Synthetic CDO’s and Basket Default Swaps in a Fixed Income Credit Portfolio

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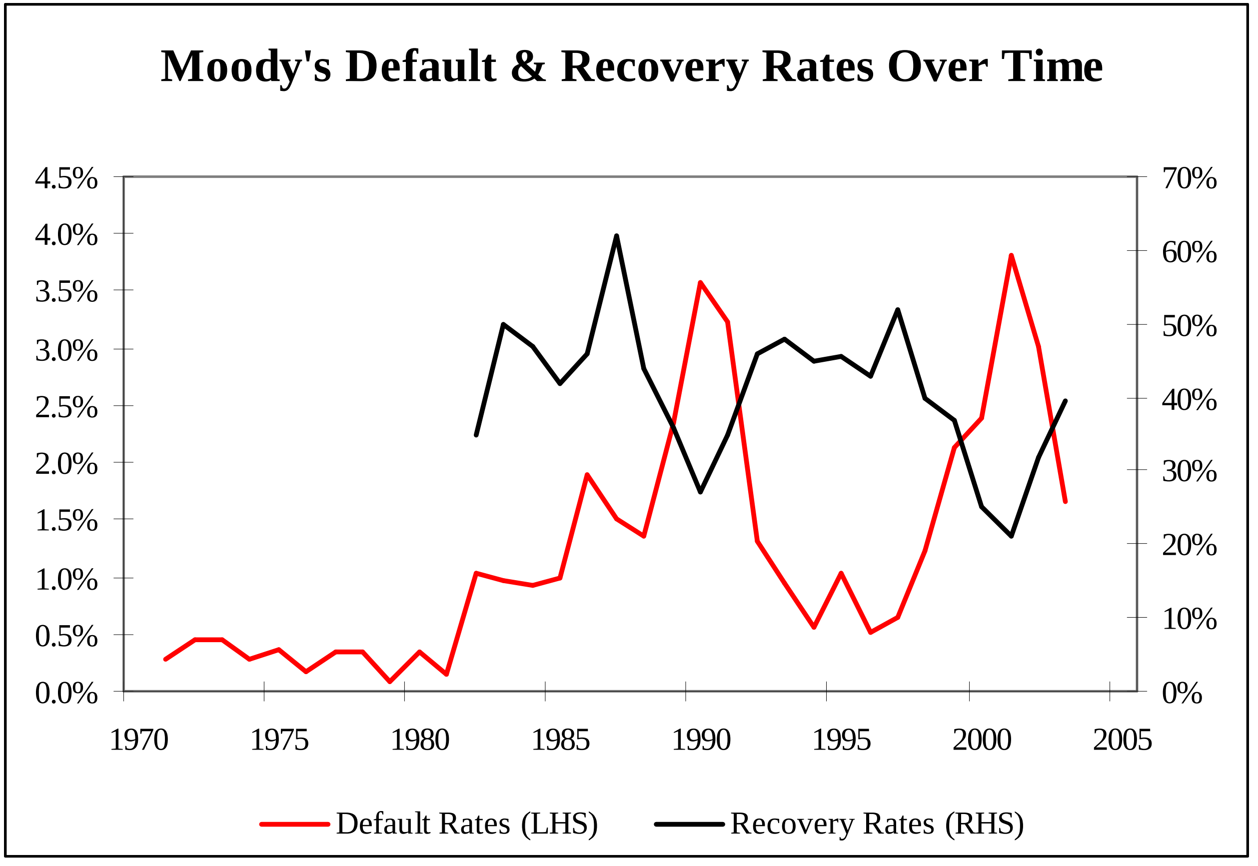
June 2005

