overall, our estimation suggests that the lower mean CDS spreads for low-liquidity firms can be attributed to two separate driving forces. First, the market regards low-liquidity firms as having less default risk. Second, the market discounts the value of low-liquidity contracts more heavily. On the other hand, the flatter mean term structure for low-liquidity firms is mainly due to the positive pricing of the liquidity risk for CDS contracts.

6. Conclusion

Using a large data set on CDS spread quotes, we perform a joint analysis of the term structure of interest rates, credit spreads, and liquidity premia. Our construction and estimation of several dynamic term structure models suggest that quadratic models perform better and more uniformly across the term structure of interest rates and credit spreads than affine models. We also find that default risk dynamics differ across different industry sectors and credit rating groups, but in all cases they show intricate interactions with the interest-rate dynamics. Specifically, we find that financial firms are more sensitive to rating changes than non-financial firms.

Our estimation also suggests that within the same industry sector and credit rating class, firms with active CDS trading activities tend to also have higher credit risks than firms with low CDS trading activities. Finally, low-liquidity firms induce heavier discounting on the yield curve and generate lower CDS spreads. Positive market pricing on the liquidity risk further renders the mean term structure of CDS spreads flatter on low-liquidity firms.

References

Andrews, D., 1991, “Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation,” *Econometrica*, 59, 817–858.

Blanco, R., S. Brennan, and I. W. Marsh, 2004, “An Empirical Analysis of the Dynamic Relationship Between Investment-Grade Bonds and Credit Default Swaps,” *Journal of Finance*, forthcoming.

Collin-Dufresne, P., R. S. Goldstein, and J. Helwege, 2003, “Is Credit Event Risk Priced? Modeling Contagion via the Updating of Beliefs,” working paper, Washington University St. Louis.

Collin-Dufresne, P., R. S. Goldstein, and J. S. Martin, 2001, “The Determinants of Credit Spread Changes,” *Journal of Finance*, 56(6), 2177–2207.

Delianedis, G., and R. Geske, 2001, “The Components of Corporate Credit Spreads: Default, Recovery, Tax, Jumps, Liquidity, and Marker Factors,” working paper, UCLA.

Duffee, G. R., 1999, “Estimating the Price of Default Risk,” *Review of Financial Studies*, 12, 197–226.

Duffie, D., 1998, “Defaultable Term Structure Models with Fractional Recovery of Par,” working paper, Stanford University.

Duffie, D., and R. Kan, 1996, “A Yield-Factor Model of Interest Rates,” *Mathematical Finance*, 6(4), 379–406.

Duffie, D., J. Pan, and K. Singleton, 2000, “Transform Analysis and Asset Pricing for Affine Jump Diffusions,” *Econometrica*, 68(6), 1343–1376.

Duffie, D., L. H. Pedersen, and K. J. Singleton, 2003, “Modeling Sovereign Yield Spreads: A Case Study of Russian Debt,” *Journal of Finance*, 58(1), 119–160.

Duffie, D., and K. Singleton, 1997, “An Econometric Model of the Term Structure of Interest Rate Swap Yields,” *Journal of Finance*, 52(4), 1287–1322.

Duffie, D., and K. J. Singleton, 1999, “Modeling Term Structure of Defaultable Bonds,” *Review of Financial Studies*, 12(3), 687–720.

Elton, E. J., M. J. Gruber, D. Agrawal, and C. Mann, 2001, “Explaining the Rate Spread on Corporate Bonds,” *Journal of Finance*, 56, 247–277.

Eom, Y. H., J. Helwege, and J.-z. Huang, 2003, "Structural Models of Corporate Bond Pricing," *Review of Financial Studies*, forthcoming.

Houweling, P., and T. Vorst, 2003, "Pricing Default Swaps: Empirical Evidence," *Journal of International Money and Finance*, forthcoming.

Huang, J.-z., and M. Huang, 2003, "How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?," working paper, Penn State University.

Hull, J., M. Predescu, and A. White, 2004, "The Relationship Between Credit Default Swap Spreads, Bond Yields, and Credit Rating Announcements," *Journal of Banking and Finance*, forthcoming.

Jarrow, R. A., and S. M. Turnbull, 1995, "Pricing Derivatives on Financial Securities Subject to Credit Risk," *Journal of Finance*, 50(1), 53–85.

Jones, E. P., S. P. Mason, and E. Rosenfeld, 1984, "Contingent Claim Analysis of Corporate Capital Structures: An Empirical Investigation," *Journal of Finance*, 39, 611–625.

Lando, D., 1998, "On Cox Processes and Credit Risky Securities," *Review of Derivatives Research*, 2, 99–120.

Leippold, M., and L. Wu, 2002, "Asset Pricing under the Quadratic Class," *Journal of Financial and Quantitative Analysis*, 37(2), 271–295.

Liu, J., F. A. Longstaff, and R. E. Mandell, 2000, "The Market Price of Credit Risk: An Empirical Analysis of Interest Rate Swap Spreads," Working paper, UCLA.

Longstaff, F. A., S. Mithal, and E. Neis, 2004, "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit-Default Swap Market," working paper, UCLA.

Longstaff, F. A., and E. Schwartz, 1995, "A Simple Approach to Valuing Risky Fixed and Floating Rate Debt," *Journal of Finance*, 50(3), 789–819.

Newey, W. K., and K. D. West, 1987, "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55(3), 703–708.

Skinner, F. S., and A. Diaz, 2003, "An Empirical Study of Credit Default Swaps," *Journal of Fixed Income*, 13(1), 28–38.

Vuong, Q. H., 1989, “Likelihood Ratio Tests for Model Selection and Non-Nested Hypotheses,” *Econometrica*, 57(2), 307–333.

Table 1

Summary Statistics of the Quote Updating Frequency on Credit Default Swap Spreads

Entries report the number of firms within each sector and credit rating class, as well as summary statistics (Median, Maximum, Minimum) on the quote updating frequency. We take daily differences on the quotes of each series and define the updating frequency as the number of days that the daily differences are nonzero, and hence the quotes are updated, for the seven series on each firm.

Sector	Rating	Number of Firms	Median	Maximum	Minimum
Financial	A	28	176	580	11
Financial	BBB	6	423	610	229
Corporate	A	57	191	686	21
Corporate	BBB	66	324	938	35
Average		39	278	704	74

Table 2

Summary statistics of credit default swap spreads

Entries report the summary statistics of the credit default swap spreads (in basis points) at the seven fixed maturities under each credit rating class, industry sector, and liquidity groups. Mean, Std, Skew, Kurtosis, and Auto denote the sample estimates of the mean, standard deviation, skewness, excess kurtosis, and the first-order autocorrelation, respectively. Data are daily from May 21, 2003 to May 12, 2004.

