Evolving Risk Aversion and the Evidence On Constant Relative Risk Aversion

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Abstract

Published studies based on expected utility assume that the utility function of the decision maker at the time that uncertainty is resolved is known at the time the decision is made. State dependent utility models admit that the utility function at the end may be a function of the result of the decisions. This paper begins to explore the consequences of allowing the utility function to evolve over the period from decision to resolution of uncertainty. It does so by assuming that the decision maker has an underlying utility with constant absolute risk aversion but the coefficient of absolute risk aversion at the time of resolution of uncertainty is not known at the time the decision must be made. The analysis presented assumes that this evolution is independent of the outcome of the decision. The results show that under these conditions the individual behaves as though the coefficients of absolute risk aversion, prudence, and temperance were decreasing as wealth increases. Different distributions of the terminal coefficient of absolute risk aversion give rise to different shapes for the effective utility.

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1. Introduction

The economics of decisions under uncertainty were many decades dominated by the model of maximization of expected utility. Much has been written from that perspective, and it has proved a fertile ground for ideas. A substantial volume of literature, both theoretical and empirical, also exists on the form of the utility curve and on the behavior of the Arrow-Pratt coefficients of risk aversion. In particular, the empirical literature appears to be solidly behind the notion that the coefficient of absolute risk aversion decreases with wealth whereas the coefficient of relative risk aversion appears to increase with wealth.

The framework used for these analyses is one in which the utility curve is stationary. In principle the utility function whose expected value is to be maximized is the function at the end of the study period, that is, when all uncertainty has been resolved. That is clear from the existence of models with "state dependent utility functions" in which the utility is different in the different terminal states of the world. State dependent utility functions, however, depend only on the terminal state and are fixed by that state. A more general formulation might allow events that occur between the time of decision and the time of resolution of uncertainty, other than the terminal state, to affect the utility function. We might entertain the hypothesis that events such as development of technology, advertising, wars, or periods of economic boom or depression might affect at least the parameters of the utility function, if not its form. If the form of the utility function at the time of resolution of uncertainty is itself uncertain at the time the decision must be made, then the expected utility must be computed by taking expectation not only over the possible outcomes resulting from the decision but also over the possible utility functions that might emerge in the interval.

This paper simplifies matters considerably by assuming that the *form* of the utility function is time-invariant and represents constant absolute risk aversion. Hence I use the terminology of evolving risk aversion rather than the more general terminology of evolving utility functions [1]. With that broad simplifying assumption it is sufficient to extend the expectation of the coefficient of risk aversion. That leads to some tractable results for the general case of an unspecified distribution of the future coefficient of absolute risk aversion. Notably, under these conditions the decision maker behaves as though the coefficient of absolute risk aversion decreases with wealth. These will be presented in the next section. Section 3 presents some simple examples of the distribution of the terminal risk aversion coefficient and relates these to specific forms of utility functions with constant coefficients. Section 4 uses these examples to illustrate the way in which constant absolute risk aversion with evolving coefficients might be distinguished from the identical form with constant coefficients. Section 5 points out a flaw in the usual empirical analyses from which we infer decreasing absolute risk aversion.

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[1] Keeney and Raiffa, 1976 briefly discuss the role of evolving utility in decisions and the reasons for anticipating such evolution.

2. Decisions with Evolving Risk Aversion

2.1 Evaluating the Expected Terminal Utility

Suppose a decision maker with constant absolute risk aversion wants to maximize utility at the end of the period. The individual's utility function, given the coefficient of risk aversion, is:[2]

$$(1) \qquad \mathrm{U}(W) = \frac{1-\mathrm{e}^{-\rho W}}{\rho}$$

The twist is that the decision maker knows that his risk aversion coefficient may change in response to increasing biological age[3], changing needs, advertising, and other events[4] that take place during the interval between decision and resolution. Under those conditions we may first express the expected utility given the terminal value of $\rho(\tau)$, that is, the value of the absolute coefficient of risk aversion when the outcome of the decision is known:

$$(2) \qquad \mathrm{E}[\mathrm{U}(W) \mid \rho(\tau)] = \int_W \mathrm{U}(W)\mathrm{dF}_W = \int_W \frac{1-\mathrm{e}^{-\rho(\tau)W}}{\rho(\tau)}\mathrm{dF}_W .$$