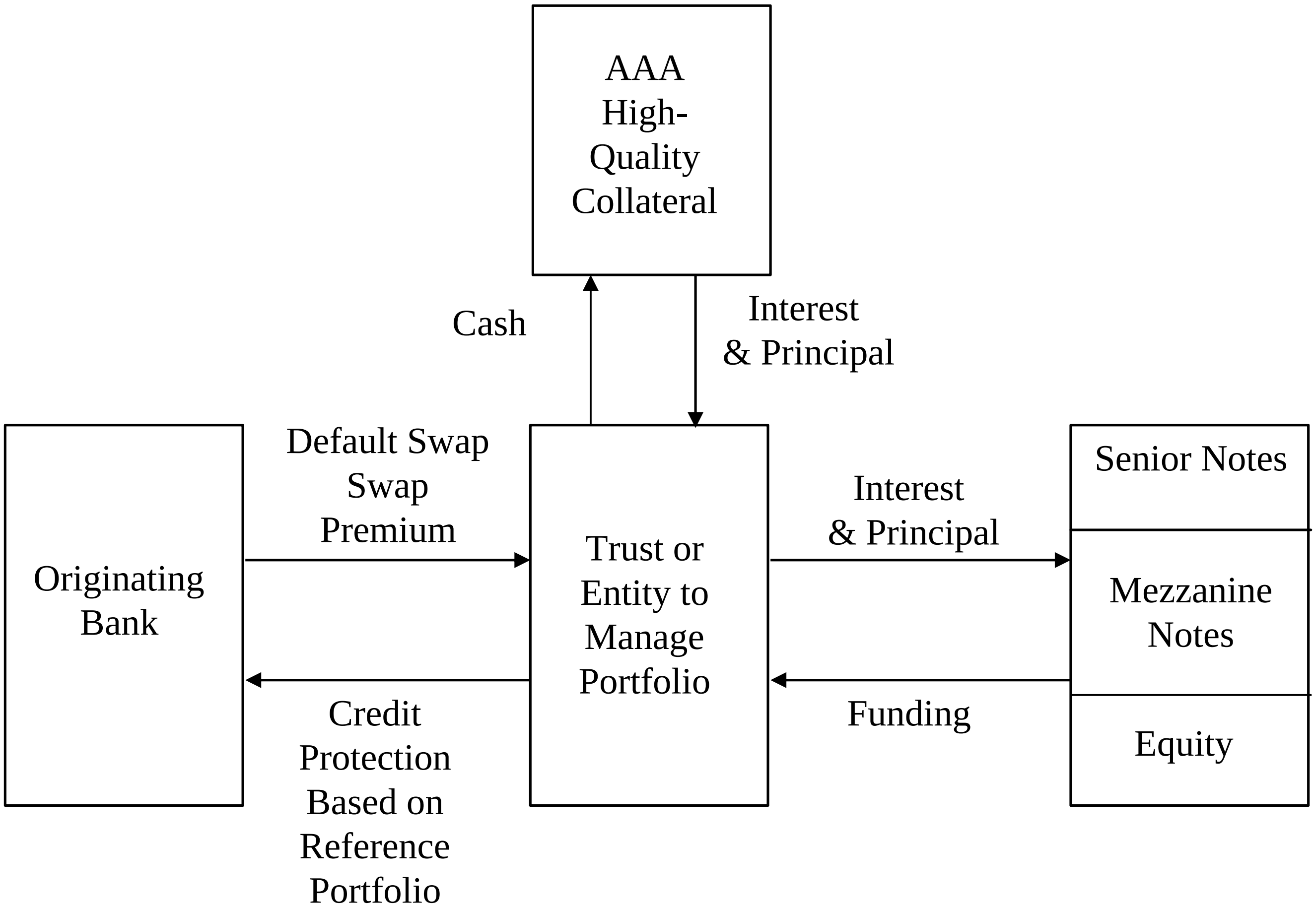
Credit Derivative Products

- CDO Notes
  - Cash & Synthetic CDO's, various tranches
  - Investment Grade Corporate names, High Yield Corporates, Emerging Market Credits, Corporate Loans
- Credit Derivatives
  - Single name default swaps
  - Basket Loss Protection
  - First to Default, Second to Default, …
- Portfolio Management
  - Synthetic CDO provide leverage and enhanced spreads
  - Model Correlation for Relative Value Trading
  - Elements of gap risk and convexity risk
  - Stress Tests: what happens to tranches when one or several names go into financial distress?