Maturity	High Liquidity					Low Liquidity				
Years	Mean	Std	Skew	Kurtosis	Auto	Mean	Std	Skew	Kurtosis	Auto
(i) Sector: Financial; Rating: A										
1	30.16	8.72	1.00	-0.33	0.98	14.53	4.70	1.59	2.46	0.97
2	39.41	10.34	0.95	-0.38	0.98	20.13	6.27	1.51	2.20	0.97
3	43.03	11.04	0.96	-0.36	0.98	22.08	6.85	1.47	2.13	0.97
4	49.69	12.41	0.91	-0.43	0.98	25.75	7.31	1.54	2.57	0.97
5	54.42	13.36	0.89	-0.49	0.98	28.35	7.67	1.58	2.75	0.97
7	61.29	13.43	0.84	-0.39	0.98	31.92	7.85	1.55	2.96	0.97
10	68.68	13.67	0.71	-0.57	0.98	35.30	8.16	1.47	2.95	0.97
(ii) Sector: Financial; Rating: BBB										
1	92.68	25.42	0.27	-1.16	0.98	51.64	12.62	0.76	-0.62	0.98
2	100.30	25.51	0.39	-1.13	0.98	55.65	14.09	0.74	-0.77	0.98
3	103.69	25.87	0.39	-1.13	0.98	56.89	14.73	0.73	-0.81	0.98
4	108.55	25.00	0.46	-1.08	0.98	58.50	12.94	0.82	-0.54	0.98
5	111.83	24.61	0.48	-1.04	0.98	60.01	11.76	0.88	-0.32	0.98
7	117.29	22.51	0.32	-1.17	0.98	60.96	9.23	0.85	0.04	0.97
10	123.18	20.52	0.16	-1.28	0.98	63.56	7.04	0.28	-0.22	0.96
(iii) Sector: Corporate; Rating: A										
1	28.85	8.17	0.78	-0.43	0.98	26.93	5.07	0.97	0.92	0.98
2	39.67	9.55	0.64	-0.77	0.98	31.37	5.39	0.54	0.23	0.98
3	43.77	10.01	0.64	-0.79	0.98	32.93	5.54	0.44	0.09	0.98
4	48.88	10.38	0.72	-0.56	0.98	36.52	5.78	0.32	-0.04	0.98
5	52.34	10.62	0.76	-0.42	0.98	39.06	5.92	0.23	-0.15	0.98
7	58.29	10.31	0.67	-0.42	0.98	41.78	5.62	-0.07	-1.02	0.99
10	64.46	9.57	0.46	-0.50	0.98	45.25	6.11	0.34	-1.06	0.99
(iv) Sector: Corporate; Rating: BBB										
1	63.35	17.89	1.30	0.88	0.98	39.55	6.49	0.51	0.05	0.98
2	75.09	19.31	1.22	0.64	0.98	46.26	7.04	0.37	0.12	0.98
3	79.17	19.76	1.20	0.58	0.98	48.61	7.16	0.31	0.12	0.98
4	83.66	19.28	1.15	0.38	0.98	52.21	6.92	0.00	-0.11	0.98
5	86.71	19.00	1.11	0.28	0.98	54.79	6.82	-0.16	-0.31	0.98
7	91.97	17.14	1.08	0.28	0.98	58.41	6.51	-0.31	-0.94	0.98
10	97.23	15.31	0.97	0.19	0.98	62.89	7.13	-0.00	-1.19	0.98

Table 3

Summary statistics of pricing errors on the libor and swap rates

Entries report the summary statistics of the pricing errors on the U.S. dollar libor and swap rates under the two-factor Gaussian affine model (left hand side under "Affine") and the two-factor Gaussian quadratic model (left hand side under "Quadratic"). We estimate both models by using quasi-maximum likelihood method joint with unscented Kalman filter. We define the pricing error as the difference between the observed interest rate quotes and the model-implied fair values, in basis points. The columns titled Mean, Std, Auto, Max, and VR denote, respectively, the sample mean, the standard deviation, the first-order autocorrelation, the maximum absolute error, and the explained percentage variance, defined as one minus the ratio of pricing error variance to interest rate variance, in percentages. The last row reports the maximized log likelihood for each model.

Maturity	Affine					Quadratic				
Years	Mean	Std	Auto	Max	VR	Mean	Std	Auto	Max	VR
1	7.56	17.35	0.91	46.89	5.08	-1.53	7.99	0.78	33.21	79.85
2	-0.23	4.09	0.88	10.38	98.46	-0.23	2.81	0.66	11.56	99.27
3	-0.05	0.22	0.16	1.11	99.99	0.30	1.59	0.57	7.33	99.84
4	0.08	0.95	0.41	5.05	99.95	-0.13	1.15	0.40	7.81	99.93
5	0.36	0.85	0.30	6.51	99.96	0.06	0.70	0.34	5.75	99.97
7	-0.72	1.03	0.74	3.87	99.94	-0.48	1.15	0.60	4.46	99.93
10	0.50	1.85	0.76	6.60	99.78	0.70	2.10	0.70	7.57	99.72
Average	1.07	3.76	0.59	11.49	86.17	-0.19	2.50	0.58	11.10	96.93
$\mathcal{L}$			5067.1					5229.7		

Table 4
Summary statistics of pricing errors on credit default swap spreads with one credit risk factor
Entries report the summary statistics of the pricing errors on the credit default swap spreads under both affine and quadratic specifications. Both specifications use one credit risk factor to price the high-liquidity credit-default swap spread at each industry and credit rating class. We estimate both models by using quasi-maximum likelihood method joint with unscented Kalman filter. We define the pricing error as the difference between the spread quotes and the model-implied fair values, in basis points. The columns titled Mean, Std, Auto, Max, and VR denote, respectively, the sample mean, the standard deviation, the first-order autocorrelation, the maximum absolute error, and the explained percentage variance, defined as one minus the ratio of pricing error variance to interest rate variance.

