The Pricing of Securitization of Life Insurance under Mortality Dependence

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Abstract

This article studies the pricing of securitization of life insurance under mortality dependence and stochastic interest rate. Following the empirical results of Brown and McDaid (2003), this paper models stochastic mortality intensity and then considers certain important risk factors (income, gender, age, and others) affecting mortality rate to derive the mortality probability for each policyholder. Further, multiple Clayton copula is used to measure the mortality dependence of multiple life insurance policies. Death time of each policyholder can be projected through multiple Clayton copula. This paper applies the estimated death time to design and price Collateralized Insurance Obligation under mortality dependence. The numerical results of Monte Carlo simulation show that the independence assumption tends to overestimate the premium of equity tranche and underestimate the premiums of mezzanine tranche and senior tranche.

Keywords: Securitization; Mortality dependence; Clayton copula; Stochastic mortality intensity; Monte Carlo simulation; Collateralized Insurance Obligation

1. Introduction

To simplify actuarial calculations in the pricing of life policies portfolios, it has traditionally been assumed that individual mortalities of policyholders are independent. Nevertheless, it may make the intuitive sense that the mortality risk of policyholders is correlated, called *mortality dependence*. There have been some studies supporting mortality dependence on pairs of individuals. For example, Parkes et al. (1969) show that there is a 40% increase in mortality among the widowers during the first few months after the death of their wives; see also Ward (1976). While the independence assumption tends to overestimate the mortality cost for joint first death insurance, it underestimates the cost for joint last survivor policies. This unrealistic assumption could have a large financial impact on the insurance industry as their mortality cost or reserves could not be projected accurately. Hence, *mortality dependence* should be emphasized when reference entities of the contract are linked to a portfolio of life insurance. Meanwhile, the joint mortality probability of the portfolio of life insurance should be projected accurately in order to obtain correct pricing for the securitization of life insurance with *mortality dependence*.

This paper uses a *copula function,* which is a mathematical function that combines marginal mortality probability into a joint mortality distribution, to measure the *mortality dependence* of multiple life insurances. Bassan and Spizzichino (2005) consider bivariate survival models characterized by the condition that bivariate aging function is a Clayton copula. This paper extends Bassan and Spizzichino (2005) to obtain joint mortality probability of multiple life insurances by using multiple Clayton copulas.

For the mortality rate of marginal mortality probability, some recent investigations almost assume that the mortality rate involves age-dependency and time-dependency terms, such as Lee and Carter (1992), Lee (2000), Olivieri (2001), Olivieri and Pitacco (2002), and Renshaw and Haberman (2003). However, for the insurers, since the information set of mortality factors is not detailed and wealthy, uncertainty of the future development may not clearly be delineated. Further, as opposed to e.g. Lee and Carter (1992), Lee (2000) this study focus on the pricing and risk management application in mind rather than the time series properties of mortality. In contrast to Milevsky and Promislow (2001), Dahl (2004), Biffis (2005), and Schrager (2006) to consider the mortality intensity for all ages simultaneously, this paper follows Biffis (2005) and Schrager (2006) to consider the mortality intensity as a hazard rate in the context of the Cox-process approach developed by Lando (1998).

Brown and McDaid (2003) review 45 recent research papers that look at factors that affect mortality after retirement. They show that, in addition to age and gender, mortality intensity is also affected by race and ethnicity, education, income, occupation, marital status, religion, health behaviors, smoking, alcohol, and obesity. Especially, Pappas et al. (1993) discover that not only do poor people have higher mortality rate than wealthy people, but also mortality rate fall consistently when levels of income goes up. Similar findings are reported in other papers, such as Sorlie et al. (1995), Williams and Gollins (1995), Deaton and Paxson (1999), and Attanasio and Emmerson (2001). Hence, income is a strong predictor of mortality. To be consistent with empirical results above, this paper proposes that the risk factors governing mortality intensity function are income, gender, age, and other factors such as marital status, religion, health behaviors, smoking, alcohol, and obesity. Importantly, this paper assumes that income follows the stochastic differential equation and it applies the extended Vasicek model proposed by Hull and White (1990) to describe the randomness of interest rate[1]. This allows us to capture the force

[1] Extended Vasicek model proposed by Hull and White (1990) allows an exact fit to the initial term structure of interest rate and interest rate volatility compared with Vasicek model proposed by Vasicek (1977).

of mortality intensity and uncertainty of its future development more accurately. Furthermore, by using multiple Clayton copulas with the mortality intensity function we constructs, this study could obtain death time of each policyholder for multiple life insurance. Once death time of each policyholder for multiple life insurance is projected precisely, it could be used to calculate the premium, reserves, and value of insurance securitization. In short, this paper attempts to apply death time of each policyholder to price the securitization of life insurance with mortality dependence under the case of multiple Clayton copulas.

The first significant development in hedging mortality and longevity risk is the Swiss Re mortality bond issued in December 2003[2]. In November 2004, a 25-year longevity bond was issued by the European Investment Bank.[3] Unfortunately, there is a "basis risk[4]" in the structure of these two products. Blake and Burrows (2001) first provide the longevity bonds to deal with longevity risk where coupon payments are linked to the number of survivors in a given cohort. Lin and Cox (2005) price mortality bonds and swaps under the following three assumptions. Firstly, the mortality distributions of policyholders are independent. Secondly, the underlying annuity insurances in the mortality bonds or swaps face the same insurance amount. Finally, the mortality bond just links to all mortality risk of the underlying annuity insurances and this infers that all bond investors with different risk preferences face the same mortality risk on the underlying annuity insurances.

This main contribution of this paper is to relax three assumptions mentioned by Lin and Cox (2005) and is the first study to design and price an alternative mortality-linked security with *mortality dependence* and stochastic interest risk, called Collateralized Insurance Obligation

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[2] This 3-year maturity mortality bond is based on a mortality index of the general population of the United States, United Kingdom, France, Italy and Switzerland rather than the portfolio of Swiss Re life insurance policy.
[3] Coupon payments are linked to the proportion of the population who were age 65 in 2003 and are still alive at the coupon date.
[4] Basis risk refers to the risk that the losses that the portfolio of Swiss Re life insurance policy will not have an anticipated correlation with the mortality index of the general population of the United States, United Kingdom, France, Italy and Switzerland.