Correlation of Default and Correlation Between Default and Recovery



Typical Synthetic CDO Structure



Example of a Synthetic CDO Structure

Managed Synthetic CDO, Partially Funded, $250MM Collateral (Reference Portfolio, $1,000MM)

Tranche	Losses From	To	Cap. Structure	Spread	Rating
Supersenior swap	250MM	1,000MM	25%-100%	0.03%	AAAA
Mezzanine I	136MM	250MM	13.6%-25%	0.65%	AAA
Mezzanine II	96MM	136MM	9.6%-13.6%	1.00%	AA
Mezzanine III	71MM	96MM	7.1% -9.6%	1.75%	A
Mezzanine IV	32.5MM	71MM	3.25%-7.1%	2.85%	BBB
First Loss	0	32.5MM	0%-3.25%	Equity	

Quotes for Tranched Dow-Jones Trac-X 5 Year March 2004

Tranche	Rating	Bid	Offer	Correlation (Old)
Trac-X N.A. 5y II	A- to BBB+	69	70	-
0%-3%	Equity *	44.4%	48.4%	20%
3%-7%	BBB-	424	464	3%
7%-10%	AA+	138	158	16%
10%-15%	AAA	60	70	21%
15%-30%	AAA+	12	15	26%

* Equity premium is an upfront premium, plus 500 bp running.

Quotes for Tranched CDX 5 Year (U.S.) December 2004

Tranche	Rating	Bid	Offer	Correlation (Base)
CDX N.A. 5y	A- to BBB+	46 bp		
0%-3%	Equity *	30.3%	31.3%	19%
3%-7%	BBB-	179	188	30%
7%-10%	AA+	67	72	35%
10%-15%	AAA	22	26	44%
15%-30%	AAA+	8	9	66%

* Equity premium is an upfront premium, plus 500 bp running.

Quotes for Tranched CDX 5 Year (U.S.)
June 2005

Tranche	Rating	Bid	Offer	Correlation (Base)
CDX N.A. 5y	A- to BBB+	56 bp		
0%-3%	Equity *	47.0%	47.3%	10%
3%-7%	BBB-	157	161	28%
7%-10%	AA+	47	48	37%
10%-15%	AAA	19	21	49%
15%-30%	AAA+	10	11	73%

* Equity premium is an upfront premium, plus 500 bp running.

Quotes for Tranched CDX 10 Year (U.S.)
December 2004

Tranche	**Rating**	**Bid**	**Offer**	**Correlation (Base)**
CDX N.A. 10y	A- to BBB+	68		
0%-3%	Equity *	52.5%	56.0%	18%
3%-7%	BBB-	500	535	26%
7%-10%	AA+	185	210	32%
10%-15%	AAA	85	98	42%
15%-30%	AAA+	30	35	63%

* Equity premium is an upfront premium, plus 500 bp running.

Quotes for Tranched CDX 10 Year (U.S.)
June 2005

Tranche	Rating	Bid	Offer	Correlation (Base)
CDX N.A. 10y	A- to BBB+	80		
0%-3%	Equity *	64.0%	65.0%	8%
3%-7%	BBB-	745	755	14%
7%-10%	AA+	173	180	25%
10%-15%	AAA	49	53	42%
15%-30%	AAA+	27	33	74%

* Equity premium is an upfront premium, plus 500 bp running.