The conditional expected value theorem ($\mathrm{E}[\mathrm{U}(W)] = \mathop{\mathrm{E}}_{\rho(\tau)}\left[\mathrm{E}\left(\mathrm{U}(W \mid \rho(\tau))\right)\right]$) allows us to express the unconditional expected utility in terms of the conditional expected utility:

$$(3) \qquad \mathrm{E}[\mathrm{U}(W)] = \int_{\rho(\tau)} \mathrm{E}[\mathrm{U}(W) \mid \rho(\tau)]\mathrm{dG}(\rho(\tau)) = \int_{\rho(\tau)}\int_W \frac{1-\mathrm{e}^{-\rho(\tau)W}}{\rho(\tau)}\mathrm{dF}_W \mathrm{dG}(\rho(\tau))$$

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[2] This formulation has utility of zero and marginal utility of one when the wealth is zero and has the virtue that $\lim_{\rho\to 0}\mathrm{U}(W) = W$. It differs from the more usual $\mathrm{e}^{-\rho W}$, which does not share these properties, by what might be thought to be an affine transformation. Since the analysis views the parameter as a random variable, however, the two do not lead to identical coefficients of risk aversion; they differ by a constant. The expression used here leads to a simpler set of results.

[3] Chronological age is deterministic, so its effects could be predicted with certainty. Biological aging includes the effect of random insults to health and so it and its effects are not exactly predictable.

[4] I assume that risk aversion does not depend on the outcome of his decisions. Interdependence would require a more complex formulation because the joint distribution of outcome and terminal risk aversion would have to be considered.

Interchanging the order of integration we find:

$$(4) \qquad \mathrm{E}[\mathrm{U}(W)] = \int_W \left[\int_{\rho(\tau)} \frac{1-\mathrm{e}^{-\rho(\tau)W}}{\rho(\tau)} \mathrm{dG}(\rho(\tau)) \right] \mathrm{dF}_W$$

The interpretation of this relation is interesting: under evolving risk aversion the decision maker with constant absolute risk aversion will behave as though her utility function is:

$$(5) \qquad \mathrm{U}^*(W) = \int_{\rho(\tau)} \frac{1-\mathrm{e}^{-\rho(\tau)W}}{\rho(\tau)} \mathrm{dG}(\rho(\tau))$$

2.2 Dependence of the Effective Utility Function on Wealth

From this relation we can infer how the decision maker's actions will appear to an observer that does not take into account the uncertainty in the terminal coefficient of absolute risk aversion. Taking derivatives of the effective utility function with respect to wealth we find that

$$(6) \qquad \frac{\partial^n \mathrm{U}^*(W)}{\partial W^n} = -\int_{\rho(\tau)} \rho^{n-1}(\tau)\, \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau))$$

I define the n^{th} characteristic of the utility function U^* as $\mathrm{C}_n(\mathrm{U}^*) = -\frac{\partial^{n+1}\mathrm{U}^*(W)/\partial W^{n+1}}{\partial^n \mathrm{U}^*(W)/\partial W^n}$,

so that $\mathrm{C}_1(\mathrm{U}^*)$ is the risk aversion coefficient, $\mathrm{C}_2(\mathrm{U}^*)$ is the prudence, $\mathrm{C}_3(\mathrm{U}^*)$ is the temperance of the utility function $\mathrm{U}^*(W)$. From the above it follows that:

$$\mathrm{C}_n\left(\mathrm{U}^*\right) = -\frac{\int_{\rho(\tau)} \rho^n(\tau)\, \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau))}{\int_{\rho(\tau)} \rho^{n-1} \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau))}$$