(CIO), which is the generalization of the securitization of Lin and Cox (2005). Comparing with mortality swap and mortality bond discussed by Blake and Burrows (2001) and Lin and Cox (2005), the best advantage of CIO is that it links to a portfolio of life insurance with different insurance amount and different degree of mortality risk. Thus, there is no basis risk for this structure and CIO could be more attractive to investors because that the investors can invest various tranches of CIO which are discriminated by the mortality risk according to their risk preference[5].

This paper is organized as follows. Section 2 models stochastic mortality intensity and uses risk factors to derive the mortality probability of each policyholder. Section 3 introduces the Clayton copula to capture mortality dependence and simulates the death time of each policyholder for multiple life insurance. Section 4 designs and prices an alternative mortality-linked security, CIO. Section 5 provides the numerical analysis of CIO. Section 6 concludes this paper.

2. Model

2.1. Stochastic mortality intensity

In life insurance, actuaries have traditionally calculated premiums and reserves using deterministic mortality intensity, which is only a function of the age of the insured. This study models the mortality intensity as a stochastic process. The advantage of this setting is to allow us to capture the features of time dependency and uncertainty of the future development. Consider n life insurances and let $\kappa_i : \Omega \rightarrow \mathbb{R}_+, i = 1, 2, \ldots n,$ be a strictly positive random variable, and it is referred to the death time of the policyholder i, for i=1,2,...n , which is generated by a

[5] The structure of CIO is similar to Collateralized Debt Obligation (CDO). In recent years, CDO has been grown rapidly in financial market and become main tool to hedge credit risk for the banks. According to the Bond Market Association, gross global issuance of all types totaled USD 157 billions in 2004, USD 251 billions in 2005, and USD 71 billions in the first quarter of 2006.

filtered probability space (Ω, G, P); P is an equivalent martingale measure, the enlarged filtration $\mathrm{G}=(G_t)_{t\geq 0}$ satisfies $\mathrm{G} \ =\mathrm{F}^{\mathrm{W}} \vee \mathrm{H}$; i.e., $G_t = F_t \ \vee H_t$ for any $t \in \mathbb{R}_+$. The death time of a policyholder is modeled as stopping time κ_i with respect to some filtration, G_t. Then we define that:

$$\kappa_i = \inf\left\{ t : \int_t^T \lambda_i(X_s)ds \geq E_i \right\}, \text{ for } \ i = 1, 2, \ldots n, \tag{1}$$

where we assume that counting process $N^i(t)$ is a doubly stochastic Poisson processes of death (Cox process) with a random intensity $\lambda_i(X_s)$. The assumption that the intensity is a function of the current level of the state variables, and not the whole history, is convenient in applications, but it is not necessary from mathematical point of view. $(X_t)_{t=0}^{T^*}$. is a right continuous with left limits $\mathbb{R}^d$-valued process and represents d state variables, such as the gender, age, or other risk factors deemed relevant for predicting the likelihood of mortality. E_i is unit exponential random variables which is independent of state variables and λ_i. The death time κ_i can be correspondingly considered as the first jump-time of the Cox processes $N^i(t)$ with non-negative stochastic intensity process $\lambda_i(X_t)$ and are conditional independent with respect to the filtration generated by $\mathbf{X}$ under P. Hence, the information at time t is:

$$G_t = F_t \vee H_t$$

where $F_t = \sigma(X(s), 0 \leq s \leq t)$ and represents the information generated by the observations of the gender, age, or other variables up to time *t,* respectively. $H_t = \sigma(1_{\{\kappa_i \leq s\}}, s \leq t)$ and stands for the natural filtration of death time κ_i, and G_t then corresponds to knowing the evolution of these variables up to time t and whether the policyholder is alive or not. As the intensity of mortality becomes large, the integrated mortality may rise faster and touches the level of the independent exponential variable sooner. Eventually, it leads the random time to be small and results to the mortality probability higher.

2.2. Risk factors of mortality intensity and mortality probability of each policyholder

Brown and McDaid (2003) review 45 recent papers on the topic about factors affecting retirement mortality. They find that the mortality rate is influenced by age, gender, race, ethnicity, education, income, occupation, marital status, religion, health behaviors, smoking, alcohol, and obesity. Especially, income is a strong predictor of mortality. Pappas et al.(1993) show that not only do poor people have higher mortality rate than wealthier people, but also mortality rate fall consistently as levels of income rise up. Further, the mortality disparity by income is widening. Hence, income is a strong predictor of mortality. Similar finding are reported by Attanasio and Emmerson (2001), Williams and Gollins (1995), Sorlie et al. (1995), and Deaton and Paxson (1999). Consequently, for simplify, this paper assumes that the risk factors governing mortality intensity function are income, gender, age, and others such as marital status, religion, health behaviors, smoking, alcohol, and obesity. This research assumes the intensity functions of gender, age, and other factors are deterministic functions. To portray the dependence on mortality process on the state of these risk factors, we introduce the enlarged filtration $G = (G_t)_{t \geq 0}$ to satisfies $G = F^I \vee H$; ie., $G_t = F_t^I \vee H_t$ for any t$\in \mathbb{R}_+$, where $F_t^I = \sigma(I(s), 0 \leq s \leq t)$ contains complete information on the personal income and $H_t^i = \sigma(1_{\{\kappa_i \leq s\}}, s \leq t)$, i =1,2,…n. $1_{\{\cdot\}}$ is the indicator function. $I(t)$ refers to the time-t personal income. Consequently, for a policyholder of age $x+t$, the mortality probability from time t to T given the information until time t is modeled as:

$$P\left(\kappa_i \le T \middle| F_T^I \vee H_t^i\right) = 1 - \exp\left(-\int_{x+t}^{x+T} \lambda_i[I(s)]ds\right) = 1 - \exp\left(-\int_t^T \lambda_i[I(s)]ds\right) \qquad (2)$$