Maturity	Affine					Quadratic				
Years	Mean	Std	Auto	Max	VR	Mean	Std	Auto	Max	VR
(i) Sector: Financial; Rating: A										
1	-0.35	3.86	0.97	12.55	80.41	-1.31	2.69	0.83	18.98	90.49
2	1.33	2.07	0.97	5.48	96.01	1.58	1.74	0.57	17.96	97.15
3	-1.37	1.36	0.96	6.07	98.47	-1.16	1.50	0.35	22.59	98.14
4	-0.00	0.02	0.66	0.08	100.00	-0.06	1.14	0.09	18.07	99.16
5	0.25	1.00	0.96	2.48	99.44	-0.04	1.19	0.27	15.85	99.21
7	0.02	1.26	0.93	3.59	99.12	-0.35	1.21	0.43	13.07	99.19
10	-0.26	2.44	0.95	7.77	96.81	0.09	2.20	0.84	10.89	97.40
(ii) Sector: Financial; Rating: BBB										
1	-2.26	4.00	0.96	15.81	97.52	-1.19	6.37	0.79	47.22	93.73
2	0.33	2.32	0.95	7.69	99.17	0.37	3.62	0.41	45.79	97.99
3	-0.82	2.08	0.96	7.64	99.35	-1.01	3.30	0.38	44.58	98.37
4	0.01	0.01	0.84	0.04	100.00	-0.14	2.47	0.08	39.28	99.03
5	-0.26	1.25	0.96	3.86	99.74	-0.26	2.44	0.24	34.00	99.02
7	-0.57	2.97	0.97	6.92	98.26	-0.21	2.57	0.45	29.23	98.70
10	-0.40	5.52	0.97	12.38	92.75	0.05	3.78	0.79	23.08	96.61
(iii) Sector: Corporate; Rating: A										
1	-4.66	1.88	0.96	8.89	94.72	-3.66	2.23	0.75	20.74	92.55
2	0.10	0.84	0.91	2.51	99.23	0.33	1.40	0.39	17.53	97.86
3	-0.85	0.66	0.93	2.93	99.56	-0.62	1.37	0.35	19.63	98.13
4	0.00	0.01	0.80	0.02	100.00	0.19	1.11	0.19	16.72	98.86
5	-0.13	0.35	0.90	1.40	99.89	-0.06	0.94	0.07	14.91	99.22
7	0.03	1.28	0.95	4.36	98.47	-0.03	1.04	0.39	13.14	98.98
10	-0.24	3.23	0.97	8.73	88.59	0.23	1.97	0.86	11.00	95.78
(iv) Sector: Corporate; Rating: BBB										
1	-5.81	2.68	0.96	11.90	97.75	-2.98	5.54	0.58	63.04	90.40
2	0.19	1.29	0.95	3.91	99.56	0.55	3.72	0.23	54.39	96.29
3	-0.51	1.09	0.95	4.35	99.70	-0.57	3.23	0.16	49.68	97.33
4	0.00	0.00	0.65	0.01	100.00	-0.09	2.88	0.11	45.70	97.76
5	-0.27	0.68	0.96	2.18	99.87	-0.32	2.62	0.13	40.86	98.10
7	-0.17	2.88	0.98	6.62	97.18	-0.07	2.65	0.38	34.19	97.61
10	-0.16	5.38	0.98	12.13	87.64	0.32	3.35	0.73	26.09	95.22

Table 5

Summary statistics of pricing errors on credit default swap spreads with two credit risk factor

Entries report the summary statistics of the pricing errors on the credit default swap spreads under both affine and quadratic specifications. Both specifications use two credit risk factors to price the high-liquidity credit-default swap spread at each industry and credit rating class. We define the pricing error as the difference between the spread quotes and the model-implied fair values, in basis points. The columns titled Mean, Std, Auto, Max, and VR denote, respectively, the sample mean, the standard deviation, the first-order autocorrelation, the maximum absolute error, and the explained percentage variance, defined as one minus the ratio of pricing error variance to interest rate variance.

Maturity	Affine					Quadratic				
Years	Mean	Std	Auto	Max	VR	Mean	Std	Auto	Max	VR
(i) Sector: Financial; Rating: A										
1	0.01	0.08	0.43	0.38	99.99	-0.40	1.64	0.85	5.45	96.46
2	1.39	0.72	0.91	2.62	99.52	1.26	1.09	0.72	6.36	98.89
3	-1.38	0.80	0.94	4.11	99.48	-1.44	1.02	0.61	12.27	99.15
4	0.00	0.00	0.22	0.00	100.00	-0.05	0.57	0.08	8.96	99.79
5	0.27	0.76	0.93	2.09	99.68	0.21	0.93	0.65	8.02	99.52
7	0.05	0.75	0.89	2.02	99.68	-0.04	0.50	0.06	7.95	99.86
10	-0.20	1.72	0.95	3.88	98.42	-0.25	1.50	0.85	9.58	98.80
(ii) Sector: Financial; Rating: BBB										
1	-0.64	1.59	0.93	3.85	99.61	-0.10	1.83	0.74	10.63	99.48
2	0.07	0.11	0.39	0.53	100.00	-0.00	0.44	0.02	6.74	99.97
3	-1.42	1.20	0.94	4.59	99.78	-1.18	1.32	0.90	6.41	99.74
4	-0.32	1.06	0.93	2.84	99.82	-0.00	0.25	0.01	3.68	99.99
5	-0.16	1.68	0.97	3.13	99.53	0.03	0.82	0.91	1.74	99.89
7	0.01	0.03	0.41	0.20	100.00	-0.05	1.53	0.91	4.21	99.54
10	-0.51	1.56	0.94	3.97	99.43	-0.27	3.05	0.94	7.00	97.79
(iii) Sector: Corporate; Rating: A										
1	-0.44	1.88	0.98	3.98	94.72	-0.65	2.16	0.69	19.36	93.00
2	1.94	0.92	0.95	3.64	99.07	1.73	1.56	0.39	18.15	97.34
3	-0.28	0.53	0.93	2.01	99.72	-0.41	1.37	0.21	21.07	98.12
4	0.00	0.00	0.50	0.01	100.00	-0.02	1.15	0.08	18.24	98.77
5	-0.30	0.30	0.94	0.81	99.92	-0.25	1.04	0.09	16.18	99.03
7	0.01	0.09	0.43	0.38	99.99	0.08	0.96	0.17	13.89	99.13
10	-0.14	0.98	0.96	2.39	98.95	-0.02	1.10	0.51	12.05	98.69
(iv) Sector: Corporate; Rating: BBB										
1	-3.39	4.22	0.98	15.16	94.44	-0.60	3.14	0.52	37.52	96.92
2	1.18	1.75	0.98	3.46	99.18	1.92	2.11	0.36	26.61	98.81
3	-0.24	0.63	0.96	2.06	99.90	-0.19	1.38	0.13	21.21	99.51
4	0.00	0.00	0.04	0.00	100.00	-0.05	1.12	0.06	17.76	99.67
5	-0.30	0.56	0.98	1.04	99.91	-0.31	1.03	0.27	14.29	99.70
7	0.00	0.00	0.08	0.01	100.00	-0.03	0.72	0.07	11.42	99.82
10	-0.00	0.54	0.92	2.40	99.88	-0.05	0.95	0.66	9.01	99.62

