HIGHER ORDER SYSTEMATIC CO-MOMENTS AND ASSET-PRICING: NEW EVIDENCE

Duong Nguyen*
Tribhuvan N. Puri*

Address for correspondence:
Duong Nguyen, PhD
Assistant Professor of Finance, Department of Accounting and Finance
Charlton College of Business
University of Massachusetts Dartmouth
285 Old Westport Road
North Dartmouth, MA 02747
Voice: 508-999-8070
dnguyen2@umassd.edu

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*University of Massachusetts, Dartmouth

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Abstract:

In this paper, we provide evidence supporting Rubinstein (1973)'s theoretical model that if returns do not follow normal distribution, measuring risk requires more than just measuring covariance, higher order systematic co-moments should be important to risk averse investors who are concerned about the extreme outcomes of their investments. Our paper also provides a contribution to the existing literature that not only Fama-French factors (SMB, HML) but also the momentum and the market liquidity factors are simply proxies for higher order co-moments. Using portfolios sorted by several criteria (size, book-to-market, and momentum), we find that including a set of 10 or higher systematic co-moments reduce the explanatory power of these factors to insignificance in almost every case. Our results are consistent in several robustness checks.

I. Introduction

The capital asset pricing model (CAPM), developed by Sharpe (1964), Lintner (1965) and its zero-beta version derived by Black (1972), have been widely investigated in theoretical as well as empirical studies. The appealing feature of CAPM is that it provides a very powerful and intuitively explanation with sound theoretical background about risk measurement and relation between risk and expected return. The CAPM has several assumptions but a crucial assumption is that the returns are normally distributed, therefore, the first two moments (i.e., mean and variance) alone are sufficient to explain the distribution. However, there is ample evidence that suggest otherwise (see for example, Fama (1965), Arditti (1971), Singleton and Wingender (1986), and more recently, Chung, Johnson, and Schill (2006)). This implies that the higher moments of return distribution are relevant to investors' decisions and should not be neglected.

Chung et al. (2006) argue that each co-moment may individually be unreliable, but a set of co-moments taken together, should not be. In fact, the set of co-moments is a measure of the likelihood of extreme outcomes, a matter of great importance to risk-averse investors. They sort stocks into size and book-to-market portfolios and find that, using a set of systematic co-moments 3^{rd} through 10^{th} substantially reduces the level of significance of Fama-French factors. One might argue that any set of variables would be able to reduce the significant levels of Fama-French factors if enough of them are included, however, when Chung et al use a set standard moment (not systematic co-moments), the significance levels of SMB (the return on a portfolio of small stocks less the return on a portfolio of big stocks) and HML (the return on a portfolio of high book-to-market stocks less the return on a portfolio of low book-to-market stocks) remain the same throughout almost all cases. Therefore, they conclude that the SMB and HML factors are simply proxies for higher systematic co-moments. Their results support Rubinstein (1973)'s

model in that measuring risk requires more than just measuring covariance and higher order co-moments (not standard moments) matter to risk averse investors.

Our paper builds on Chung et al. (2006). We argue that Fama-French factors SMB and HML are not the only non-market factors that affect stock returns. Gundy and Martin (2001), Fama and French (1996, 2004) suggest that one of the most serious problems to asset pricing is the momentum effect of Jagadeesh and Titman (1993, 2001). The stocks that do well relative to the market over the last three to twelve months tend to do well in the next few months and stocks that do poorly continue to do poorly. This momentum effect is distinct from value effect captured by stock characteristics and is not explained by the Fama-French three-factor model as well as by the CAPM.

The empirical finance literature has also documented another source of risk of concern to investor: liquidity risk. Studies by Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2001), and Huberman and Halka (2001) provide evidence of the existence of commonality across stocks in liquidity fluctuations. Their findings have initiated a new research hypothesis that if liquidity shocks are non-diversifiable and have a varying impact across individual securities, the more sensitive a stock's return to such shocks, the greater its expected return should be. This hypothesis has been supported by Pastor and Stambaugh (2003) and Acharya and Pedersen (2005). In particular, Pastor and Stambaugh (2003) develops a measure of aggregate liquidity, based on daily price reversal, and shows that stocks whose returns are more sensitive to market liquidity factor command higher required rate of return than stocks whose returns are less sensitive to market liquidity factor. Acharya and Pedersen (2005) employ the liquidity measure of Amihud (2002) to show that expected stock returns are a function of stock

liquidity level and several liquidity betas (covariances between stock return and market liquidity; stock liquidity and market return; stock liquidity and market liquidity).

The purpose of our paper is to examine whether the momentum and market liquidity factors are independent risk factors in their own right or merely proxies for higher order systematic co-moments as in the case of SMB and HML found in Chung et al. (2006). Our motivation is from the theoretical model of Rubinstein (1973) and empirical evidence in Chung et al. We argue that if the returns are not normal, measuring risk requires higher order systematic co-moments, not just covariance, and when higher order co-moments are considered, other factors such as SMB, HML, momentum, and market liquidity should not matter to investors.