We assume that the point processes governing mortality for the gender, age, income, and other risk factors is:

$$\lambda(t) = f\left(others(t), Z_{gender}(t), age(t), \log\left[\frac{I(t)}{B(t)}\right]\right)$$

where $Z_{gender}(t)$ is a dummy variable; it equals 1 if female or 0 otherwise. $B(t)$ refers to the saving account and then $\log\left[\frac{I(t)}{B(t)}\right]$ represents the present logarithmic personal income in the cause of being consistent with the empirical evidence that the resulting relation between health and income produces a curve line instead of a straight line. This could result from non-linear relations between health and commodities or environmental factors that affect health (Godfrey, 1989) or it could arise from non-linear associations between income commodities or environmental factors that affect health (Edwards, Babor, 1994 and Atkinson, Gomulaka, and Stern, 1990). For simplicity, hence, the linear mortality rate function admits the following representation:

$$\lambda_i(t) = \lambda_i^o(t) + \lambda_i^g(t) + \lambda_i^a(t) + \lambda_i^I \log\left[\frac{I(t)}{B(t)}\right] \qquad (3)$$

where λ_i^g measures the sensitivity of entity i to the difference of gender, λ_i^a represents the sensitivity of entity i to the level of age, and λ_i^I estimates the sensitivity of entity i to the logarithmic personal income. λ_i^o represents the sensitivity of entity i to the others variables such as marital status, religion, health behaviors, smoking, alcohol, and obesity. According to the results of Brown and McDaid (2003), λ_i^o, λ_i^g, and λ_i^a should be positive, whereas λ_i^I

should be negative. For simplicity, following the certain studies such as Koo (1998), Nejadmalayeri and Patrick (2003), and Venegas-Martínez (2006), this study assumes that personal income $I(t)$ follows a geometric Brownian motion, and then the stochastic differential equation is given by:

$$dI(t) = r(t)I(t)dt + \sigma_I I(t)\, dW_t^I\,, \tag{4}$$

where $r(t)$ is the instantaneous return of personal income. W_t^I standards for Brownian motion with respect to F_t, and σ_I denotes the volatilities of return on personal income. Therefore, it is noteworthy that under the setup of equality (3), applying equation (4), using the iterated expectations, the conditional mortality probability of κ_i up to time $T \geq t$, on the set $\{\kappa_i > t\}$ is presented in the following proposition.

Proposition: *The conditional mortality probability of each policyholder* κ_i *admits the following representation:*

$$\mathbf{P}(\kappa_i \leq T | G_t) = 1 - \exp\left[-\int_t^T \left(\lambda_i^o(s) + \lambda_i^g(s) + \lambda_i^a(s)\right) ds\right] \exp\left[\frac{(T-t)^2}{4}\lambda_i^I \sigma_I^2 + \frac{(T-t)^3}{6}(\lambda_i^I)^2 \sigma_I^2\right] \tag{5}$$

Proof.

$$\mathbf{P}(\kappa_i \leq T | G_t) = \mathbf{E}\left[\mathbf{P}(\kappa_i \leq T \left| F_T^I \vee H_t^i\right.) | G_t\right] = 1 - \mathbf{E}\left[\exp(-\int_t^T \lambda^i(s)\, ds) | G_t\right] \tag{6}$$

Substitution of the linear intensity $\lambda_i(s) = \lambda_i^o(s) + \lambda_i^g(s) + \lambda_i^a(s) + \lambda_i^I \log[\frac{I(s)}{B(s)}]$ into (6), we obtain:

$$\mathbf{E}\left[\mathbf{P}\left(\kappa_i \leq T \left| F_T^I \vee H_t^i\right.\right) \middle| G_t\right] = 1 - \mathbf{E}\left\{\exp\left(-\left[\int_t^T \left(\lambda_i^o(s) + \lambda_i^g(s) + \lambda_i^a(s) + \lambda_i^I \log[\frac{I(s)}{B(s)}]\right) ds\right]\right) \middle| G_t\right\}$$

Let $Y_1 \equiv -\int_t^T \lambda_3 \log[\frac{I(s)}{B(s)}]ds$, then we have:

$$\begin{aligned}
Y_1 &= -\int_t^T \lambda_i^I \log[\frac{I(s)}{B(s)}]ds \\
&= -\int_t^T \lambda_i^I \left(\log[\frac{I(t)}{B(t)}] - \frac{1}{2}\sigma_I^2(s-t) + \sigma_I \int_t^s dW_v^I \right) ds \\
&= -\lambda_i^I \log[\frac{I(t)}{B(t)}](T-t) + \frac{1}{2}\lambda_i^I \sigma_I^2 \int_t^T (s-t)ds - \lambda_i^I \sigma_I \int_t^T \int_t^s dW_v^I \, ds \\
&= -\lambda_i^I \log[\frac{I(t)}{B(t)}](T-t) + \frac{1}{2}\lambda_i^I \sigma_I^2 \int_t^T (s\text{-}t)ds \text{ - } \lambda_i^I \sigma_I \int_t^T \int_v^T ds \, dW_v^I \\
&= -\lambda_i^I \log[\frac{I(t)}{B(t)}](T-t) + \frac{1}{2}\lambda_i^I \sigma_I^2 \int_t^T (s-t)du - \lambda_i^I \sigma_I \int_t^T (T-v) dW_v^I
\end{aligned}$$

Without loss of generality, it assumes that $I(t) = B(t) = 1$, and then the following result will be obtained:

$$Y_1 = \frac{(T-t)^2}{4}\lambda_i^I \sigma_I^2 - \lambda_i^I \sigma_I \int_t^T (T-v) \, dW_v^I$$

Therefore, we have:

$$\begin{aligned}
\mathbf{P}(\kappa_i \le T \,|G_t) &= 1 - \exp\left[-\int_t^T \left(\lambda_i^o(s) + \lambda_i^g(s) + \lambda_i^a(s) \right) ds \right] \mathbf{E}\left[\exp(Y_1) | G_t \right] \\
&= 1 - \exp\left[-\int_t^T \left(\lambda_i^o(s) + \lambda_i^g(s) + \lambda_i^a(s) \right) ds \right] \exp\left[\mathbf{E}(Y_1 | G_t) + \frac{1}{2}\mathbf{V}(Y_1 | G_t) \right] \\
&= 1 - \exp\left[-\int_t^T \left(\lambda_i^o(s) + \lambda_i^g(s) + \lambda_i^a(s) \right) ds \right] \exp\left[\frac{(T-t)^2}{4}\lambda_i^I \sigma_I^2 - \frac{(T-t)^3}{6}(\lambda_i^I)^2 \sigma_I^2 \right] \\
&\equiv Q_i
\end{aligned}$$

where $\mathrm{E}(\cdot)$ and $\mathrm{V}(\cdot)$ are respectively the conditional expectation and variance with respect to G_t , respectively.