Table 6
Summary statistics of pricing errors on the low-liquidity credit default swap spreads
Entries report the summary statistics of the pricing errors on the low-liquidity credit default swap spreads. In addition to two interest rate factors and two credit risk factors that have been identified using the benchmark interest rates and the high-liquidity credit default swap spreads, we add one additional idiosyncratic credit risk factor and a liquidity risk factor to account for the credit spread movements in the low-liquidity groups. We define the pricing error as the difference between the spread quotes and the model-implied fair values, in basis points. The columns titled Mean, Std, Auto, Max, and VR denote, respectively, the sample mean, the standard deviation, the first-order autocorrelation, the maximum absolute error, and the explained percentage variance, defined as one minus the ratio of pricing error variance to interest rate variance.

Maturity	Affine					Quadratic				
Years	Mean	Std	Auto	Max	VR	Mean	Std	Auto	Max	VR
(i) Sector: Financial; Rating: A										
1	2.82	1.29	0.96	5.93	97.82	0.10	1.59	0.97	3.77	96.69
2	2.13	0.78	0.95	3.62	99.43	0.80	0.87	0.95	2.53	99.29
3	-0.37	0.32	0.93	1.27	99.92	-0.78	0.35	0.94	1.76	99.90
4	-0.00	0.00	0.29	0.02	100.00	-0.00	0.00	0.01	0.04	100.00
5	0.07	0.28	0.94	0.66	99.96	0.17	0.31	0.95	0.82	99.95
7	0.02	0.17	0.88	0.45	99.98	0.02	0.20	0.89	0.46	99.98
10	-0.00	0.04	0.49	0.15	100.00	0.00	0.01	0.07	0.05	100.00
(ii) Sector: Financial; Rating: BBB										
1	-1.45	2.54	0.99	5.92	99.00	-1.05	2.53	0.99	5.50	99.01
2	0.00	0.16	0.13	2.23	100.00	0.01	0.18	0.17	2.52	100.00
3	-0.29	1.39	0.97	2.70	99.71	-0.29	1.39	0.97	2.78	99.71
4	0.00	0.10	0.03	1.53	100.00	0.01	0.09	0.12	1.42	100.00
5	0.21	0.77	0.98	1.25	99.90	0.19	0.76	0.98	1.32	99.90
7	-1.63	2.81	0.99	6.00	98.45	-1.68	2.82	0.98	6.14	98.43
10	-3.52	4.85	0.98	11.41	94.41	-3.37	5.01	0.98	11.68	94.04
(iii) Sector: Corporate; Rating: A										
1	-0.08	3.47	0.99	7.27	81.96	0.62	2.82	0.98	5.91	88.14
2	-0.06	1.93	0.99	4.61	95.93	0.29	1.82	0.98	4.20	96.35
3	-1.33	0.78	0.98	3.48	99.39	-1.23	0.76	0.98	3.35	99.42
4	0.00	0.01	0.49	0.10	100.00	-0.00	0.02	0.55	0.16	100.00
5	0.60	0.69	0.98	2.27	99.57	0.58	0.69	0.98	2.28	99.58
7	0.02	0.10	0.53	0.88	99.99	0.03	0.09	0.56	0.77	99.99
10	-0.24	0.73	0.96	1.49	99.42	-0.35	0.65	0.95	1.41	99.55
(iv) Sector: Corporate; Rating: BBB										
1	0.66	2.87	0.98	7.72	97.43	0.38	1.71	0.91	7.98	99.09
2	1.33	1.34	0.98	3.46	99.51	0.77	1.09	0.96	2.91	99.68
3	-0.50	0.40	0.95	1.45	99.96	-0.76	0.29	0.92	1.57	99.98
4	-0.00	0.00	0.27	0.03	100.00	0.00	0.00	0.23	0.04	100.00
5	0.15	0.36	0.97	0.88	99.96	0.26	0.35	0.96	1.19	99.97
7	0.00	0.05	0.57	0.20	100.00	0.02	0.07	0.35	0.53	100.00
10	0.01	0.28	0.91	0.95	99.97	-0.16	0.42	0.92	1.00	99.93

Table 7

Dynamic and term structure of the benchmark libor interest rates

Entries report the parameter estimates and the absolute magnitudes of the *t*-statistics (in parentheses) that determine the dynamics and term structure of the benchmark libor interest rates. The estimations are based on 12-month libor and swap rates of two, three, five, seven, and ten years, with quasi-maximum likelihood method.

Model	κ_x		γ_{x0}	γ_{x1}	a_r	b_r
	0.2365	0	−0.1987	0.0819		0.0000
Affine	(5.22)	−−	(4.45)	(0.05)	0.0046	(0.04)
	−0.9338	0.3073	−0.9752	−0.0321	(1.04)	0.0116
	(12.87)	(5.35)	(3.77)	(0.00)		(20.2)
	0.7597	0	1.1885	0.7581		0.0006
Quadratic	(64.5)	−−	(15.7)	(3.79)	0.0081	(8.20)
	−0.6567	0.1196	−1.6100	0.0774	(79.7)	0.0025
	(38.17)	(26.6)	(25.8)	(0.06)		(22.9)

Table 8
One-factor default arrival dynamics and the term structure of credit spreads

Entries report the second-stage parameter estimates and the absolute magnitudes of the t-statistics (in parentheses) that determine the one-factor default arrival dynamics and the term structure of credit spreads. The estimations are based on high-liquidity credit default swap spreads at each of the four industry and credit rating classes with quasi-maximum likelihood method.