Using Fama-Macbeth (1973) procedure for portfolios sorted by size, book-to-market, and momentum for the period 1970-2005, we find that adding a set of systematic co-moments of order 3 through 10 or higher reduces the explanatory power of SMB, HML, momentum, and Pastor-Stambaugh market liquidity factors to insignificance in almost all cases. Also, consistent with Chung et al (2006), we do not find the similar results when adding a set of standard moments of order 3 through 10 or higher. To check the stability of the results, we divide the sample into two sub-samples: 1970-1987 and 1988-2005, and perform similar analyses. The results still hold in both sub-periods. Our findings lend support to Chung et al (2006) and Rubinstein (1973) in that higher systematic co-moments are true factors in asset pricing and other factors such as SMB, HML, momentum, and market liquidity are simply proxies for these true factors.

Our paper is organized as follows. The next section discusses the data and methodology employed in the paper. Section III presents the empirical findings and evidence on stability of the results and section IV concludes the paper.

II. Data and Methodology

Following Chung et al (2006), we employ the two-step Fama-MacBeth procedure for our analyses. We sort all ordinary common stocks (with the CRSP share code = 10 and 11) in three stock exchanges, NYSE, AMEX, and NASDAQ, into 50 portfolios based on size, book-to-market, and momentum. In particular, for size portfolios, at the end of each calendar year in the period 1965-2005, all stocks are ranked based on their market capitalization and sorted into 50 portfolios of equal number of stocks. For book-to-market portfolios, all stocks are ranked by their beginning-of-period book-to-market ratios and then divided into 50 portfolios of equal size. The momentum portfolios are constructed as follows. At the end of each month t, all stocks are sorted into 50 portfolios of equal size based on their prior compound return from month t-2 to t-12. Using portfolios constructed above, we compute equally weighted monthly returns for each of the 50 portfolios. We subtract the 30-day Treasury bill yield to obtain the excess portfolio return. The size and book-to-market portfolios are rebalanced every year. The momentum portfolios are rebalanced every month.[1]

Once we have constructed the portfolios, we apply the two-step Fama-Macbeth (1973) procedure to our empirical asset pricing tests. In particular, first we test the five-factor model that includes Fama-French, momentum, and Pastor-Stambaugh factors as follows:

$$r(j,t) = a_0 + a_{rmrf} b_{rm}(j,t) + a_{smb} b_s(j,t) + a_{hml} b_h(j,t) + a_{mom} b_m(j,t) + a_{liq} b_l(j,t) + e(j,t) \qquad (1)$$

where $r(j,t)$ is the excess return of portfolio j in month t *and* $b_{rm}(j,t)$, $b_s(j,t)$, $b_h(j,t)$, $b_m(j,t)$, $b_l(j,t)$ are factor loadings for excess return of portfolio j on factors (*Rm-Rf*), SMB and

[1] The momentum portfolios are constructed along the line with the procedure in Ken French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french

HML, momentum (MOM), and Pastor-Stambaugh market liquidity (LIQ) respectively, in month *t*.[2]

For each month *t*, the factor loadings are computed by regressing portfolio returns over the past five years on the market factor (*Rm-Rf*), SMB, HML, MOM, and LIQ, respectively. The result is a time series for each factor loading from 1970 to 2005 (we lose the first five-year data in the original sample in order to estimate the factor loadings). Once the factor loadings are computed, for each month *t*, we perform cross-sectional regressions of the period portfolio returns on those loadings as in equation (1). Repeating this process for all months in the period 1970-2005, we have 432 sets of coefficient estimates. Following Fama-MacBeth, we average these estimates to get the average coefficients.

Our second analysis is to examine whether the above factor loadings are still significant when a set of systematic co-moments is added to the model. In particular, for each month t, we perform cross-sectional regressions of excess portfolio returns on the loadings of SMB, HML, MOM, and LIQ, and on the systematic co-moments as follows

$$r(j,t) = a_0 + a_{smb}b_s(j,t) + a_{hml}b_h(j,t) + a_{mom}b_m(j,t) + a_{liq}b_l(j,t) + \sum_{i=2}^{n} a_i b(i,j,t) + e(j,t) \qquad (2)$$

where $b(i,j,t)$ is the i th systematic co-moment of portfolio j in month t.[3] We compute the systematic co-moments in month t using the past 60 months of portfolio returns as follows:

$$b(i,j,t) = \frac{\sum_{\tau=1}^{60}\left[r(j,t-\tau) - \frac{1}{60}\sum_{k=1}^{60} r(j,t-k)\right]\left[r(m,t-\tau) - \frac{1}{60}\sum_{k=1}^{60} r(m,t-k)\right]^{i-1}}{\sum_{\tau=1}^{60}\left[r(m,t-\tau) - \frac{1}{60}\sum_{k=1}^{60} r(m,t-k)\right]} \qquad (3)$$

[2] We thank the Wharton Research Database Service (WRDS) for proving us the data on these factors.
[3] We include the loading on the market risk premium factor in the set of systematic co-moment since the loading is the 2nd systematic co-moment.