Using the chain rule of differentiation and the properties of $\lambda_i^I < 0$ and $\lambda_i^o, \lambda_i^g, \lambda_i^a > 0$, we have:

$$\frac{\partial Q_i}{\partial \sigma_I^2} = -\frac{Q_i (T - t)^2}{2}\left[\frac{1}{2}\lambda_i^I - \frac{(T - t)}{3}(\lambda_i^I)^2\right] > 0 ,$$

$$\frac{\partial Q_i}{\partial T} = -Q_i\left[-\left(\lambda_i^o(T) + \lambda_i^g(T) + \lambda_i^a(T)\right) + \frac{(T-t)\sigma_i^2}{2}\left(\lambda_i^I - (T-t)(\lambda_i^I)^2\right)\right] > 0 . \tag{7}$$

The results of equation (7) indicate that the mortality probability of policyholder i is increasing with the volatilities of return on personal income σ_I^2 and this result is consistent with Brenner (2005). He indicates that volatility of changes in that rapid economic growth was – in the very short-term – a source of increased mortality experienced of the United States 1901–2000. Further, the mortality probability of policyholder i is also increasing with the maturity T of contract. This makes sense that when the transaction time of contract is longer, the higher probability that policyholders maybe die during the transaction period.

3. Mortality dependence

Recall the literatures about securitization of mortality risk, Lin and Cox (2005) assume that the mortality distributions of policyholders are independent when pricing mortality bonds and swaps with embedded option. However, *mortality dependence* for the portfolio of life insurance plays an important role when forecasting improvements in mortality is under consideration seriously. Furthermore, when the structured mortality product is priced, mortality dependence for the portfolio of life insurance affects the likelihood of extreme outcomes in the portfolio of life insurance. Thus *mortality dependence* for the portfolio of life insurance should also be emphasized and the joint mortality probability for the portfolio of life insurance should be projected accurately in hedging or pricing the securitization of mortality risk. To estimate the joint mortality probability for the portfolio of life insurance, this study will present a useful instrument, called copula[6] for the joint mortality probability, to capture the characteristics of *mortality dependence*.

3.1. Multivariate copula

A copula function, dnoted by $C(u_1,u_2,...,u_n)$, represents the joint cumulative distribution function (c.d.f.) of n standard uniform random variables $U_1,U_2,...,U_n$:

$$C(u_1,u_2,...,u_n) = P(U_1 \le u_1, U_2 \le u_2,...,U_n \le u_n).$$

Let $\kappa_1,\kappa_2,...,\kappa_n$ be death time of n policyholders, with $F_1,F_2,...,F_n$ their marginal c.d.f.'s. Considering the joint mortality probability of n life insurance and strictly increasing monotonicity for c.d.f.'s, we have:

$$\begin{aligned} F(T_1,T_2,...,T_n) &= \mathrm{P}(\kappa_1 \le T_1, \kappa_2 \le T_2,...,\kappa_n \le T_n) \\ &= \mathrm{P}(F_1(\kappa_1) \le F_1(T_1), F_2(\kappa_2) \le F_2(T_2),...F_n(\kappa_n) \le F_n(T_n)) \end{aligned}$$

By the version of Sklar's Theorem (1959), for any multivariate distribution function $F(T_1,T_2,...T_n)$ with marginal c.d.f. $F_1,F_2,...,F_n$, there exists a Copula C unique on $\mathrm{Ran}\,F_1 \times \mathrm{Ran}\,F_2 \times ..... \times \mathrm{Ran}\,F_n$ such that:

$$F(T_1,T_2,...T_n) = C(F_1(T_1),F_2(T_2),...,F_n(T_n)),$$

where

$$\begin{aligned} F_i(T_i) &= \mathrm{P}(\kappa_i \le T_i \,|\, G_t) \\ &= 1-\exp\left[-\int_t^{T_i}\left(\lambda_i^o(s)+\lambda_i^g(s)+\lambda_i^a(s)\right)ds\right]\exp\left[\frac{(T_i-t)^2}{4}\lambda_i^I\sigma_I^2 - \frac{(T_i-t)^3}{6}(\lambda_i^I)^2\sigma_I^2\right], \quad i=1,2,...,n. \end{aligned}$$

Further, a corollary of Sklar's Theorem (1959) is that:

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[6] Copulas are a very general tool to describe the interrelation of several random variables. Although the immense generality is also the drawback of copulas (detailed discussions see Rank (2006)), it is remarkable that the use of copulas has greatly improved the modeling of dependence in practice. For example, in contrast to linear correlation, the use of copulas avoids typical pitfalls and therefore leads to a mathematically consistent modeling of dependence. For an extensive discussion of copulas, we would like refer the reader to Nelsen (1999) for a formal framework.

$$C(u_1,u_2,...,u_n)=F(F_1^{-1}(u_1),F_2^{-1}(u_2),...,F_n^{-1}(u_n)),$$

where $F_1^{-1},F_2^{-1},...,F_n^{-1}$ are quasi-inverses of $F_1, F_2, ..., F_n$.

3.2. Multivariate Clayton copula

Among the copula family, Clayton copula has been frequently applied in the actuarial field. For instance, Bassan and Spizzichino (2005) consider bivariate survival models characterized by the condition that bivariate aging function is a Clayton copula. Clayton copula can characterize tail dependence in multidimensional data and allows for asymmetric tail dependence. Hence, this paper extends Bassan and Spizzichino (2005) bivariate survival model to obtain multivariate life insurance by using multivariate Clayton copula.

Let φ be a generator: $[0,1]\to[0,\infty]$, which is continuous, strictly decreasing $\varphi'(u)<0$ for all $u\in[0,1]$. Then the function $C:[0,1]^n\to[0,1]$

$$C(u_1,u_2,...,u_n)=\varphi^{-1}(\varphi(u_1)+\varphi(u_2)+...+\varphi(u_n))$$

is the Archimean n-variate copula with generator φ.

Clayton copula function is one of one-parameter Archimedean copula. Let $\varphi(u)=u^{-\theta}-1$ be a continuous strictly deceasing function such that $\varphi(0)=\infty$ and $\varphi(1)=0$, then $\varphi^{-1}(u)=(u+1)^{-\frac{1}{\theta}}$ is the inverse of $\varphi(u)$. Then, for all $n\geq 2$, the function $C_\theta:[0,1]^n\to[0,1]$ defined as

$$C_\theta(u_1,u_2,...,u_n)=\exp\left\{-\left[\sum_{i=1}^{n}(-\ln u_i)^\theta\right]^{\frac{1}{\theta}}\right\}$$

is an n-dimensional Clayton copula, where $\theta>1$ is the dependence parameter.