Θ		κ_{xy}		κ_y	γ_{y0}	γ_{y1}	a_i	$b_i^\top$		c_i
					(i) Affine Models					
Financial	A	-0.0363	0.1033	0.0001	-0.0230	0.0232	0.0143	-0.0014	0.0015	0.0038
		(1.00)	(5.57)	(0.03)	(0.07)	(0.04)	(1.62)	(5.39)	(41.5)	(19.5)
Financial	BBB	0.0486	-0.0421	0.0001	-0.0175	0.0176	0.0369	0.0017	0.0021	0.0137
		(2.13)	(5.28)	(0.02)	(0.01)	(0.01)	(1.71)	(1.53)	(16.3)	(18.1)
Corporate	A	0.1974	0.0618	0.0001	-0.0226	0.0227	0.0128	0.0002	0.0015	0.0035
		(7.72)	(6.49)	(0.03)	(0.05)	(0.04)	(1.69)	(0.67)	(15.0)	(23.1)
Corporate	BBB	-0.1196	0.0735	0.0001	-0.0224	0.0225	0.0308	-0.0029	0.0008	0.0056
		(2.90)	(3.55)	(0.01)	(0.08)	(0.04)	(6.36)	(7.08)	(10.0)	(14.4)
					(ii) Quadratic Models					
Financial	A	0.3922	-0.0211	0.0718	0.0181	0.0537	0.0061	0.0006	-0.0007	0.0032
		(28.7)	(11.3)	(9.24)	(0.00)	(0.02)	(28.1)	(23.00)	(12.6)	(20.7)
Financial	BBB	0.2768	0.0947	0.2043	0.1493	0.0551	0.0186	0.0009	-0.0030	0.0112
		(24.2)	(32.1)	(28.9)	(0.02)	(0.02)	(47.0)	(5.32)	(45.7)	(16.4)
Corporate	A	0.1108	0.0079	0.1396	0.0875	0.0521	0.0050	-0.0008	-0.0010	0.0083
		(20.5)	(5.94)	(24.5)	(0.00)	(0.01)	(27.6)	(45.9)	(20.7)	(44.0)
Corporate	BBB	0.1906	0.0646	0.2056	0.1536	0.0520	0.0125	-0.0014	-0.0020	0.0140
		(24.9)	(48.8)	(30.3)	(0.01)	(0.01)	(21.6)	(19.8)	(20.2)	(17.3)

Table 9

Two-factor default arrival dynamics and the term structure of credit spreads

Entries report the second-stage parameter estimates and the absolute magnitudes of the t-statistics (in parentheses) that determine the two-factor default arrival dynamics and the term structure of credit spreads. The estimations are based on high-liquidity credit default swap spreads at each of the four industry and credit rating classes with quasi-maximum likelihood method.

Θ		κ_{xy}		κ_y		γ_{y0}	γ_{y1}	a_i	b_i	c_i
				(i) Affine Models						
Financial	A	-0.1694	0.0686	0.0001	0	0.0287	-0.0060	0.0069	0.0005	0.0003
		(3.36)	(3.43)	(0.01)	—	(0.42)	(0.06)	(1.27)	(0.41)	(0.13)
		0.2931	-0.2120	0.4474	0.7912	-1.9590	0.7815	—	0.0004	0.0067
		(1.95)	(4.90)	(2.58)	(8.57)	(1.92)	(0.02)	—	(2.69)	(4.07)
Financial	BBB	0.6387	0.1686	0.4582	0	-0.2490	0.4515	0.0342	0.0044	0.0089
		(7.80)	(5.37)	(10.29)	—	(0.15)	(0.02)	(0.25)	(2.33)	(3.99)
		0.2125	0.0112	0.1718	0.0947	0.0573	0.0873	—	0.0071	0.0172
		(2.13)	(0.39)	(3.60)	(11.6)	(0.19)	(0.03)	—	(12.5)	(10.1)
Corporate	A	-0.1975	0.6094	0.0005	0	-1.8568	-0.0019	0.0103	0.0002	0.0012
		(0.61)	(5.27)	(0.01)	—	(7.00)	(0.00)	(1.79)	(0.32)	(0.30)
		0.4848	-0.1206	0.0965	0.0418	-0.1767	0.0325	—	0.0034	0.0055
		(3.32)	(0.28)	(3.15)	(0.60)	(0.13)	(0.01)	—	(19.5)	(5.74)
Corporate	BBB	-0.3278	0.2385	0.0006	0	-0.5814	-0.0041	0.0320	-0.0029	0.0011
		(4.37)	(5.70)	(0.01)	—	(4.64)	(0.00)	(0.45)	(6.34)	(0.52)
		0.0480	0.0277	0.1494	0.0147	-0.1889	0.0104	—	0.0016	0.0055
		(0.47)	(0.34)	(7.50)	(0.27)	(0.05)	(0.01)	—	(17.2)	(12.4)
				(ii) Quadratic Models						
Financial	A	-0.0407	-0.0320	0.5577	0	0.5019	0.5520	0.0041	0.0003	0.0067
		(1.74)	(2.03)	(9.85)	—	(4.93)	(0.02)	(12.2)	(6.48)	(9.05)
		-0.4381	0.0742	-0.1412	0.0120	-1.2139	0.0032	—	-0.0011	0.0021
		(21.3)	(21.1)	(2.57)	(1.31)	(21.6)	(0.00)	—	(68.4)	(20.5)
Financial	BBB	0.0159	0.0628	0.1034	0	-0.2342	0.0980	0.0092	-0.0008	0.0002
		(0.37)	(13.8)	(14.5)	—	(2.30)	(0.13)	(7.3)	(4.68)	(1.01)
		-0.0261	-0.2045	0.4805	1.6345	-1.5258	1.6254	—	-0.0015	0.0325
		(0.96)	(7.89)	(9.31)	(14.2)	(8.9)	(0.07)	—	(49.6)	(12.1)
Corporate	A	0.6625	-0.0392	0.0004	0	1.1627	-0.0054	0.0039	-0.0010	0.0006
		(28.0)	(7.08)	(0.06)	—	(17.1)	(0.00)	(23.3)	(59.0)	(11.95)
		-0.2741	0.0274	-0.0621	0.1090	-0.7897	0.1001	—	-0.0008	0.0049
		(13.8)	(6.40)	(7.25)	(5.99)	(18.1)	(0.02)	—	(105.7)	(21.2)
Corporate	BBB	-0.0569	0.0087	0.0846	0	-0.1380	0.0846	0.0098	0.0006	0.0001
		(1.60)	(1.36)	(6.44)	—	(1.83)	(0.07)	(15.0)	(9.46)	(0.74)
		-0.1874	-0.0738	0.1988	0.4640	-0.8735	0.4640	—	-0.0011	0.0120
		(9.76)	(8.12)	(12.3)	(17.6)	(11.0)	(0.02)	—	(79.9)	(11.4)

Table 10
Idiosyncratic credit and liquidity risk
Entries report the third-stage parameter estimates and *t*-statistics (in parentheses) that determine the idiosyncratic credit and liquidity risk dynamics in accounting for the idiosyncratic credit spreads embedded in the low-liquidity credit default swaps. The parameters are estimated using quasi-maximum likelihood method.