where $r(m,t)$ is the return of the CRSP value weighted index. We compute the systematic co-moments up to order 10th as in Chung et al (2006). We also experiment a set of systematic co-moments up to order 15th to see whether the findings in Chung et al are robust to higher order of co-moments or they are only chance results. Our findings which are described in the next section show that the two set of systematic co-moments give similar results

Since one might argue that including any set of variables would be able to reduce the significant levels of the factors, to address this issue, we also include a set of standard moments (not systematic co-moments) as follows

$$r(j,t) = a_0 + a_{rmrf} b_{rm}(j,t) + a_{smb} b_s(j,t) + a_{hml} b_h(j,t) + a_{mom} b_m(j,t) + a_{liq} b_l(j,t) + \sum_{i=3}^{n} a_i m(i,j,t) + e(j,t) \quad (4)$$

where $m(i,j,t)$ is the standard moment order *i* of portfolio *j* in month *t*. We also use the past 60 months of portfolio returns to compute $m(i,j,t)$ as follows

$$m(i,j,t) = \frac{1}{60} \sum_{\tau=1}^{60} [r(j,t-\tau)]^i \quad (5)$$

III. Empirical findings

Table 1 provides a summary statistics for the distributions of portfolios returns for size-sorted, book-to-market sorted, and momentum sorted portfolios. We use three statistics, namely Kolmogorov-Smirnov, Cramer-von Mises, Anderson-Darling, to test the normality of the portfolio returns. In all cases, the normality is strongly rejected. Since the returns do not follow normal distribution, the first two moments alone should not be sufficient to characterize the return distribution hence higher order moments should be considered.

We report the two-step Fama-MacBeth (1973)'s procedure applied to the five factor model (2^{nd} systematic co-moment, SMB, HML, MOM, and LIQ) in table 2. For each month, factors loadings: s, h, m, l, are computed by regressing portfolio returns on the SMB, HML, MOM, LIQ, respectively. Then, in each month, portfolio returns are regressed on these factor loadings (also including the 2^{nd} systematic co-moment, which is the loading on the market factor) to get the Fama-MacBeth coefficients. Panel A presents the results for all months including January. As can be seen from the table, at least two out of four factor loadings are significant for all portfolios. The factor loadings on HML and MOM: h and m, respectively, are significant in all cases, while the factor loading on market liquidity factor LIQ is significant only in the case of book-to-market sorted portfolios. We compute the F-statistics to examine whether the coefficients on SMB, HML, MOM, and LIQ are jointly different from zero. In all sorting criteria, the F-statistics reject the null hypothesis that all factor loadings are jointly equal to zero, suggesting at least one of the factors is significant in explaining cross sectional stock returns.

We also look at January returns and find that for all month including January, the SMB coefficient is statistically significant, however, when we exclude the months of January, the coefficient is no longer significant (see table 2, panel B). This implies that the size effect might be attributable to the January effect. The effects of HML, MOM, and LIQ remain unchanged.

In the next analysis, we examine how the significant levels of SMB, HML, MOM, and LIQ change when the systematic co-moments are added to the model. Table 3, panel A reports the results for size sorted portfolios. We find that the significance levels of factor loadings on SMB, HML, MOM, and LIQ start decreasing when we add more systematic co-moments and become insignificant when a set of 10 co-moments is included.[4] We also experiment with a set of 15 co-moments instead of a set of 10 co-moments, the results are not different. More

[4] The significant level of loading on LIQ factor basically remains insignificant throughout all cases.

interestingly, the magnitude of the F-statistics that test the joint significance of coefficients of the factors decrease when more systematic co-moments are added and become insignificant when a set of 10 or 15 co-moments is included. This implies that if a sufficient number of co-moments are considered, the factor loadings become insignificant in explaining cross sectional stock returns. Panel B and C report the results for book-to-market and momentum sorted portfolios. The findings are very similar to those in Panel A.

Since one may argue that any set of variable would be able to reduce the significance levels of the factor loadings if a sufficient number of them are included. We show that it is not the case when we add standard moments (not systematic co-moments) to the model. Standard moments are computed as in equation (5). The results are reported in Table 4. In all cases (for size, book-to-market, and momentum portfolios), whether we add a set of 10 or15 standard moments, the explanatory powers of factor loadings on SMB, HML, MOM, and LIQ remain almost unchanged compared with the original levels when no standard moment is included. The F-statistics are significant and their magnitudes are similar in all cases. The findings imply that standard moments do not reduce the significance of common factors, but the systematic co-moments do. This is consistent with Rubinstein (1973)'s model in that if return is not normal, risk averse investors should be concerned about higher order of co-moments.

Another important observation that our results are not driven by imprecision in estimation caused by adding more variables is that some factor loadings are insignificant before adding higher co-moments, they remain insignificant after adding higher systematic co-moments or even higher standard moments. It appears that a set of systematic co-moments only affects the factors that are significant and does not affect the factors that are not significant before adding more variables. A set of standard moments does not reduce the significant levels of the factors

regardless they are significant or insignificant at the original levels. If adding more variables causes imprecision in estimation, this should not be the case.

