Fuzzy Multi-Criteria Decision Making to Select Mutual Funds Investment Style

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Abstract

The investment of mutual funds which investors are often required evaluating the investment strategies according to their own subjective preferences in terms of numerical values from various criteria. This situation can be regarded as a fuzzy multiple criteria decision-making (MCDM) problem. The purpose of this study attempts to propose an alternative approach, fuzzy multiple criteria decision making with fuzzy integral, relaxing the independence among considered criteria for evaluation of MCDM problems, which is oftentimes the basic assumption in applying AHP, for evaluating the strategies of selecting the mutual funds investment style in Taiwan. We also employ triangular fuzzy numbers to represent the decision makers' subjective preferences on the considered criteria, as well as for the criteria measurements to evaluate mutual funds investment style for investors. First, in this study we employ factor analysis to extract four independent common factors from those criteria. Second, we construct the evaluation frame using AHP composed of the above four common factors with sixteen evaluated criteria, and then derive the relative weights with respect to considered criteria. Third, the synthetic utility value corresponding to each mutual fund investment style is aggregated by the fuzzy weights with fuzzy performance values. Finally, we compare with empirical data and find that the model of FMCDM predict the rate of return very exactly in certain range λ , hence the non-additive fuzzy integral technique is an effective method to predict the mutual funds performance, meanwhile it can help investors to make decision in different conditions (λ).

Keywords: Mutual Fund, Fuzzy Multiple Criteria Decision Making (FMCDM), Triangular Fuzzy Number, Non-additive Fuzzy Integral.

1. Introduction

Mutual funds have become a popular avenue for investors and the net assets of mutual funds have grown exponentially from a mere \$17 billion in 1960 to over \$6 trillion in 2003. The number of mutual funds has increased to nearly ten thousand, exceeding the number of stocks listed on the organized exchange, making the selection of mutual funds an onerous task for the investor. In addition, the mutual funds are moving rapidly towards financial market development in response to increasing market demand and the mutual funds industry have emerged as a major player in the financial system with net assets of over $6 trillion and serving nearly 100 million investors. Therefore the mutual funds have huge market potential, have been gaining momentum in the financial market. The complexities are numerous, and overcoming these complexities to offer successful selections is a mutual fund manager's challenge.

The mutual fund managers need to evaluate aquatic return so as to reduce its risk to find the optimal combination of invested stocks out of many feasible stocks and allocating the amount of investing funds to many stocks. Because of the limit amount of funds to invest into mutual funds, the solution of the portfolio selection problem proposed by Markowitz [20] has a tendency to increase the number of stocks selected for mutual funds. In a real investment, a fund manager first makes a decision on how much proportion of the investment should go to the market, and then he invests the fund to which stocks which is the stock selection ability. After that, many researchers explained in the presence of market-timing ability that actions will affect the performance of mutual funds. When invest mutual funds, some reports also point out that there are 90% investors will consider the rate of return firstly, next is the reputation of Mutual Funds Corporation and investment risk. Maximizing the mutual fund performance is the primary goal of mutual fund managers in a corporation. Usually, the mutual fund return reflects the financial performance of a fund corporation for operating and development. This paper explores which criteria can lead to high mutual fund performance.

In real world systems, the decision-making problems are very often uncertain or vague in a number of ways. Due to lack of information, the future state of the system might not be known completely. This type of uncertainty has long been handled appropriately by probability theory and statistics. However, in many areas of daily life, such as mutual fund, stock, debt, derivatives and others, human judgment, evaluation, and decisions often employ natural language to express thinking and subjective perception. In these natural languages the meaning of words is often vague and might be well defined, but when using the word as a label for a set, the boundaries within which objects do or do not belong to the set become fuzzy or vague. Furthermore, human judgment of events may be significantly different based on individual's subjective perceptions or personality, even using the same words. Fuzzy numbers are introduced to appropriately express linguistic variables. Therefore the investment of mutual funds which investors are often required evaluating the investment strategies according to their own subjective preferences in terms of numerical values from various criteria. This situation can be regarded as a fuzzy multiple criteria decision-making (MCDM) problem.

In this paper the fuzzy hierarchical analytic approach was used to determine the weights of criteria from subjective judgment, and a non-additive integral technique was utilized to evaluate the performance of investment style for mutual funds. Traditionally, researchers have used additive techniques to evaluate the synthetic performance of each criterion. In this article, we demonstrate that the non-additive fuzzy integral is a good means of evaluation and appears to be more appropriate, especially when the criteria are not independent situations. The conceptual investment of mutual funds is discussed in the next section. The fuzzy hierarchical analytic approach and non-additive fuzzy integral evaluation process for multi-criteria decision-making (MCDM) problems are derived in the subsequent section. Then an illustrative example is presented, applying the FMCDM methods for mutual funds, after which we discuss and show how the FMCDM methods in this paper are effective. Finally, the conclusions are presented.