		Credit Risk					Liquidity Risk				
Θ		κ_m	γ_{m0}	γ_{m1}	a_m	c_m	κ_q	γ_{q0}	γ_{q1}	a_q	b_q
(i) Affine Models											
Financial	A	0.0010	-0.0275	-0.0500	-0.0062	0.0030	0.3312	-0.1299	0.2832	0.3423	0.2822
		(0.06)	(0.51)	(0.01)	(13.9)	(26.3)	(9.55)	(0.47)	(0.16)	(1.48)	(7.89)
Financial	BBB	0.0001	-0.0732	-0.0381	-0.0150	0.0093	0.9375	-0.4433	0.9374	2.9285	2.7536
		(0.00)	(2.53)	(0.01)	(1.24)	(30.7)	(10.6)	(0.75)	(0.11)	(2.44)	(9.07)
Corporate	A	0.0012	-1.0741	-0.0577	-0.0011	0.0028	0.8452	4.9606	0.8448	6.0187	0.9083
		(0.01)	(1.58)	(0.01)	(2.00)	(9.92)	(36.34)	(4.14)	(1.38)	(6.08)	(17.1)
Corporate	BBB	0.0009	-0.0991	-0.0434	-0.0176	0.0042	0.3376	0.6776	0.2911	1.0670	0.3245
		(0.10)	(5.08)	(0.19)	(9.67)	(27.9)	(15.6)	(3.48)	(0.44)	(7.73)	(11.8)
(ii) Quadratic Models											
Financial	A	0.0008	-0.5700	-0.0502	-0.0067	0.0038	0.9494	1.8194	0.9003	2.3761	0.8999
		(0.02)	(6.08)	(0.01)	(1.08)	(45.2)	(28.0)	(2.72)	(0.45)	(5.82)	(8.67)
Financial	BBB	0.0001	-0.0902	-0.0381	-0.0166	0.0105	0.9070	-0.4327	0.9070	2.9863	2.6709
		(0.00)	(2.52)	(0.01)	(4.01)	(26.5)	(17.0)	(0.91)	(0.12)	(3.50)	(11.6)
Corporate	A	0.0102	-0.2178	-0.0399	-0.0043	0.0052	0.6745	2.9258	0.6735	4.2633	0.8047
		(0.11)	(1.40)	(0.01)	(1.49)	(17.6)	(28.3)	(3.46)	(1.60)	(5.38)	(14.0)
Corporate	BBB	0.0068	-0.2343	-0.0087	-0.0229	0.0082	0.5273	3.3088	0.5204	2.7831	0.3963
		(0.22)	(6.28)	(0.00)	(0.56)	(17.8)	(39.4)	(8.57)	(1.61)	(19.1)	(19.1)

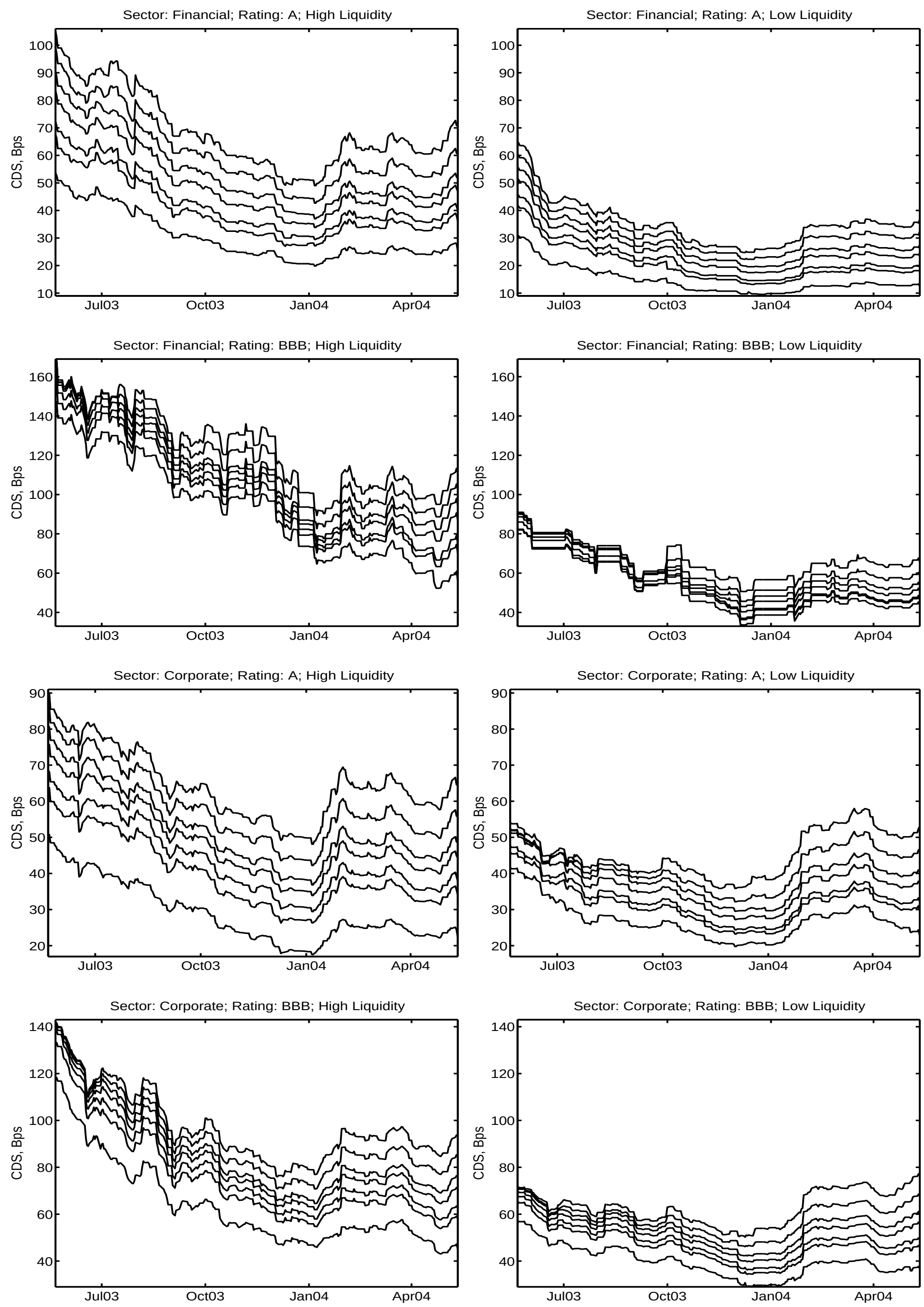

Figure 1

Time series of credit default swap spreads.