Stability of the results

Chung et al (2006) consider different return horizons: daily, weekly, monthly, quarterly, and semi-annually. However, since the Pastor and Stambaugh market liquidity factor is constructed using monthly data only, the data of this factor is not available for other horizons. Therefore, to check robustness of the results, we divide the sample into two sub-periods: (1) 1970-1987 and (2) 1988-2005. The results are reported in table 5 and 6. Since the results with a set of 10 systematic co-moments (or standard moments) are very similar to those with a set of 15 systematic co-moments (or standard moments), we report only those with the 10 co-moments (or standard moments) to be consistent with Chung et al (2006).

As can be seen from table 5, for size portfolios, when the model includes only the 2^{nd} systematic co-moment (covariance), factor loadings on SMB, HML, MOM, LIQ, the coefficients of these loadings are all significant. In particular, the t-statistics of *s, h, m, l* are 1.80, -6.55, 5.21, 3.42, respectively. The F-statistics for the joint significance of these loadings is 16.15, which is significant at 1 percent or below. However, when a set of 10 systematic co-moments is included, the t-statistics of *s, h, m, l,* are 0.55, -0.22, -0.53, 1.28, respectively, which are insignificant. The F-statistics reduces to only 1.21, which is also insignificant. The results with book-to-market portfolios are very similar to those with size portfolios. For the momentum portfolios, the loading on momentum factor, m, is significant at the original level (t-stat = 3.20), but becomes insignificant when adding a set of 2^{nd} to 10^{th} co-moments is added. The F-stat also reduces from 3.32 (significant at 5 percent level) to 0.83 (insignificant).

The results with standard moments in panel B, table 5 are also consistent to those in the whole sample in that adding a set of standard moment does not reduce the explanatory power of factor loadings for all size, book-to-market, and momentum portfolios. For the momentum portfolios, the coefficients of *s* and *l* are not significant. However, this is not the result of adding standard moments since they are insignificant even before we include the set of standard moments. The magnitudes of F-statistics remain almost unchanged before and after adding the set.

Table 6 presents the results for the second sub-period: 1988-2005. The findings are generally consistent with those in the first sub-period as well as in the whole sample. Only in the case of size portfolio, adding systematic co-moments does not reduce substantially the significance levels of all factor loadings. SMB and HML remain significant and the magnitude of F-stat does not reduce substantially. However, in other cases, the results are in line with previous findings.

We also look at January returns, since from table 2, the coefficient of SMB is significant (t-stat = 2.45) when January is included, but becomes insignificant when January is excluded (t-stat = 0.96). Therefore, one might argue that the effect of SMB on stock returns can be explained by January effect. To check this, we perform Fama-MacBeth regressions for the months of January only. The results are not tabulated but can be briefly described as follows. The t-statistic of SMB = 4.37 (when only the 2^{nd} systematic co-moment is included) reduces to 0.87 (when a set of 2^{nd} to 10^{th} systematic co-moments are included). However, adding a set of 3^{rd} to 10^{th} standard moments seems to be less successful at reducing the significance level of SMB (t-stat = 1.37). Overall, our robustness checks are consistent with the previous findings that adding a set

of systematic co-moments reduces substantially the significance levels of factor loadings: SMB, HML, MOM, and LIQ.

IV. Conclusion

In this paper, we attempt to provide evidence to support Rubinstein (1973)'s model that if returns are not normal, risk averse investors should be concerned about extreme outcomes of their investments which are measured by higher order systematic co-moments. Therefore, measuring risk requires more than just measuring covariance risk and when higher order systematic co-moments are considered, other common factors such as SMB, HML, MOM, LIQ, should not matter to investors.

In general, our empirical findings support the Rubinstein's model. For all sorting criteria (size, book-to-market, and momentum), we find that adding a set of 10 or 15 systematic co-moments reduces substantially the significance levels of the factors: SMB, HML, MOM, and LIQ and causes them to become insignificant in most cases. One might argue that the results are driven by imprecision in estimation due to adding more independent variables, we show that it is not the case. We perform a similar analysis with a set of 10 or 15 standard moments and find that the explanatory powers of the factors remain the same after including the set of standard moments. Also, in some cases, some factors remain consistently insignificant before and after adding the set of variables. Thus, it does not appear that our results are being driven by simply adding more explanatory variables. Our results are also consistent in both sub-periods as well as in January months.

Our paper also provides an extension to Chung et al (2006). We show that not only SMB, HML but also the momentum and market liquidity factors might simply be the proxies for higher

order systematic co-moments. The practical implication is that in a well-diversified portfolio, the idiosyncratic moments (e.g., standard deviation, skewness, kurtosis, etc.) are eliminated, investors only earn compensation for exposure to systematic co-moments (e.g., co-variance, co-skewness, co-kurtosis, etc.), and these systematic co-moments should be priced when investors consider their investments.

Reference:

Acharya, V.V. and L.H. Pedersen, 2005. Asset Pricing with Liquidity Risk, *Journal of Financial Economics* 77, 375-410.

Amihud, Y., 2002. Illiquidity and Stock returns: Cross Section and Time Series Effects, *Journal of Financial Markets*, 5, 31 – 56.