In a representative set of well-known one-parameter systems of copulas, includes Clayton copula, there exists a one-to-one relationship between the real-valued dependence parameter, θ, and Kendall's non-parametric measure of association, τ. For Clayton copula C_θ, one has[7]:

$$\tau = 1 + 4\int_0^1 \frac{\varphi(u)}{\varphi^{-1}(u)} du = \frac{\theta}{\theta+2}$$

It gives the product copula (independence; $\tau = 0$) if $\theta = 0$, the lower Frechet bound (perfect negative dependence; $\tau = -1$) when $\theta = -1$, and the upper one (perfect positive dependence; $\tau = 1$) for $\theta \to \infty$.

3.3. Simulate the death time via Clayton copula approach

As suggested by Cherubini, Luciano, and Vecchiato (2004), a general method to simulate death time drawn from the Clayton copula is formulated by using a conditional approach. Since $U_1,...,U_n$ has the joint distribution function $C(u_1,u_2,..,u_n) = \varphi^{-1}(\varphi(u_1)+\varphi(u_2)+...+\varphi(u_n))$, then the conditional distribution of U_k given the values of $U_1,...,U_{k-1}$ is given by:

$$C(u_k \,|u_1, u_2,...,u_{k-1}) = \mathrm{P}(U_k \le u_k \,|U_1 \le u_1,...,U_{k-1} \le u_{k-1})$$

$$= \frac{\partial^{k-1} C_k(u_1,..,u_k) \big/ \partial u_1..\partial u_{k-1}}{\partial^{k-1} C_{k-1}(u_1,..,u_{k-1}) \big/ \partial u_1..\partial u_{k-1}}$$

$$= \frac{\varphi^{-1(k-1)}(\varphi(u_1)+\varphi(u_2)+...\varphi(u_k))}{\varphi^{-1(k-1)}(\varphi(u_1)+\varphi(u_2)+...\varphi(u_{k-1}))} = \frac{\varphi^{-1(k-1)}(c_k)}{\varphi^{-1(k-1)}(c_{k-1})},$$

where $c_k = \sum_{i=1}^{k} \varphi(u_i)$ and $k = 2,..,n$.

Hence, the general procedure for simulating the death time via Clayton copula approach in a multivariate setting is as follows:

- Simulate n independent random variables $(v_1, v_2,...,v_n)$ from $U(0,1)$.

[7] The most widely known scale-invariant measures of association are Kendall's tau and Spearman's rho. Because that Spearman's rho of the Clayton copula shows complicated expression, this paper uses Kendall's tau as the dependence measure of the Clayton copula. (See Cherubini, Luciano, and Vecchiato (2004), p.126).

- Set $u_1 = v_1$.

- Set $v_2 = C_2(u_2|v_1) = \mathrm{P}(U_2 \le u_2 | V_1 = v_1) = \dfrac{\varphi^{-1(1)}(c_2)}{\varphi^{-1(1)}(c_1)}$, where $c_1 = \varphi(u_1) = u_1^{-\theta} - 1$ and $c_2 = \varphi(u_1) + \varphi(u_2) = u_1^{-\theta} + u_2^{-\theta} - 2$.

  Hence, $v_2 = \left(\dfrac{u_1^{-\theta} + u_2^{-\theta} - 1}{u_1^{-\theta}}\right)^{-\frac{1}{\theta}-1}$ and then $u_2 = \left(v_1^{-\theta}(v_2^{-\frac{\theta}{\theta+1}} - 1) + 1\right)^{-\frac{1}{\theta}}$.

- Set $v_3 = C_3(u_3|v_1, v_2) = \mathrm{P}(U_3 \le u_3 | V_1 = v_1, |V_2 = v_2) = \dfrac{\varphi^{-1(2)}(c_3)}{\varphi^{-1(2)}(c_2)} = \left(\dfrac{u_1^{-\theta} + u_2^{-\theta} + u_3^{-\theta} - 2}{u_1^{-\theta} + u_2^{-\theta}}\right)^{-\frac{1}{\theta}-2}$,

  and then $u_3 = \left(\left[(\mathrm{u}_1^{-\theta} + \mathrm{u}_2^{-\theta}\right](v_3^{-\frac{\theta}{2\theta+1}} - 1) + 2\right)^{-\frac{1}{\theta}}$.

- Using the same way, we can solve u_n through the following equation:

$$v_{\mathrm{n}} = \left(\frac{u_1^{-\theta} + u_2^{-\theta} + \ldots + u_n^{-\theta} - n + 1}{u_1^{-\theta} + u_2^{-\theta} + \ldots + u_{n\text{-}1}^{-\theta} - n + 2}\right)^{-\frac{1}{\theta}-n+1},$$

  and get:

$$u_n = \left(\left[(u_1^{-\theta} + u_2^{-\theta} + \ldots + u_{n\text{-}1}^{-\theta} - n + 2\right](v_n^{-\frac{\theta}{\theta(1\text{-}n)\text{-}1}} - 1) + 1\right)^{-\frac{1}{\theta}}.$$

- Get the death time of policyholder i: $\kappa_i = F^{-1}(u_i),\ i = 1, \ldots, n$,

  where $u_i = 1 - \exp\left[-\int_t^{T_i}\left(\lambda_i^o(s) + \lambda_i^g(s) + \lambda_i^a(s)\right)ds\right]\exp\left[\dfrac{(T_i - t)^2}{4}\lambda_i^I\sigma_I^2 - \dfrac{(T_i - t)^3}{6}(\lambda_i^I)^2\sigma_I^2\right]$

  That is:

$$\kappa_i = \inf\left(t : 1 - \exp\left[-\int_t^{T_i}\left(\lambda_i^o(s) + \lambda_i^g(s) + \lambda_i^a(s)\right)ds\right]\exp\left[\frac{(T_i - t)^2}{4}\lambda_i^I\sigma_I^2 - \frac{(T_i - t)^3}{6}(\lambda_i^I)^2\sigma_I^2\right] \le u_i\right)$$

Hence, given the coefficients of $\lambda_i^o, \lambda_i^g, \lambda_i^a, \sigma_I^2$, the implied death time κ_i can be solved.

When death time of each policyholder for multiple life insurance is obtained precisely, it is very useful and favorable for the insurer to compute some important subjects such as the premium, reserves, and price of insurance securitization. This paper attempts to apply death time of each policyholder to the securitization of life insurance with mortality dependence by adopting multiple Clayton copulas. In the following section, we will design and price a new mortality-linked security called Collateralized Insurance Obligation, which is the generalization of the securitization of Lin and Cox (2005) and is different from mortality swap and mortality bond illustrated in the prior studies.