The behavior of these characteristics as wealth increases have major effects on economic behavior. For CARA utilities, all these characteristics are equal to the coefficient of risk aversion and are independent of wealth. By taking the derivative with respect to W we find:

$$(9) \quad \frac{\partial \mathrm{C}_n}{\partial W} = \frac{\left(\int\limits_{\rho(\tau)} \rho^n(\tau)\, \mathrm{e}^{-\rho(\tau)W}\, \mathrm{dG}(\rho(\tau))\right)^2 - \int\limits_{\rho(\tau)} \rho^{n-1} \mathrm{e}^{-\rho(\tau)W}\, \mathrm{dG}(\rho(\tau)) \int\limits_{\rho(\tau)} \rho^{n+1}(\tau)\, \mathrm{e}^{-\rho(\tau)W}\, \mathrm{dG}(\rho(\tau))}{\left(\int\limits_{\rho(\tau)} \rho^{n-1} \mathrm{e}^{-\rho(\tau)W}\, \mathrm{dG}(\rho(\tau))\right)^2}$$

Letting $\rho^{n-1}\mathrm{e}^{-\rho(\tau)W} = X_n{}^2$ and $\rho^{n+1}(\tau)\mathrm{e}^{-\rho(\tau)W} = Y_n{}^2$ we express this as:

$$(10) \quad \frac{\partial \mathrm{C}_n}{\partial W} = \frac{\left(\mathrm{E}[X_n Y_n]\right)^2 - \mathrm{E}[X_n{}^2]\mathrm{E}[Y_n{}^2]}{\left(\mathrm{E}[X_n{}^2]\right)^2}$$

It follows from the Cauchy-Schwarz inequality that $\frac{\partial \mathrm{C}_n}{\partial W} \leq 0$ for all *n*, with the equality holding only if the distribution is degenerate. Hence in particular the coefficients of absolute risk aversion, prudence and temperance are all decreasing in wealth[5] for every non-degenerate distribution if the underlying utility function is CARA. It follows that the decision maker with constant absolute risk aversion whose coefficient may evolve through the period before uncertainty is resolved will appear to act as though her risk aversion, prudence and temperance were all decreasing with increasing wealth.

Moreover, we have:

[5] The fact that the risk aversion coefficient is is decreasing in wealth could be inferred directly from the results of Pratt, 1964, which shows that mixtures of utility functions with constant absolute risk aversion have decreasing absolute risk aversion. The proof given above generalizes that result to other characteristics of mixtures of utility functions with constant absolute risk aversion.

$$\mathrm{C}_n - \mathrm{C}_{n-1} = \frac{\int\limits_{\rho(\tau)} \rho^n(\tau)\, \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau))}{\int\limits_{\rho(\tau)} \rho^{n-1} \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau))} - \frac{\int\limits_{\rho(\tau)} \rho^{n-1}(\tau)\, \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau))}{\int\limits_{\rho(\tau)} \rho^{n-2} \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau))}$$

$$= \frac{\int\limits_{\rho(\tau)} \rho^n(\tau)\, \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau)) \int\limits_{\rho(\tau)} \rho^{n-2}(\tau)\, \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau)) - \left(\int\limits_{\rho(\tau)} \rho^{n-1}(\tau)\, \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau)) \right)^2}{\int\limits_{\rho(\tau)} \rho^{n-1} \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau)) \int\limits_{\rho(\tau)} \rho^{n-2} \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau))}$$

If we let $\quad \rho^n(\tau)\, \mathrm{e}^{-\rho(\tau)W} = U^2$

and $\quad \rho^{n-2}(\tau)\, \mathrm{e}^{-\rho(\tau)W} = V^2$

then $\quad \rho^{n-1}(\tau)\, \mathrm{e}^{-\rho(\tau)W} = UV$

it follows that:

$$\mathrm{C}_n - \mathrm{C}_{n-1} = \frac{\mathrm{E}(U_n{}^2)\mathrm{E}(V_n{}^2) - \mathrm{E}^2(U_n V_n)}{\int\limits_{\rho(\tau)} \rho^{n-1} \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau)) \int\limits_{\rho(\tau)} \rho^{n-2} \mathrm{e}^{-\rho(\tau)W} \mathrm{dG}(\rho(\tau))}$$

The denominator is positive since the integrands are positive over the support for $\rho(\tau)$. The denominator is negative by the Cauchy-Schwartz inequality. This proves that for this family of effective utility functions $\mathrm{C}_n - \mathrm{C}_{n-1} \geq 0$ for all *n*.