Arditti, F., 1971. Another Look at Mutual Fund Performance, *Journal of Financial and Quantitative Analysis* 6, 909-912.

Black, F. 1972. Capital Market Equilibrium with Restricted Borrowing, *Journal of Business*, 45, 444-454.

Chordia, T., R. Roll, and A. Subrahmanyam, 2000. Commonality in liquidity. *Journal of Financial Economics* 56, 3-28.

Chung, Y.P., H. Johnson, and M. Schill, 2006. Asset Pricing when Returns are Nonnormal: Fama-French factors vs higher-order systematic co-moments, J*ournal of Business*, 79, 923-940.

Fama, E., 1965, "The Behavior of Stock Market Prices," *Journal of Business*

Fama, E. and K. French, 1996. Multifactor Explanations of Asset Pricing Anomalies, *Journal of Finance* 51, 55-84.

Fama, E. and K. French, 2004. The Capital Asset Pricing Model: Theory and Evidence, *Journal of Economic Perspectives*, 18, 25-46.

Fama, E. and J. D. MacBeth, 1973. Risk, Return and Equilibrium: Empirical Tests, Journal of Political Economy 81, 607–636.

Gundy, B. and J. S. Martin, 2001. Understanding the nature of risks and the source of the rewards to momentum investing, *Review of Financial Studies* 14, 29-78.

Hasbrouck, J., and D. J. Seppi, 2001. Common factors in prices, order flows, and liquidity, *Journal of Financial Economics* 59, 383-411.

Huberman, Gur, and Dominika Halka, 2001. Systematic liquidity, *Journal of Financial Research* 24, 161-178.

Jegadeesh, N. and S. Titman, 1993. Return to Buying Winners and Selling Loser: Implications for Stock market Efficiency, *Journal of Finance*, v48, 65-91.

Jegadeesh, N. and S. Titman, 2001. Profitability of Momentum Strategies: An Evaluation of Alternative Explanations, *Journal of Finance*, v56 (2, Apr), 699-720.

Lintner, J., 1965. The valuation of risk assets and selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 47, 13-37.

Pástor, L. and R.F. Stambaugh, 2003. Liquidity risk and expected stock returns. *Journal of Political Economy* 111, 642-685.

Rubinstein M., 1973, The Fundamental Theorem of Parameter-preference Security Valuation, *Journal of Financial and Quantitative Analysis* 8, 61-69.

Sharpe, W.F., 1964. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, *Journal of Finance* 19, 425-442.

Singleton, J.C. and J. Wingender 1986. Skewness Persistence in Common Stock Returns, *Journal of Financial and Quantitative Analysis*, 21, 335-341.

Table 1: Summary statistics of portfolio return

The table reports the summary statistics for portfolio returns (size, book-to-market, and momentum) under analysis. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. The size and book-to-market portfolios are rebalanced every year. The momentum portfolios are rebalanced every month. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2002 are used in computation. Kolmogorov-Smirnov, Cramer-von Mises, Anderson-Darling statistics are used to test normality of portfolio returns.
***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

	Size portfolios	Book-to-market portfolios	Momentum portfolios
Number of portfolio-period observation	21600	21600	21600
Mean	0.0079	0.0083	0.0082
Variance	0.0045	0.0046	0.0047
Skewness	0.0474	0.0517	1.0992
Kurtosis	3.0200	3.1312	16.8590
Kolmogorov-Smirnov	0.0345***	0.0371***	0.0639***
Cramer-von Mises	10.978***	11.192***	40.647***
Anderson-Darling	79.937***	80.874***	259.654***

Table 2: Fama-Macbeth regression results

This table reports Fama-MacBeth regression estimates for size, book-to-market, and momentum portfolios. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. The size and book-to-market portfolios are rebalanced every year. The momentum portfolios are rebalanced every month. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2002 are used in computation. In each month, portfolio returns are regressed on the factor loadings *b, s, h, m, l*. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks (based on their prior compound return from month t-2 to t-12). LIQ is the Pastor-Stambaugh market liquidity factor. The mean coefficient estimates across the sample period are reported with their t-statistics. The F-statistics test the joint significance of the *s, h, m, l* estimates. Panel A reports the Fama-MacBeth regression results for all months including January, while Panel B presents the results for all months excluding January.
***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Panel A: all months including January

Portfolios	b	s	h	m	l	Mean adj. R^2 [F-stat]
Size	0.0356 (8.94)***	0.0041 (2.45)**	-0.0127 (-5.60)***	0.0129 (5.54)***	0.0049 (1.29)	0.46 [13.31]***
Book-to-market	0.0355 (9.01)***	0.0026 (1.53)	-0.0076 (-3.82)***	0.0077 (3.39)***	0.0110 (3.14)***	0.44 [7.39]***
Momentum	0.0011 (0.26)	-0.0005 (-0.28)	-0.0045 (-2.18)**	0.0101 (4.13)**	-0.0005 (-0.13)	0.43 [6.03]***