The seven lines in each panel plot the time-series of the average quotes on credit default swap spreads at seven fixed maturities for each industry sector, credit rating class, and liquidity group. Data are daily from May 21, 2003 to May 12, 2004, 256 days per series. For ease of comparison, we use the same scaling for the two liquidity groups under each industry sector and credit rating class.

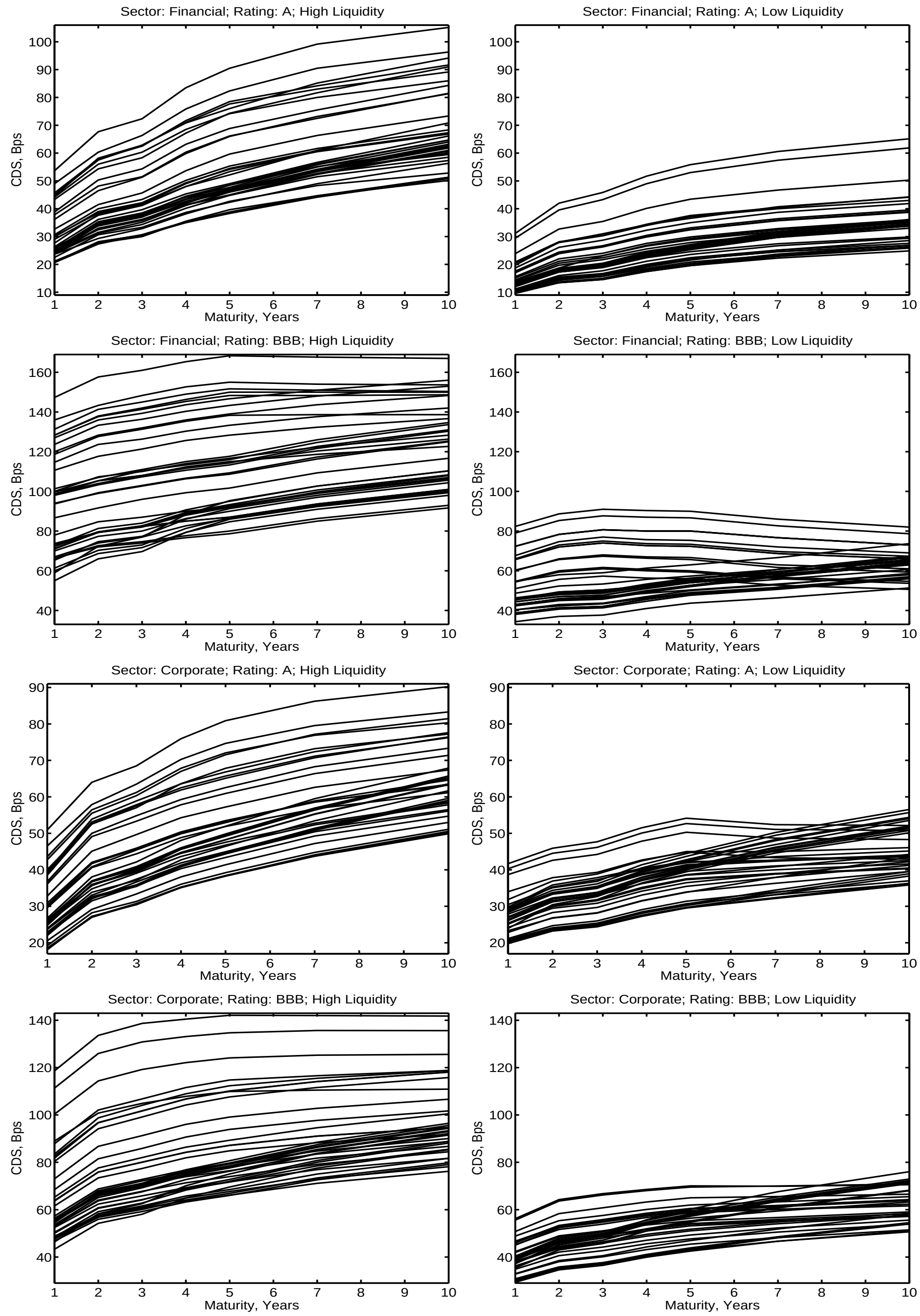

Figure 2

Term structure of credit default swap spreads.

Lines in each panel plot the term structure of the average quotes on credit default swap spreads at different days for each credit rating class, industry sector, and liquidity group. Data are daily from May 21, 2003 to May 12, 2004, 256 days per series. To reduce clustering, we plot one term structure every seven days for each panel. For ease of comparison, we use the same scaling for the two liquidity groups under each industry sector and credit rating class.

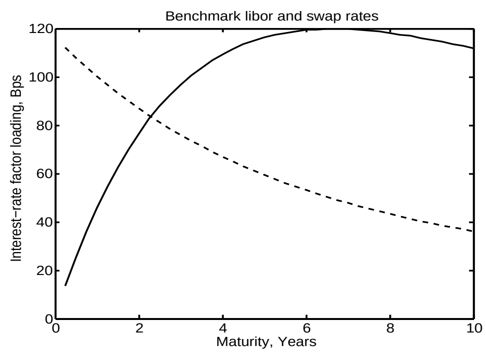

Figure 3
Benchmark interest rate factor loading.
Solid line denotes the contemporaneous response of the continuously compounded benchmark spot rate to unit shocks from the first interest-rate factor. The dashed line plots the response to unit shocks from the second factor. The loadings are computed based on the estimated affine model of benchmark interest rate.