3. Some Examples of Effective Utility Functions

The results shown above indicate that any distribution of the terminal coefficient of risk aversion will generate an effective utility function with decreasing absolute risk aversion, provided the expectation converges. A few examples are useful in showing the correspondences.

3.1 Isolated Support Points for the Terminal Coefficient

Consider a decision maker with constant absolute risk aversion who believes that her terminal risk aversion coefficient at time τ will be $\rho_i(\tau)$ with probability $p_i(\tau)$. The notation emphasizes that both the support points and their probability could be functions of time scale of the decision. The effective utility function for this decision maker will be:

$$\text{(11)} \qquad \mathrm{U}^*(W)=\sum_{\mathrm{n=1}}^{\mathrm{N}} p_n(\tau)\frac{1-\mathrm{e}^{-\rho_n(\tau)W}}{\rho_n(\tau)}$$

Thus the effective utility function corresponding to multiple support points is the weighted average of utilities with different coefficients of constant absolute risk aversion.

3.2 Gamma Distribution for the Terminal Coefficient

Suppose the decision maker believes that the parameter at the end of the period will have a gamma distribution with expected value of $\hat{\rho}(\tau)$ and standard deviation of $\hat{\sigma}(\tau)$. Hence:

$$\text{(12)} \qquad g\big(\rho(\tau)\big)=\frac{\nu^{\alpha}}{\Gamma(\alpha)}\rho(\tau)^{\alpha\text{-}1}\mathrm{e}^{-\nu\rho}$$

where $\nu=\dfrac{\hat{\rho}(\tau)}{\hat{\sigma}^2(\tau)}$ and $\alpha=\dfrac{\hat{\rho}^2(\tau)}{\hat{\sigma}^2(\tau)}$.[6]

After some calculations we find that

$$\text{(13)} \qquad \mathrm{U}^{*'}(W)=\frac{\nu^{\alpha}}{\Gamma(\alpha)}\int \mathrm{e}^{-\rho(\tau)W}\rho^{\alpha\text{-}1}(\tau)\mathrm{e}^{-\nu\rho(\tau)}\mathrm{d}\rho(\tau)=\frac{\nu^{\alpha}}{\Gamma(\alpha)}\frac{\Gamma(\alpha)}{(\nu+W)^{\alpha}} \quad \text{and}$$

$$\text{(14)} \qquad \mathrm{U}^{*''}(W)=-\frac{\nu^{\alpha}}{\Gamma(\alpha)}\int \mathrm{e}^{-\rho(\tau)W}\rho^{\alpha}(\tau)\mathrm{e}^{-\nu\rho(\tau)}\mathrm{d}\rho(\tau)=\frac{\nu^{\alpha}}{\Gamma(\alpha)}\frac{\Gamma(\alpha+1)}{(\nu+W)^{\alpha+1}}$$

From these relations we obtain:

$$\text{(15)} \qquad \mathrm{EC}_{ARA}=-\frac{U^{*''}(W)}{U^{*'}(W)}=\frac{\Gamma(\alpha+1)}{\left(\nu+W\right)^{\alpha+1}}\frac{\left(\nu+W\right)^{\alpha}}{\Gamma(\alpha)}=\frac{\alpha}{\nu+W}=\frac{\hat{\rho}^2(\tau)}{\hat{\rho}(\tau)+W\hat{\sigma}^2(\tau)}$$

and, correspondingly,

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[6] Note that if $\rho_0>\sigma_0^2$ then $\alpha<1$ and the integral defining the effective utility does not converge.

$$(16) \qquad \mathrm{EC}_{RRA} = \frac{W\alpha}{\nu + W} = \frac{W\hat{\rho}^2(\tau)}{\hat{\rho}(\tau) + W\hat{\sigma}^2(\tau)}$$