4. Application: Collateralized Insurance Obligation

4.1. Motivation

Lin and Cox (2005) assume that the underlying annuity insurances in the mortality bonds or swaps face the same insurance amount. In addition, the mortality bond links to all mortality risk of the underlying annuity insurances and this infers that all bond investors with different risk preferences face the same mortality risk on the underlying annuity insurances. On the contrary, Collateralized Insurance Obligation is linked to a portfolio of life insurance with different insurance amount and degree of mortality risk. Hence, the investors can invest various tranches of CIO which are discriminated by the mortality risk according to their risk preferences. Further, reinsurers can avoid the counterparty risk that may arise with traditional reinsurance. In practice, CIO may address some important market imperfections. First, securitization of mortality risk for insurance policies could reduce insurer’s reserves and let the allocation of capital becomes more flexible. Second, insurance policies may be illiquid and lead to a reduction in their market values, while securitization may improve liquidity, and thereby raise the total value for the issuer of the CIO structure.

4.2. The structure of CIO

CIO can be viewed as the form of liability-backed bonds whose underlying collateral is typically a pool of life insurances. The structure and cash flow diagram of the CIO is showed in Figure 1, and is similar to that of Collateralized Debt Obligation (CDO). The insurance company pays the premiums P to Special Purpose Vehicle (SPV) for mortality swap. SPV issues three tranches (senior tranche, mezzanine tranche, and equity tranche) of the CIO to investors with different degree of mortality risk preferences. SPV invests the swap premium P and cash from the sale of bond, $\mathrm{A}=\sum_{i=1}^{n} A_i$, in default-free bonds with coupon rate $D(t)$. The different tranche investors receive different regular tranche premiums W, which is equial to swap premium P plus coupon rate of default-free securities $D(t)$, and receive the full nominal value of contract A. If the loss on the underlying collateral portfolio occurs, the tranche investor will receive the residual nominal value of contract $\mathrm{A}-L(t)$.

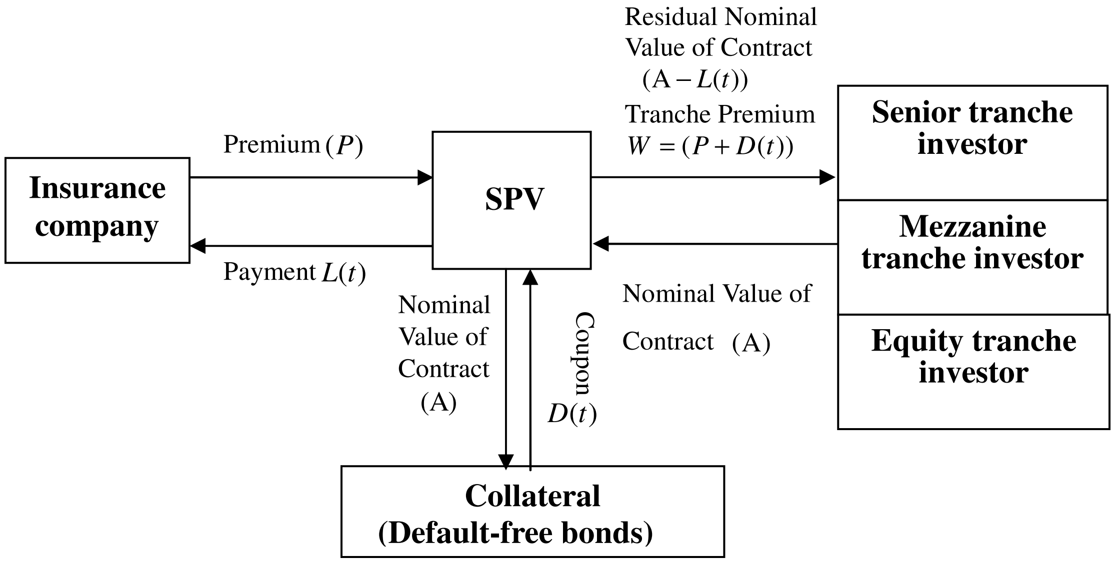

Fig. 1. The structure and cash flow diagram of the CIO

4.3. The payoff of CIO

Let us consider the reference portfolio including n life insurance policies. Let L_i denote the net loss of issuer as the policyholder i dies. Set $N_i(t) = 1_{\{\kappa_i \le t\}}$ be the counting process which jumps from 0 to 1 at death time of the policyholder i. Furthermore, $L(t)$ displays the total cumulative loss amounts of policyholders portfolio at t and can be expressed as:

$$L(t) = \sum_{i=1}^{n} L_i \, N_i(t)$$

which is thus a pure jump process.

Let us consider a tranche of a CIO, where the death payment pays all losses that occur on the collateral portfolio, above a threshold C and below a threshold D where $0 \le C \le D \le \sum_{i=1}^{n} A_i = \mathrm{A}$. It is called the equity tranche when $C = 0$; if $C > 0$ and $D < \sum_{i=1}^{n} A_i$, it is defined as the mezzanine tranche, and as $D = \sum_{i=1}^{n} A_i$, it is regarded as senior or super-senior tranches. $M(t)$ denotes the cumulative losses on a given tranche. These losses are equal to zero if $L(t) \le C$, is equal to $L(t) - C$, if $C < L(t) \le D$ and is equal to $D - C$ if $L(t) > D$. This can be summarized as:

$$M(t) = \big(L(t) - C\big)1_{\{C,D\}} L(t) + (D - C)1_{\left\{D, \sum_{i=1}^{n} A_i\right\}} L(t).$$

We notice that both $L(t), M(t)$ are a pure jump process.

Under the risk-neutral probability measure P, we can write the price of the death payment of the given tranche as:

$$\mathbf{DP} = E^{\mathbf{P}}\left[\int_0^T \exp(-\int_0^t r(u)\,du)\,dM(t)\right] = E^{\mathbf{P}}\left[\int_0^T B(0,t)\,dM(t)\right]$$

where $r(u)$ denotes the spot rate, and T denotes the maturity of the CIO.

It can be considered that we only need the information of the first moment of the cumulative loss on the tranche. This work can be computed if the distribution of total losses has been simulated by Monte-Carlo simulation. We apply the extended Vasicek model to describe the randomness of spot rate $r(u)$, then the stochastic differential equation is given by:

$$d\,r(t) = [\theta(t) - \alpha(t)\,r(t)]\,dt + \sigma_r\,dW_t^r,$$

where $\theta(t)$ represents the long-term equilibrium value of the process; $\alpha(t)$ is a nonnegative mean reversion speed; and σ_r is the volatilities of spot rate. W_t^r standards for Brownian motion with respect to F_t.