2. Review of Mutual Fund Investment

With the number of mutual funds growth and variation, the investors concern about the subject is how to select the mutual funds that can enjoy the advantage and make money. Jensen [12] demonstrated that, in the presence of market-timing ability. Grant [10] explained how market-timing actions will affect the results of empirical tests that focus only on microforecasting skills. Treynor and Mazuy [28] added a quadratic term to the Jensen function to test for market-timing ability. Jensen [14] developed theoretical structures for the evaluation of micro and macroforecasting performance of fund managers where the basis for evaluation is a comparison of the ex post performance of the fund manager with the returns on the market. Merton [21] and Henriksson's model differs from the Jensen [14] formulation in that their forecasters follow a more qualitative approach to market timing. Chang and Lewellen [19] and Henriksson [11] employed the Merton-Henriksson model in evaluating mutual fund performance and found no evidence of market timing by fund managers. Bhattacharya and Pfleiderer [2] extended the work of Jensen [14]. By correcting an error made in Jensen, they show that one can use a simple regression technique to obtain accurate measures of timing and selection ability.

The investment performance of mutual fund managers has been extensively examined in the finance literature. Most of these studies employed a method developed by Jensen [12, 13] and later refined by Black, Jensen and Scholes [3], Blume and Friend [4]. Such a method compares a particular manager performance with that of a benchmark index fund. Connor and Korajczyk [8] developed a method of portfolio performance measurement using a competitive version of the arbitrage pricing theory (APT). However, they ignored any potential market timing by managers. One weakness of the above approach is that it fails to separate the aggressiveness of a fund manager from the quality of the information he possesses. It is apparent that superior performance of a mutual fund manager occurs because of his ability to "time" the market and his ability to forecast the returns on individual assets. Fama [9] indicates that there are two ways for fund managers to obtain abnormal returns. The first one is security analysis, which is the ability of fund managers to identify the potential winning securities. The second one is market timing, which is the ability of portfolio managers to time market cycles and takes advantage of this ability in trading securities.

Lehmann and Modest [18] combined the APT performance evaluation method with the Treynor and Mazuy [28] quadratic regression technique. They found statically significant measured abnormal timing and selectivity performance by mutual funds. They also examined the impact of alternative benchmarks on the performance of mutual funds finding that performance measures are quite sensitive to the benchmark chosen and finding that a large number of negative selectivity measures. Also, Henriksson [11] found a negative correlation between the measures of stock selection ability and market timing. Lee and Rahman [17] empirically examine market timing and selectivity performance of mutual funds. It is important that fund managers be evaluated by both selection ability and market timing skill. They concentrate on a fund manager's security selection and market timing skills. However, in mutual fund areas, external evaluation, human judgment and subjective perception also affect the performance of mutual funds. In actual the performance of mutual funds involve many criteria, in this article we will discuss these criteria and performance at the same time.

3. The Method of Fuzzy Multi-Criteria Decision-Making

In this section we employ factor analysis to extract four independent common factors from those criteria. At the same time we construct the evaluation frame using AHP (Analytic Hierarchy Process) composed of the above four common factors with sixteen evaluated criteria, and derive the relative weights with respect to considered criteria. Then the synthetic utility value corresponding to each mutual fund investment style is aggregated by the fuzzy weights with fuzzy performance values. Traditional AHP is assumed that there is no interaction between any two criteria within the same hierarchy. However, a criterion is inevitably correlated to another one with the degrees in reality. Sugeno [24] introduced the concept of fuzzy measure and fuzzy integral, generalizing the usual definition of a measure by replacing the usual additive property with a weak requirement, i.e. the monotonic property with respect to set inclusion. In this section, we give a brief to some notions from the theory of fuzzy measure and fuzzy integral. We describe a fuzzy hierarchical analytic approach to determine the weighting of subjective judgments. Since investors can not clearly estimate each considered criterion in terms of numerical values for the anticipated alternatives/strategies, fuzziness is considered to be applicable.

3.1 General fuzzy measure

The fuzzy measure is a measure for representing the membership degree of an object in candidate sets. It assigns a value to each crisp set in the universal set and signifies the degree of evidence or belief of that element's membership in the set. Let X be a universal set. A fuzzy measure is then defined by the following function g: $\aleph \rightarrow [0, 1]$

That assigns each crisp subset of X a number in the unit interval [0, 1]. The definition of function g is the power set $\aleph$. When a number is assigned to a subset of X, $A \in \aleph$, $g(A)$, this represents the degree of available evidence or the subject's belief that a given element in X belongs to the subset A. This particular element is most likely found in the subset assigned the highest value.

In order to quantify a fuzzy measure, function g needs to conform to several properties. Normally function g is assumed to meet the axiom of the probability theory, which is a probability theory measurement. Nevertheless, actual practice sometimes produces a result against the assumption. This is why the fuzzy measure should be defined by weaker axioms. The probability measure will also become a special type of fuzzy measure. The axioms of the fuzzy measures include:

(1) $g(\phi)=0$, $g(X)=1$ (boundary conditions);

(2) $\forall A,B \in \aleph$, if $A \subseteq B$ then $g(A) \le g(B)$ (monotonicity).

Once the universal set is infinite, it is required to add continuous axioms [16].