From these we find that

$$(17) \qquad \frac{\partial \mathrm{EC}_{ARA}}{\partial W} = -\frac{\hat{\rho}^2(\tau)\hat{\sigma}^2(\tau)}{\left(\hat{\rho}(\tau) + W\hat{\sigma}^2(\tau)\right)^2} < 0$$

$$(18) \qquad \frac{\partial \mathrm{EC}_{RRA}}{\partial W} = \frac{\hat{\rho}^3(\tau)}{\left(\hat{\rho}(\tau) + W\hat{\sigma}^2(\tau)\right)^2} > 0$$

The effective coefficient of absolute risk aversion declines with wealth whereas the effective coefficient of relative risk aversion increases with wealth.

The effective utility function belongs to the family with hyperbolic absolute risk aversion coefficients (HARA family). If the standard deviation of the terminal risk aversion coefficient increases with time more rapidly than the expected value, this effective utility function approaches one with constant relative risk aversion as the time horizon for the decision becomes longer.

If the variance of the risk aversion coefficient is small that then the effective coefficient of absolute risk aversion is $\mathrm{EC}_{ARA} = \frac{\hat{\rho}^2(\tau)}{\hat{\rho}(\tau) + W\hat{\sigma}^2(\tau)} \approx \hat{\rho}(\tau)$ and the effective coefficient of relative risk aversion is $\mathrm{EC}_{RRA} = \frac{W\hat{\rho}^2(\tau)}{\hat{\rho}(\tau) + W\hat{\sigma}^2(\tau)} \approx W\hat{\rho}(\tau)$. So if the variance is very much smaller than the expected value, the traditional view provides a reasonable approximation. On the other hand, if $\hat{\sigma}^2(\tau)\mathrm{Max}(W) >> \hat{\rho}(\tau)$ then $\mathrm{EC}_{ARA} \approx \frac{1}{W}\frac{\hat{\rho}^2(\tau)}{\hat{\sigma}^2(\tau)}$ and $\mathrm{EC}_{RRA} \approx \frac{\hat{\rho}^2(\tau)}{\hat{\sigma}^2(\tau)}$ so the world is turned on end and what we might interpret as constant *relative* risk aversion is actually evidence of an underlying utility with constant *absolute* risk aversion.

The rates at which the effective risk aversion coefficients change as the parameters of the distribution change are of interest. For the effective coefficient of absolute risk aversion we have: $\frac{\partial \, \mathrm{EC}_{ARA}}{\partial \hat{\rho}(\tau)} = \frac{\hat{\rho}^2(\tau) + 2W\hat{\rho}(\tau)\hat{\sigma}^2(\tau)}{\left(\hat{\rho}(\tau) + W\hat{\sigma}^2(\tau)\right)^2} > 0$ and $\frac{\partial \, \mathrm{EC}_{ARA}}{\partial \hat{\sigma}(\tau)} = -\frac{2\hat{\rho}^2(\tau)\hat{\sigma}(\tau)W}{\left(\hat{\rho}(\tau) + W\hat{\sigma}^2(\tau)\right)^2} < 0$. Thus at small values of wealth a small change in the expected value of the future underlying risk aversion coefficient changes the effective coefficient of absolute risk aversion by virtually the same amount, whereas at high levels of wealth the increase in the effective coefficient is much smaller than the change in the expected value. At low levels of wealth, small increases in the estimated standard deviation of the underlying coefficient reduce the effective coefficient by only small amounts, at large levels of wealth the effect is inversely proportional to wealth.

The corresponding results for the effective coefficients of relative risk aversion are $\frac{\partial \, \mathrm{EC}_{RRA}}{\partial \hat{\rho}(\tau)} = \frac{W\hat{\rho}(\tau) + 2W\hat{\rho}(\tau)\hat{\sigma}^2(\tau)}{\left(\hat{\rho}(\tau) + W\hat{\sigma}^2(\tau)\right)^2} > 0$ and $\frac{\partial \, \mathrm{EC}_{RRA}}{\partial \hat{\sigma}(\tau)} = -\frac{2\hat{\rho}^2(\tau)\hat{\sigma}(\tau)W^2}{\left(\hat{\rho}(\tau) + W\hat{\sigma}^2(\tau)\right)^2} < 0$. Hence at low levels of wealth the effect of changes in both expected value and standard deviation of the distribution of underlying coefficients has little impact on the effective coefficient while at high levels of wealth the effects become nearly constant.