The fair spread (or equivalently the fair premium) of that tranche can be found by putting into an equivalence of the death payment to the premium payment. In this case, the premium payment can be written as follows:

$$\mathbf{PP} = E^{\mathbf{P}}\left[\lim_{\mathbf{m}\to\infty}\sum_{i=1}^{m}\Delta_{i-1,i}\, W\ B(0,t_i)\left[D-C\right]\mathrm{I}_{[L(t)\le C]} + \lim_{\mathbf{m}\to\infty}\sum_{i=1}^{m}\Delta_{i-1,i}\, W\ B(0,t_i)\left[D-L(t)\right]\mathrm{I}_{[C\le L(t)\le D]}\right]$$
$$= W\cdot E^{\mathbf{P}}\left[\int_0^T B(0,t)\ g\left(L(t)\right)dt\right]$$

where $g\left(L(t)\right)=\min\left\{\max\left[D-L(t),0\right],D-C\right\}$ and $(D-C)$ is the tranche size at inception, m denotes all premium payment dates, t_i denotes the premium payment date, $\Delta_{i-1,i}$ denotes the tenor between successive premium payment dates which take into account the day count convention, W is the fair spread, $(D-L(t))$ is the outstanding tranche notional at time $t\in[0,T]$, and, clearly $0\le M(t)\le D-C$ since $0\le L(t)\le\sum_{i=1}^{n}A_i$ for all t.

Under the condition of no arbitrage market, the initial expected return equals to expected loss, i.e., **PP=DP**, thus the fair premium W on different level of tranche can be computed as:

$$W=\frac{E^{\mathrm{P}}\left[\int_0^T B(0,t)\,dM(t)\right]}{E^{\mathrm{P}}\left[\int_0^T B(0,t)\,g\left(L(t)\right)dt\right]} \tag{8}$$

From the equation above, we know that the pricing of the CIO involves the computation of aggregate loss distributions over different time horizons $L(t)$. Thus, CIO tranche premiums depend upon the individual mortality risk of reference portfolio and the dependence structure between mortality rate of n life policies. We remark that the only thing we need in order to price each CIO tranche is to obtain the simulated counting process $N_i(t)$ via the Clayton copula framework.

5. Numerical analysis

Since there is no close-form solution for tranche premium of CIO based on the result of equation (8), this research uses Monte-Carlo simulations to estimate the tranche premium of CIO. One parameter will happen to play a key role: the measure of dependence among the policies: Kendall's tau, τ. First, this study recapitulates the values chosen for the input parameters and calculates the premium of CIO. In addition, it also examines the sensitivity of dependence among the policies to the premium of each tranche.

In order to simulate the premium for different tranches of CIO, the following assumptions are made:

5.1. Assumption for input parameters

- Maturity of CIO: $T = 10$.
- Number of life insurance policies: $n = 12$; Benefits (insurance amounts) of 12 policies are listed in Table 1.
- Maximum loss of the three tranches: equity=680 (680÷1700=40%), mezzanine=340 (340÷1700=20%), and senior=680 (680÷1700=40%). Measure of dependence among the policies, Kendall's tau, τ=0.5.
- Mortality intensity of other factors: $\lambda_0 = 0.1$; mortality intensity for male: λ_1^g=0.018 and 0.015 for female; mortality rate for different level of age: λ_2^a=0.1; and the sensitivity of the logarithmic personal income: λ_3= -0.1.
- Initial term structure is flat and satisfies $B(t,T) = \exp\left[-0.01\times(T - t)\right]$; volatilities of return on personal income σ_I is 0.01; and the mean reversion speed of Vasicek model: $\alpha(t)$=0.0254.
- Simulations: 20,000 times.

5.2. Computation of premium for each tranche

Based on the data above and the simulation process of Section 3.3, the death time of each policyholder, can be estimated and is displayed by the way that the smallest death time is on the top and the highest death time is in the button in Table 2. Table 2 represents that 5 policyholders die prior to maturity of contract (10 years) in 12 policyholders. Table 3 and Table 4 show the loss of sample path for each tranche and average nominal present value in each year, respectively. Table 3 indicates that the loss of sample path for equity tranche, mezzanine tranche, and senior tranche are 631.415, 309.406, and 208.973, respectively. Table 4 reports the total average nominal present values for equity tranche, mezzanine tranche, and senior tranche to be 4814.095, 2414.499, and 4814.095, respectively. Taking the equity tranche as an example, the death payment for equity tranche equals:

$$100\exp(-0.01\times 5.1153)+300\exp(-0.01\times 6.6236)+200\exp(-0.01\times 9.0933)+80\exp(-0.01\times 9.139)$$
$$=631.415.$$

Besides, the average nominal principal for the 10 years are 680, 680, 680, 680, 680, 591.523, 467.080, 280, 280, and 8.058, respectively. Consequently, the premium for equity tranche equals:

$$\begin{aligned}&\mathrm{W}\big[680\exp(-0.01\times 1)+680\exp(-0.01\times 2)+680\exp(-0.01\times 3)+680\exp(-0.01\times 4)\\&\quad+680\exp(-0.01\times 5)+591.530\exp(-0.01\times 6)+467.080\exp(-0.01\times 7)+280\exp(-0.01\times 8)\\&\quad+280\times\exp(-0.01\times 9)+8.058\times\exp(-0.01\times 10)\big]=4814.095\mathrm{W}\end{aligned}$$

Based on the equation (8), the premium of equity tranche *W* can be computed to be 0.13116 (631.415÷4814.093). Using the similar computation, the premiums of mezzanine and senior tranches are 0.128145 (309.406÷2414.499) and 0.043409 (208.973÷4814.095), respectively.

5.3. Sensitivity of Kendall's tau

Fig. 2 reports the tranche premium as a function of Kendall's tau for three tranches. It shows that the equity tranche has the highest premium, the premium of mezzanine tranche ranks secondary, and senior tranche has the lowest. Based on the results, it can be concluded that equity tranche has the highest mortality risk in virtue of abiding by the foremost loss, and senior trnache has the lowest mortality risk which results from bearing the loss lastly. Therefore, the investors of equity tranche require larger premium due to bearing the foremost loss. Further, Figure 2 shows that the equity tranche is negatively related to Kendall's tau, τ. This implies that the independence assumption ($\tau = 0$) tends to overestimate the premium of equity tranche. This phenomenon also can be found in the mortality swap and bond with independence assumption considered by Lin and Cox (2005). On the contrary, as Kendall's tau rises up, the tranche premiums of mezzanine increase gradually and approach the value of 0.06 and the tranche premiums of senior go up with a steeper slope than that of mezzanine. It implies that the independence assumption ($\tau = 0$) tends to underestimate both the premiums of mezzanine and senior. The reason is that low dependence among the policies reduces the probability of extreme loss and then results in low exposure in the mezzanine and senior tranches. On the other hand, low dependence among the policies will decrease the possibility of zero, thus the premium of the equity tranche decreases when Kendall's tau increases.