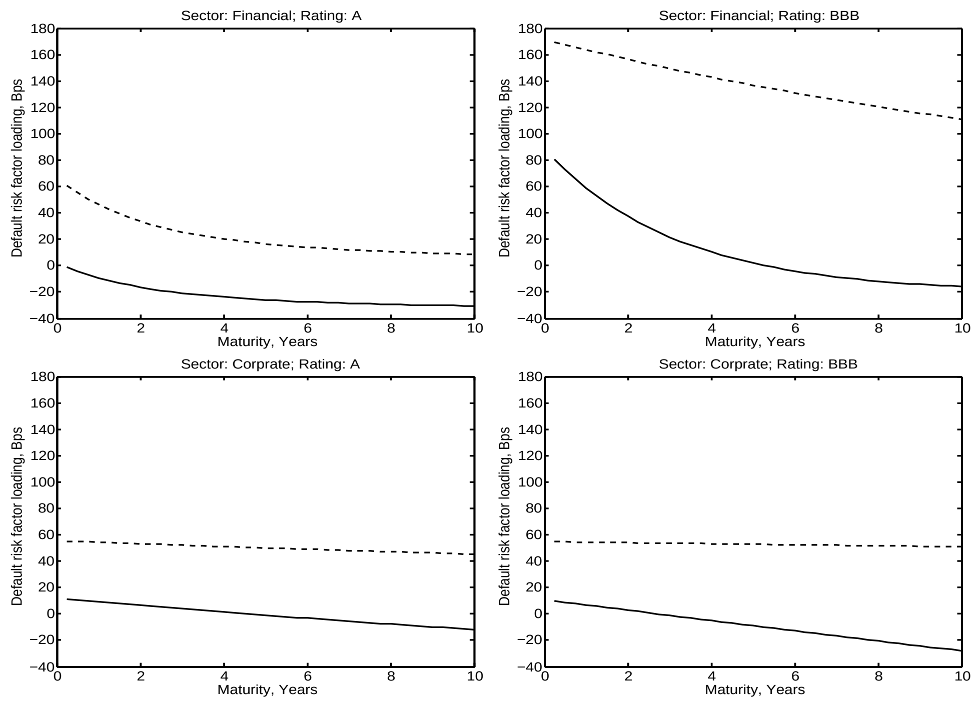

Figure 4
Default risk factor loading.
Solid lines denote the contemporaneous response of the continuously compounded corporate spot rate to unit shocks from the first default risk factor. The dashed line plots the response to unit shocks from the second default risk factor. The loadings are computed based on the parameter estimates of the two-factor affine model of default risk.

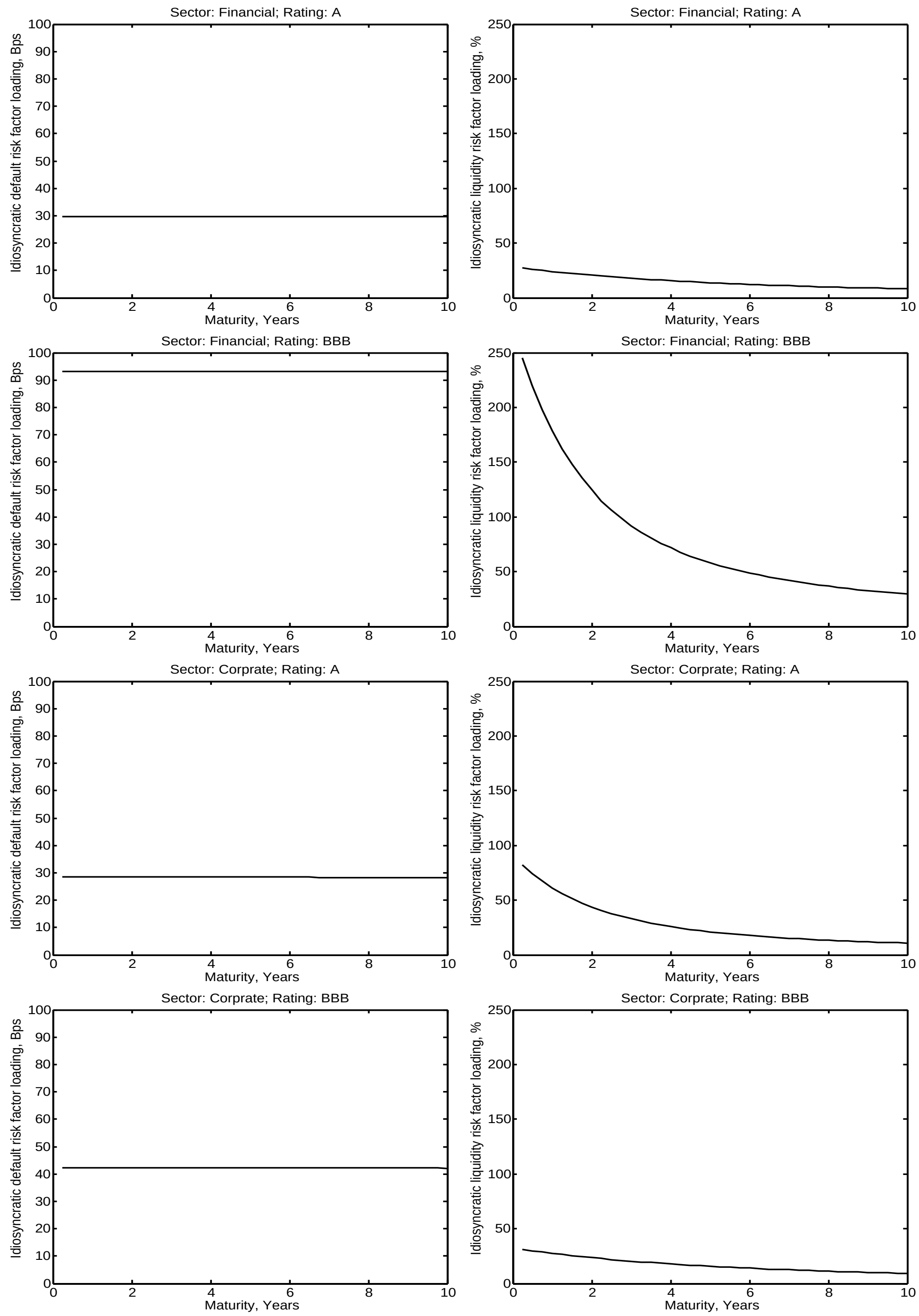

Figure 5

Idiosyncratic default risk and liquidity risk factor loading.

Lines in the left panel plot the contemporaneous response of the spot rate on the low-liquidity group to unit shocks from the idiosyncratic default risk factor. Lines in the right panel plot the response to unit shocks from the liquidity risk factor.