This suggests that empirical investigations would be more fruitful if we have longitudinal data on the behavior of individuals with large wealth. Longitudinal data from countries in which financial holdings are of public record could be used for this purpose.

4. Does it Make a Difference?

So far I have shown that an individual with constant absolute, but evolving, risk aversion will act as though she has declining risk aversion. This difference is meaningless, however, if there is no way of distinguishing between the hypothesis of evolving risk aversion and that of stationary risk aversion. To explore whether we might be able to tell the difference, we may consider the case of a gamma distribution of the terminal coefficient of risk aversion and that of a HARA utility function.

Consider asking an individual with a HARA utility with constant coefficients to make a decision, such as to allocate funds to a riskless asset and a risky asset with specified characteristics for a period τ. The allocation will depend on the wealth, the characteristics of the risky investment, and on the investment horizon in a way that can be specified once the parameters of the utility function and the characteristics of the risky investment are known. The decision can then be described as a function of the investment horizon by actually calculating the allocation. Asking for the decision maker with constant but evolving coefficient of risk aversion will produce a different function of the investment horizon because the parameters of the utility function are now explicitly functions of time.

Thus it is possible to define experiments which, in principle at least, can tell us whether the decision maker is behaving as though the utility function were HARA with constant coefficients or as though she had a constant risk aversion coefficient that is expected to evolve through time.

The distinction is more complicated, however, if the coefficient of risk aversion of the HARA decision maker is a function of age. In that case it would be necessary to specify the change with age and incorporate that into the analysis in order to determine the dependence of the decision on as a function of the investment horizon.

An interesting feature is that all functions generated by evolving risk aversion with an underlying CARA utility share the property that $C_n - C_{n-1} =\geq 0$. This provides at least one possibility for definitively rejecting the hypothesis that CARA with evolving risk aversion has explanatory power.

5. Empirical Evidence on the Relation between Coefficients of Risk Aversion and Wealth

The literature has a variety of papers which attempt to measure the behavior of risk aversion as a function of wealth. Meyer and Meyer, 2004, have pointed out the dangers of uncritical use of the reported values. They point out that some papers use utility functions whose argument is wealth, other use income as the argument, and other still use consumption. Moreover, wealth and income can be defined before or after taxes and wealth can include or exclude assets such as home ownership and retirement accounts. They point out that at least crude adjustments can be made to bring these measurements to a common basis and show that when this is done the measures of relative risk aversion of Arrow-Pratt wealth the differences among estimates decrease.

Most often the surveys include only one element of risk, such as investment risk or insurance coverage. If two risks exist simultaneously the artificiality of the survey can lead to peculiar results. For example, Venezian (2004) has shown that investment and insurable risks coexist and one of them is treated as exogenous, then the insurance decision is endowment dependent even if the decision maker has a CARA utility with known parameter. If both decisions are exogenous then the insurance decision is not dependent on the initial wealth. If nothing is said about the second risk and the respondent is aware of it, we do not know what assumptions are made in developing the response.

The major set of data analyzed by Meyer and Meyer comes from Friend and Blume, 1975. That classic study used data on the investments of 2,100 households, classified into six levels of wealth[7]. Adjusting the data on relative risk aversion coefficients to the Arrow-Pratt wealth measure Meyer and Meyer find coefficients of relative risk aversion of between 2 and 4, and generally decreasing with wealth. The original estimates are larger and not so uniformly decreasing. The most consistent set, based on wealth which includes housing but excludes human capital has standardized[8] coefficients of *relative* risk aversion of 2.76 for households of wealth from $1,000 to $10,000 and of 1.98 for households of wealth of over $1,000,000.