6. Conclusions

Being different from assumption of Lin and Cox (2005) that individual mortalities of policyholders are independent, this article attempts to propose the securitization in life insurance with morality dependence. The stochastic morality intensity of involving some risk factors (such as income, gender, age and other factors) governing mortality intensity function is also taken into consideration. By extending bivariate survivor model of Bassan and Spizzichino (2005), this study considers the correlation of multiple life insurances with Clayton copula for the joint mortality probability and then the premium for equality tranche, mezzanine tranche, and senior tranche are estimated.

It is the first study to design and price a new mortality-linked security, CIO. This security allows investors to invest various tranches of CIO which are discriminated by the mortality risk according to their risk preferences. Numerical results of CIO discover that the premium of equity tranche is a decreasing function of Kendall’s tau. This implies that the independence assumption tends to overestimate the premium of equity tranche. This phenomenon of overestimation also can be found in the mortality swap and bond with independence assumption proposed by Lin and Cox (2005). On the contrary, the premiums of mezzanine tranche and senior tranche are increasing functions of Kendall’s tau. It implies that the independence assumption tends to underestimate the premiums of mezzanine tranche and senior tranche. These results are very important because it reveals the necessity to consider the mortality dependence. Without considering mortality dependence, the insurance company will misprice the premiums of different tranches CIO. Hence, this article provides the insurance company a more accurate pricing method for CIO.

Table 1
Benefits of reference life insurance policies

Policy	Benefits	Ratio
Policy 1	450	26.47%
Policy 2	300	17.65%
Policy 3	100	5.88%
Policy 4	200	11.76%
Policy 5	25	1.47%
Policy 6	50	2.94%
Policy 7	100	5.88%
Policy 8	100	5.88%
Policy 9	50	2.94%
Policy 10	200	11.76%
Policy 11	50	2.94%
Policy 12	75	4.41%
Total	1700	100%

Table 2
The estimation death time of each policy

Policy	Benefits	Death time (year)	Whether die prior to maturity or not
Policy 8	100	5.115	yes
Policy 2	300	6.624	yes
Policy 10	200	9.093	yes
Policy 4	200	9.139	yes
Policy 1	450	9.588	yes
Policy 3	100	10.08	no
Policy 7	100	10.357	no
Policy 11	50	11.432	no
Policy 6	50	15.493	no
Policy 12	75	15.786	no
Policy 9	50	15.923	no
Policy 5	25	20.135	no

Table 3
The loss of sample path for each tranche

Year/Tranche	Equity Tranche 0~680	Mezznine Tranche 681~1020	Senior Tranche 1021~1700
1	0	0	0
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	100	0	0
7	300	0	0
8	0	0	0
9	0	0	0
10	280	340	230
Cumulative loss	680	340	230
Present value of loss	631.415	309.406	208.973

Table 4
Average nominal present value in each year

Year/Tranche	Equity Tranche 0~680	Present value	Mezznine Tranche 681~1020	Present value	Senior Tranche 1021~1700	Present value
1	680.000	673.234	340.000	336.617	680.000	673.234
2	680.000	666.535	340.000	333.268	680.000	666.535
3	680.000	659.903	340.000	329.952	680.000	659.903
4	680.000	653.337	340.000	326.668	680.000	653.337
5	680.000	646.836	340.000	323.418	680.000	646.836
6	591.530	557.082	251.530	236.882	591.530	557.082
7	467.080	435.503	149.664	160.293	467.080	435.503
8	280.000	258.473	171.916	158.698	280.000	258.473
9	280.000	255.901	171.916	157.119	280.000	255.901
10	8.058	7.291	57.009	51.584	8.058	7.291
Total average nominal present value		4814.095		2414.499		4814.095

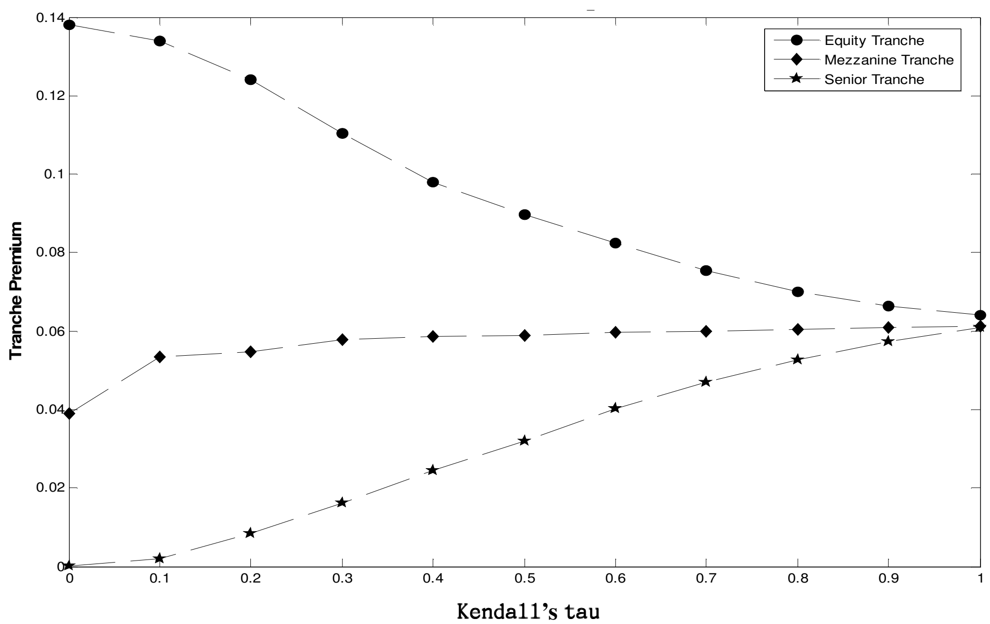

Fig. 2. Tranche premium as a function of Kendall's tau

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