Panel B: all months excluding January

Portfolios	b	s	h	m	l	Mean adj. R^2 [F-stat]
Size	0.0409 (10.42)***	0.0016 (0.96)	-0.0148 (-6.27)***	0.0147 (6.22)***	0.0037 (0.99)	0.45 [17.05]***
Book-to-market	0.0414 (10.81)***	-0.0003 (-0.17)	-0.0092 (-4.42)***	0.0091 (4.01)***	0.0096 (2.68)***	0.43 [9.25]***
Momentum	-0.0043 (-1.21)	-0.0047 (-2.56)**	-0.0030 (-1.56)	0.0156 (7.26)***	-0.0035 (-1.02)	0.42 [16.11]**

Table 3: Systematic Co-moments and Common Factors

This table reports Fama-MacBeth regression estimates for size, book-to-market, and momentum portfolios when adding systematic higher order co-moments. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. The size and book-to-market portfolios are rebalanced every year. The momentum portfolios are rebalanced every month. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2002 are used in computation. In each month, portfolio returns are regressed on the factor loadings *b, s, h, m, l*, and the respective number of systematic co-moments. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. The systematic co-moments are estimated using the same rolling five-year portfolio return with the market return. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks (based on their prior compound return from month t-2 to t-12). LIQ is the Pastor-Stambaugh market liquidity factor. The mean coefficient estimates across the sample period are reported with their t-statistics. The F-statistics test the joint significance of the *s, h, m, l* estimates. Panel A reports the results for size portfolios, Panel B presents the results for book-to-market portfolios and panel C provides the results for momentum portfolios. ***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Panel A: size portfolios

Systematic Co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
2^{nd} to 3^{rd}	0.0030	-0.0123	0.0077	-0.0076	0.46
	(1.50)	(-5.36)***	(3.04)***	(-1.52)	[11.04]***
2^{nd} to 4^{th}	-0.0007	-0.0075	0.0059	-0.0116	0.47
	(-0.31)	(-2.88)**	(2.02)**	(-2.15)**	[5.63]***
2^{nd} to 5^{th}	-0.0018	-0.0089	0.0075	-0.0129	0.48
	(-0.75)	(-2.90)**	(2.42)**	(-2.33)**	[6.46]***
2^{nd} to 6^{th}	0.0020	-0.0121	0.0088	-0.0071	0.48
	(0.73)	(-3.31)***	(2.37)**	(-1.15)	[6.11]***
2^{nd} to 7^{th}	0.0010	-0.0098	0.0095	-0.0072	0.49
	(0.33)	(-2.18)**	(2.39)**	(-1.05)	[4.58]***
2^{nd} to 8^{th}	0.0030	-0.0135	0.0116	-0.0025	0.49
	(0.74)	(-2.66)**	(2.46)**	(-0.31)	[5.39]***
2^{nd} to 9^{th}	0.0070	-0.0156	0.0012	0.0034	0.49
	(1.51)	(-2.50)**	(0.21)	(0.39)	[2.67]**
2^{nd} to 10^{th}	0.0080	-0.0121	-0.0078	0.0045	0.49
	(1.39)	(-1.77)	(-1.16)	(0.47)	[1.38]
2^{nd} to 15^{th}	0.0135	-0.0090	-0.0032	0.0106	0.45
	(1.65)	(-0.98)	(-0.36)	(0.89)	[0.95]

Panel B: book-to-market portfolios

Systematic Co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
2^{nd} to 3^{rd}	0.0015	-0.0077	0.0054	0.0043	0.45
	(0.86)	(-3.58)***	(2.22)**	(0.97)	[4.05]***
2^{nd} to 4^{th}	0.0022	-0.0084	0.0068	0.0059	0.45
	(0.99)	(-3.20)***	(2.30)**	(1.12)	[3.99]***
2^{nd} to 5^{th}	0.0006	-0.0071	0.0080	0.0031	0.46
	(0.26)	(-2.24)**	(2.61)***	(0.57)	[3.21]**
2^{nd} to 6^{th}	0.0027	-0.0074	0.0084	0.0037	0.46
	(0.97)	(-2.04)**	(2.31)**	(0.63)	[2.44]**
2^{nd} to 7^{th}	0.0043	-0.0093	0.0083	0.0095	0.46
	(1.46)	(-2.09)**	(1.98)**	(1.43)	[2.04]*
2^{nd} to 8^{th}	0.0052	-0.0087	0.0074	0.0102	0.47
	(1.39)	(-1.79)*	(1.65)	(1.29)	[1.52]
2^{nd} to 9^{th}	0.0067	-0.0084	-0.0030	0.0120	0.47
	(1.55)	(-1.36)	(-0.55)	(1.41)	[0.77]
2^{nd} to 10^{th}	0.0025	-0.0015	-0.0079	0.0058	0.47
	(0.52)	(-0.22)	(-1.28)	(0.64)	[0.57]
2^{nd} to 15^{th}	0.0031	0.0013	0.0101	0.0020	0.42
	(0.43)	(0.13)	(1.14)	(0.17)	[0.52]