Quotes for Tranched DJ Europe iTraxx-X, 5 Year December 2004

Tranche	Rating	Bid	Offer	Correlation (Base)
DJ Europe iTraxx 5y	A- to BBB+	35.25	35.75	
0%-3%	Equity *	22.9%	23.4%	19.5%
3%-6%		130	132	29.3%
6%-9%		42.5	44.5	37%
9%-12%		26	27.5	42.6%
12%-22%		13.5	14.5	56.5%

* Equity premium is an upfront premium, plus 500 bp running.

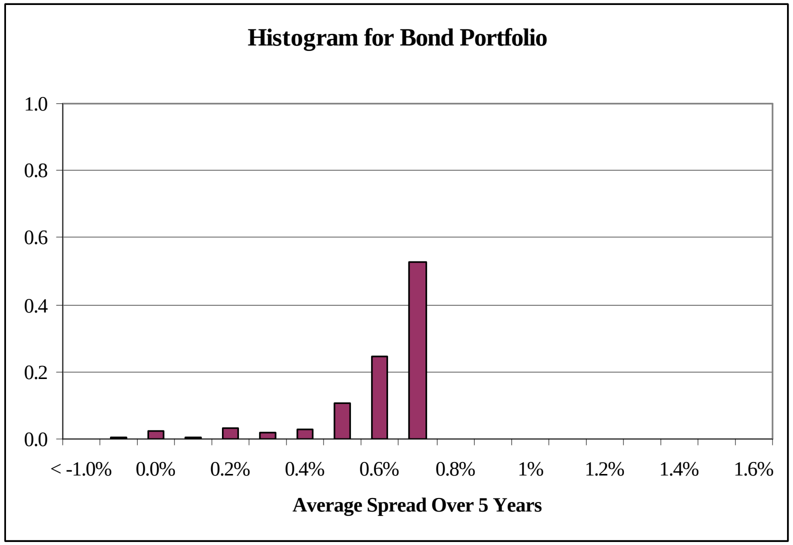
Quotes for Tranched DJ Europe iTraxx-X, 10 Year December 2004

Tranche	Rating	Bid	Offer	Correlation (Base)
DJ Europe iTraxx 5y	A- to BBB+	52	53	
0%-3%	Equity *	45.5%	46.75%	17.7%
3%-6%		362	377	26%
6%-9%		148	155	33%
9%-12%		83	88	39%
12%-22%		44	47	52.4%

* Equity premium is an upfront premium, plus 500 bp running.