Data like these have been used to argue that the coefficient of absolute risk aversion must be rapidly decreasing with wealth. It is important to realize, however, that this study, along with others like it, should not be used for that purpose. Correlation does not imply causation. The wealth of each household is not an endowment given to the household by the researcher, it is wealth that has resulted from a series of decisions made in the past by the household. Although some of the wealth may be the result of decisions of people in previous generations who endowed it with wealth it is clear that wealth at the beginning of the study is not independent of the risk aversion characteristic of the household.

If we assume, for the sake of argument, that all households used in the study have been existence for the same length of time and all received the same endowment initially, both in funds and in human capital, then the households with lower risk aversion would have invested their funds in riskier portfolios than households with high risk aversion. Since financial theory predicts that investments with higher risks have, on average, higher returns, it follows that the households with lower risk aversion would be over-represented among the wealthy households

[7] The authors use three different definitions of wealth and the separation in done with each of the measures rather than on a uniform basis.
[8] Meyer and Meyer, 2004, standardize all their results to apply to Arrow-Pratt wealth after taxes.

and under-represented among those of lower wealth. Wealth is an endogenous result of the model, depending on the original endowment, the returns to human capital, and the risk of the investment portfolio chosen by the household. We should not interpret the wealth as being exogenous and hence we cannot draw causal conclusions about the effect of wealth on risk aversion.

6. Discussion

This example shows that parameter evolution can change quite dramatically the way in which we interpret the data. If one wanted to argue for the status quo it is simple to say that the difference is merely cosmetic, that it does not matter whether the underlying model is one of hyperbolic risk aversion with constant coefficients or one of constant risk aversion with non-stationary coefficients. On the other hand, if indeed what we are observing is a world in which parameter drift is possible then the model with evolving parameters may provide some advantages.

For example, it opens the door to research on how parameters change and to the acceptance that the form that we see at one time may not be exactly the form that we see at a different time. Under those conditions, examining the empirical data over long periods of time may provide useful hints about how stable or labile the coefficients might be. I have no illusions that this will be an easy task. Inferring the shape of the risk aversion curve involves many assumptions about what variables should be viewed as endogenous or exogenous; the existing studies assume, in essence, that correlation means causation, but the alternate explanation that wealthy people become that way because they are risk-tolerant while highly risk averse people remain of modest means because they forego expected increases in wealth for the sake of safety would lead to the conclusion that wealthier groups would prove to be more risk tolerant on average than less well off groups. Under one model wealth causes reduced risk aversion, under the other it is low risk aversion that leads to higher wealth. Both models imply negative correlation between the two variables..

The utility with constant absolute risk aversion and a gamma distribution is convenient for purposes of illustration because it leads to closed form solutions. It also provides a contrast between the underlying and effective coefficients of risk aversion. Other combinations of utility function and probability distributions will lead to different relations between the two sets of coefficients. It may be interesting to ascertain the range of effective coefficients of absolute and relative risk aversion that can be obtained from the assumption that the underlying utility function exhibits constant absolute risk aversion. It is possible that appropriate choices for the distribution of the single underlying parameter could lead to an adequate range to explain the existing observations. The process converts the underlying utility function of k parameters into an effective utility function that has as many parameters as the probability function describing the uncertainty in the vector of k parameters. This usually requires a minimum of $2k$ parameters, but it may require more. The simple utility function with constant risk aversion could lead to an effective function with three or more parameters if the distribution function for the uncertain parameter involves three of more parameters; one parameter will be retained only if the distribution has a single parameter, such as the exponential distribution, for example.[9]

Non-stationary risk aversion leads to non-stationary market risk premia. Since secular changes in the market risk premium seem to be the rule, it might be fruitful to consider if the

[9] The exponential distribution would give a non-convergent integral with the utility function used in this paper. If the alternate formulation $\mathrm{U}(W) = -\mathrm{e}^{-\rho W}$ were used it would give a hyperbolic effective risk aversion coefficient.

changes in risk premium over time can be used to estimate how the mean and variance of the risk aversion coefficient have changed in the past.

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