Panel C: momentum portfolios

Systematic Co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
2^{nd} to 3^{rd}	-0.0022	-0.0038	0.0099	-0.0014	0.44
	(-0.93)	(-1.79)*	(3.82)***	(-0.27)	[4.93]***
2^{nd} to 4^{th}	-0.0044	-0.0044	0.0061	-0.0045	0.44
	(-1.65)	(-1.75)*	(1.89)*	(-0.79)	[3.23]**
2^{nd} to 5^{th}	-0.0028	-0.0059	0.0042	-0.0013	0.45
	(-0.95)	(-1.60)	(1.09)	(-0.21)	[1.67]
2^{nd} to 6^{th}	-0.0065	-0.0008	0.0063	-0.0055	0.45
	(-1.92)	(-0.19)	(1.30)	(-0.85)	[1.92]
2^{nd} to 7^{th}	-0.0023	-0.0091	0.0052	0.0036	0.45
	(-0.66)	(-1.54)	(0.94)	(0.47)	[1.76]
2^{nd} to 8^{th}	-0.0001	-0.0079	0.0054	0.0062	0.46
	(-0.01)	(-1.29)	(0.95)	(0.71)	[0.93]
2^{nd} to 9^{th}	-0.0016	-0.0091	0.0054	0.0045	0.46
	(-0.35)	(-1.11)	(0.83)	(0.48)	[0.99]
2^{nd} to 10^{th}	-0.0032	-0.0056	0.0085	0.0013	0.46
	(-0.51)	(-0.63)	(1.08)	(0.12)	[0.81]
2^{nd} to 15^{th}	-0.0033	-0.0063	0.0115	0.0005	0.42
	(-0.42)	(-0.66)	(1.34)	(0.04)	[0.94]

Table 4: Standard Moments and Common Factors

This table reports Fama-MacBeth regression estimates for size, book-to-market, and momentum portfolios when adding a set of standard moment up to order 10 or 15. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. The size and book-to-market portfolios are rebalanced every year. The momentum portfolios are rebalanced every month. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2002 are used in computation. In each month, portfolio returns are regressed on the factor loadings *b, s, h, m, l*, and the respective number of standard moments. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. The standard moments are estimated using the same rolling five-year portfolio return with the market return. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks (based on their prior compound return from month t-2 to t-12). LIQ is the Pastor-Stambaugh market liquidity factor. The mean coefficient estimates across the sample period are reported with their t-statistics. The F-statistics test the joint significance of the *s, h, m, l* estimates.
***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Standard moments	s	h	m	l	Mean adj. R^2 [F-stat]
A. Size portfolios					
3rd to 10th	0.0171 (5.95)***	-0.0170 (-6.63)***	0.0082 (2.57)**	0.0134 (2.50)**	0.55 [13.75]***
3rd to 15th	0.0160 (5.78)***	-0.0156 (-5.89)***	0.0068 (2.14)**	0.0113 (2.11)**	0.54 [13.39]***
B. Book-to-market portfolios					
3rd to 10th	0.0122 (4.76)***	-0.0134 (-5.82)***	0.0078 (2.77)***	0.0113 (2.04)**	0.52 [11.64]***
3rd to 15th	0.0130 (4.70)***	-0.0131 (-5.63)***	0.0062 (2.05)***	0.0130 (2.21)**	0.51 [10.37]***
C. Momentum portfolios					
3rd to 10th	0.0031 (1.03)	-0.0054 (-2.67)***	0.0092 (2.89)***	0.0023 (0.48)	0.53 [3.57]***
3rd to 15th	0.0025 (0.69)	-0.0048 (-2.11)**	0.0084 (2.45)**	-0.0003 (-0.07)	0.50 [2.65]**

Table 5: Sub-period 1970-1987

This table reports Fama-MacBeth regression estimates in the sub-period 1970-1987 for size, book-to-market, and momentum portfolios when adding a set of systematic co-moments or a set of standard moments of order 3 through 10. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. The size and book-to-market portfolios are rebalanced every year. The momentum portfolios are rebalanced every month. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2002 are used in computation. In each month, portfolio returns are regressed on the factor loadings *b, s, h, m, l,* and the respective number of systematic co-moments or standard moments. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. The systematic co-moments and standard moments are estimated using the same rolling five-year portfolio return with the market return. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks (based on their prior compound return from month t-2 to t-12). LIQ is the Pastor-Stambaugh market liquidity factor. The mean coefficient estimates across the sample period are reported with their t-statistics. The F-statistics test the joint significance of the *s, h, m, l* estimates. Panel A reports the results with systematic co-moments, Panel B presents the results with standard moments. In each panel, the original indicates the models includes only the 2nd systematic co-moment (covariance), and the factor loadings on SMB, HML, MOM, and LIQ. ***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Panel A: Regressions with systematic co-moments