Simulated Portfolio Returns Over 5 Years
Example from March 2004

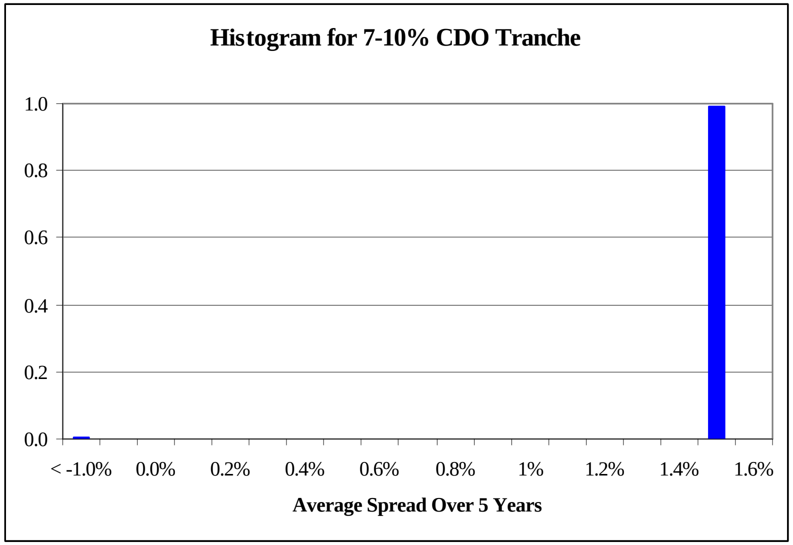
Portfolio of 100 Bonds



Average Spread 0.59%

Standard Deviation 0.24%

7-10% CDO Tranche of Portfolio

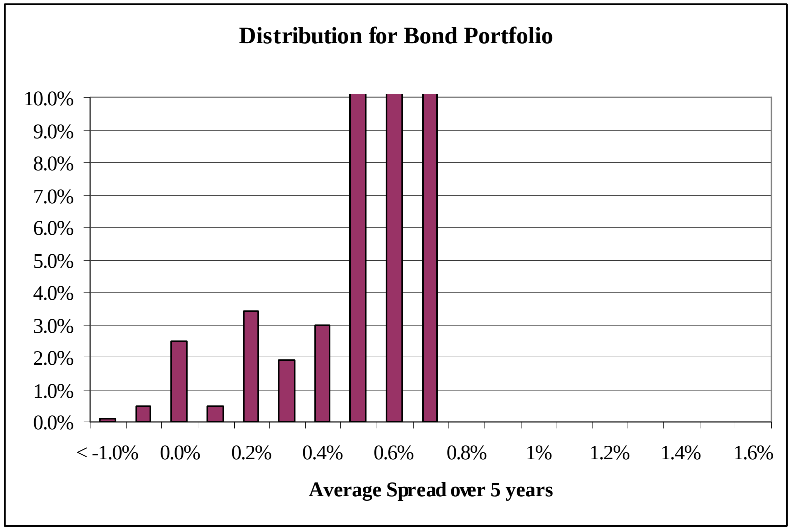


Average Spread 1.45%

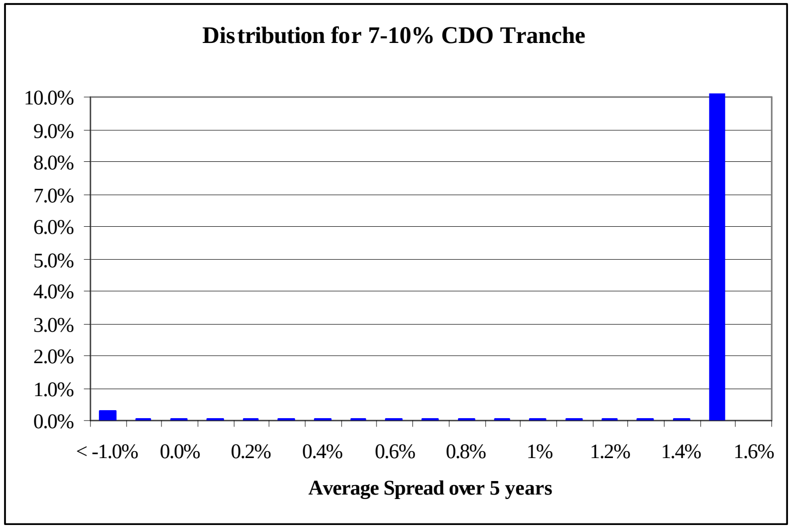
Standard Deviation 0.68%

Focus on Left Tail of Distribution

Portfolio of 100 Bonds



7-10% CDO Tranche of Portfolio



Valuation Models for Basket Trades

- CDO Notes
  - Valuation is the value of the collateral (at par) plus the value of selling basket loss protection.
- Valuation of Basket Loss Protection
  - Gaussian copula model has become the industry standard.
  - Calibrate risk neutral survival probability curves for individual names to quotes for single name default swaps.
  - Set correlation of default across names in the portfolio and simulate multivariate normal random variables.
  - For each simulation path, compute correlated uniform (0,1) simulations by applying the cumulative normal distribution function to the simulated normal random variables.
  - Use the correlated uniform random numbers and the survival probability functions to compute simulated time to default for each name in the portfolio.
  - Compute relevant cashflows and discounted present value for each tranche.
  - Simulated model is consistent with the pricing of single name default swaps.