Systematic co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
A. Size portfolios					
Original	0.0038 (1.80)*	-0.0179 (-6.55)***	0.0158 (5.21)***	0.0172 [3.42]***	0.47 [16.15]***
2nd to 10th	0.0057 (0.55)	-0.0018 (-0.22)	-0.0055 (-0.53)	0.0176 (1.28)	0.49 [1.21]
B. Book-to-market portfolios					
Original	0.0019 (0.90)	-0.0114 (-4.86)***	0.0071 (2.41)**	0.0183 (4.39)***	0.44 [10.30]***
2nd to 10th	0.0004 (0.04)	0.0048 (0.70)	-0.0048 (-0.56)	0.0107 (0.85)	0.47 (1.50)
C. Momentum portfolios					
Original	-0.0006 (-0.24)	-0.0022 (-0.99)	0.0097 (3.20)***	-0.0030 (-0.77)	0.38 [3.32]**
2nd to 10th	-0.0090 (-0.80)	0.0062 (0.68)	0.0156 (1.38)	-0.0199 (-1.30)	0.41 (0.83)

Panel B: Regressions with standard moments

Standard moments	s	h	m	l	Mean adj. R^2 [F-stat]
A. Size portfolios					
3rd to 10th	0.0134 (3.38)***	-0.0157 (-5.36)***	0.0110 (2.56)**	0.0182 (3.36)***	0.54 [9.05]***
B. Book-to-market portfolios					
3rd to 10th	0.0131 (3.38)***	-0.0124 (-4.41)***	0.0107 (2.59)**	0.0183 (3.41)***	0.50 [8.39]***
C. Momentum portfolios					
3rd to 10th	0.0038 (0.56)	-0.0071 (-1.99)**	0.0118 (2.11)**	0.0010 (0.13)	0.46 (3.32)**

Table 6: Sub-period 1988-2005

This table reports Fama-MacBeth regression estimates in the sub-period 1988-2005 for size, book-to-market, and momentum portfolios when adding a set of systematic co-moments or a set of standard moments of order 3 through 10. Size portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end market capitalization. Book-to-market portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their previous year-end book-to-market ratios. Momentum portfolios are constructed by sorting stocks into 50 equal-size portfolios based on their prior compound return from month t-2 to t-12. The size and book-to-market portfolios are rebalanced every year. The momentum portfolios are rebalanced every month. All NYSE-AMEX-NASDAQ ordinary common stocks from 1965-2002 are used in computation. In each month, portfolio returns are regressed on the factor loadings *b, s, h, m, l,* and the respective number of systematic co-moments or standard moments. These factor loadings are computed by regressing portfolio returns using the past five year data on the market premium, SMB, HML, MOM, and LIQ factors, respectively. The systematic co-moments and standard moments are estimated using the same rolling five-year portfolio return with the market return. SMB and HML are Fama-French (1993) common factors, MOM represents the return on a portfolio of winner stocks less the return on a portfolio of loser stocks (based on their prior compound return from month t-2 to t-12). LIQ is the Pastor-Stambaugh market liquidity factor. The mean coefficient estimates across the sample period are reported with their t-statistics. The F-statistics test the joint significance of the *s, h, m, l* estimates. Panel A reports the results with systematic co-moments, Panel B presents the results with standard moments. In each panel, the original indicates the model including only the 2nd systematic co-moment (covariance), and the factor loadings on SMB, HML, MOM, and LIQ. ***, **, * denote significance level at 1 percent, 5 percent, and 10 percent respectively.

Panel A: Regressions with systematic co-moments

Systematic co-moments	s	h	m	l	Mean adj. R^2 [F-stat]
A. Size portfolios					
Original	0.0045 (1.70)*	-0.0076 (-2.10)**	0.0100 (2.83)***	-0.0073 (-1.27)	0.45 [3.98]***
2nd to 10th	0.0103 (2.12)**	-0.0224 (-2.06)**	-0.0101 (-1.17)	-0.0085 (-0.64)	0.48 [3.32]***
B. Book-to-market portfolios					
Original	0.0032 (1.23)	-0.0037 (-1.17)	0.0082 (2.39)**	0.0037 (0.66)	0.43 [1.78]
2nd to 10th	0.0048 (0.95)	-0.0078 (-0.68)	-0.0110 (-1.22)	0.0008 (0.06)	0.47 [0.50]
C. Momentum portfolios					
Original	-0.0004 (-0.15)	-0.0069 (-1.95)*	0.0106 (2.73)***	0.0018 (0.23)	0.49 [3.75]***
2nd to 10th	0.0024 (0.40)	-0.0175 (-1.15)	0.0014 (0.12)	0.0226 (1.49)	0.51 [0.98]

Panel B: Regressions with standard moments

Standard moments	s	h	m	l	Mean adj. R^2 [F-stat]
A. Size portfolios					
3rd to 10th	0.0129 (4.11)***	-0.0151 (-3.75)***	0.0015 (0.37)	-0.0054 (-0.67)	0.55 [7.81]***
B. Book-to-market portfolios					
3rd to 10th	0.0113 (3.35)***	-0.0144 (-3.95)***	0.0049 (1.28)	0.0044 (0.45)	0.52 [5.75]***
C. Momentum portfolios					
3rd to 10th	-0.0026 (-0.80)	-0.0044 (-1.42)	0.0114 (2.63)***	-0.0040 (-0.50)	0.60 [2.48]**