Deficiencies of the Copula Model

- What is the correlation of default?
- The model can handle different correlations across the names in the portfolio, but how should these correlations be set? Most users set the correlations to be equal across all names.
- A single correlation is not consistent with observed market quotes for basket tranches. Typically, each tranche has its own market implied correlation.
- The model can handle random recovery, but it cannot handle correlation between default and recovery.
- Current procedures for extracting risk neutral survival probability functions do not work if default and recovery are correlated. New models would be necessary for valuing single name default swaps.

Why does the market put a pricing premium on default correlation?

- Alternative view. Finance theory tells us that markets put risk premiums into prices as compensation for risk. In a time-state preference approach to asset pricing (Arrow-Debreu), the market increases the subjective probabilities for those states in which marginal utility of wealth is high, relative to those states associated with lower marginal utility of wealth. The marginal utility of wealth for risk averse investors is high when wealth is low, or when values of market portfolios are low.

- Default rates increase during recessions when market portfolios are falling in value. Investors impute higher risk neutral probabilities for corporate defaults. Default and recovery rates are correlated so that when massive defaults occur, recovery rates will be lower.

- These rare states when default rates are high and recovery rates are low get higher risk-neutralized probabilities. The net result is higher value for default protection against these rare events.

- Copula models capture this risk premium by increasing the correlation of default, to effectively increase the probability of simulating massive defaults. But the model misses the correlation between default and recovery.

Time permitting, sketch a model with correlated default and recovery

- Model for pricing single name default swaps. Let *h* be the default intensity, so that

  $h\,\Delta t$ is the default probability over a small time interval Δt.

  Survival probabilities are $\quad \Pr(t,T) \;=\; \hat{E}_t\left(e^{-\int_t^T h(s)\,ds} \right).$

- Default swap rates are set so that the net present value of the default swap is zero. $D(t,T)$ is the discounting function, Prem is the default swap premium, and Rec is the recovery rate. Swap premiums are over N time intervals, and defaults are assumed to occur at a set of M discrete time points.

$$PV \;=\; \sum_{\mathrm{j=1}}^{N} \mathrm{Prem}(t_j)\, D(t,t_j)\, \Pr(t,t_j) \;-\; \sum_{k=1}^{M} \left[1-\hat{E}_t(\mathrm{Rec})\right] D(t,t_k)\left[\Pr(t,t_{k-1})-\Pr(t,t_k)\right] \;=\; 0$$

- To derive this model, one must assume that default intensities, interest rates, and recovery rates are independent.

The More General Pricing Model for Single Name Default Swaps

$$PV = \sum_{j=1}^{N} \text{Prem}(t_j)\, \hat{E}_t\left[e^{-\int_t^{t_j}(r_s+h_s)ds} \right] - \sum_{k=1}^{M} \hat{E}_t\left(\text{Loss}(t_k)\, e^{-\int_t^{t_k} r(s)ds} \left[e^{-\int_t^{t_{k-1}} h(s)ds} - e^{-\int_t^{t_k} h(s)ds} \right] \right) = 0$$

- If interest rates are independent, we can use the discounting function from the interest rate market.

$$PV = \sum_{j=1}^{N} \text{Prem}(t_j)\, D(t,t_j)\, \hat{E}_t\left(e^{-\int_t^{t_j} h(s)ds} \right) - \sum_{k=1}^{M} D(t,t_k)\, \hat{E}_t\left(\text{Loss}(t_k) \left[e^{-\int_t^{t_{k-1}} h(s)ds} - e^{-\int_t^{t_k} h(s)ds} \right] \right) = 0$$

Sketch of a Model with Correlated Default and Recovery

- Need to define stochastic processes for the factors that determine both default intensities (and spreads) and loss rates. Models that generate closed form solutions for single name default swaps will be easier to calibrate. Need to model correlations across spreads.

- Calibrate model to single name default swap market and quotes for basket loss protection.

- Look for relative value opportunities across different tranches and across different portfolios.