Certainly the elements in question are not within the empty set but within the universal set, regardless of the amount of evidence from the boundary conditions in Axiom 1.

The fuzzy measure is often defined with an even more general function:

g: $\beta \rightarrow [0,1]$

where $\beta \subset \aleph$ so that:

1. $\phi \in \beta$ and $X \in \beta$;
2. if $A \in \beta$, then $\overline{A} \in \beta$
3. β is closed under the operation of set function; i.e., if $A \in \beta$ and $B \in \beta$, then $A \cup B \in \beta$.

The set β is usually called the Borel field. The triple (X, β, g) is called a fuzzy measure space if g is a fuzzy measure on a measurable space (X, β).

It is sufficient to consider the finite set in actual practice. Let X be a finite criterion set, $X = \{x_1, x_2, .., x_n\}$ and the power set $\aleph$ be a class of all of the subsets of X. It can be noted that $g(\{x_i\})$ for a subset with a single element, x_i is called a fuzzy density. In the following paragraph, we use g_i to represent: $g(\{x_i\})$.

The term "general fuzzy measure" is used to designate a fuzzy measure that is only required to satisfy the boundary condition and monotonic to differentiate the λ-fuzzy measure, F-additive measure, and classical probability measure.

3.2 λ - Fuzzy measure

The specification for general fuzzy measures requires the values of a fuzzy measure for all subsets in X. Sugeno and Terano have developed the λ-additive axiom [26] in order to reduce the difficulty of collecting information. Let (X, β, g) be a fuzzy measure space: $\lambda \in (-1, \infty)$. If $A \in \beta$, $B \in \beta$; and $A \cap B = \phi$, and

$$g(A \cup B) = g(A) + g(B) + \lambda g(A) g(B) \tag{1}$$

If this holds, then fuzzy measure g is λ-additive. This kind of fuzzy measure is named λ fuzzy measure, or the Sugeno measure. In this paper we denote this λ-fuzzy measure by g_i to differentiate from other fuzzy measures. Based on the axioms above, the λ-fuzzy measure of the finite set can be derived from fuzzy densities, as indicated in the following equation:

$$g_\lambda(\{x_1, x_2\}) = g_1 + g_2 + \lambda g_1 g_2 \tag{2}$$

where g_1, g_2 represents the fuzzy density.

Let set $X = \{x_1, x_2, .., x_n\}$ and the density of fuzzy measure $g_i = g_\lambda(\{x_i\})$, which can be formulated as follows:

$$g_\lambda(\{x_1, x_2, ..., x_n\}) = \sum_{i=1}^{n} g_i + \lambda \sum_{i_1=1}^{n-1} \sum_{i_2=i_1+1}^{n} g_{i_1} g_{i_2} + \cdots + \lambda^{n-1} g_1 g_2 \cdots g_n \tag{3}$$

For an evaluation case with two criteria, A and B, there are three cases based on the above properties.

Case 1: if λ >0, i.e. $g_\lambda(A \cup B) > g_\lambda(A) + g_\lambda(B)$, implying that A and B have a multiplicative effect.

Case 2: if λ =0, i.e. $g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B)$, implying that A and B have an additive effect.

Case 3: if λ <0, i.e. $g_\lambda(A \cup B) < g_\lambda(A) + g_\lambda(B)$, implying that A and B have a substitutive effect.

The fuzzy measure is often used with the fuzzy integral for aggregating information evaluation by considering the influence of the substitutive and multiplication effect among all criteria.

3.3 Fuzzy integral [24, 25, 26]

In a fuzzy measure space(X, β, g), let h be a measurable set function defined in the fuzzy measurable space. Then the definition of the fuzzy integral of h over A with respect to g is

$$\int_A h(x)dg = \sup_{\alpha \in [0,1]} [\alpha \wedge g(A \cap H_\alpha)] \tag{4}$$

where H_α ={ x belonging to X |$h(x) \geq \alpha$ }. A is the domain of the fuzzy integral. When A=X, then A can be taken out.

Next, the fuzzy integral calculation is described in the following. For the sake of simplification, consider a fuzzy measure g of $(X, \aleph)$ where X is a finite set. Let $h: X \to [0,1]$ and assume without loss of generality that the function $h(x_j)$ is monotonically decreasing with respect to j, i.e., $h(x_1) \geq h(x_2) \geq \cdots \geq h(x_n)$. To achieve this, the elements in X can be renumbered. With this, we then have

$$\int h(x)dg = \bigvee_{i=1}^{n} \left[f(x_i) \wedge g(X_i) \right] \tag{5}$$

where $X_i = \{x_1, x_2, \cdots, x_i\}$, i= 1,2,$\cdots$,n.

In practice, h is the evaluated performance on a particular criterion for the alternatives, and g represents the weight of each criterion. The fuzzy integral of h with respect to g gives the overall evaluation of the alternative. In addition, we can use the same fuzzy measure using Choquet's integral, defined as follows [22].

$$\int hdg = h(x_n)g(X_n) + [h(x_{n-1}) - h(x_n)]g(X_{n-1}) + \cdots + [h(x_1) - h(x_2)]g(X_1) \tag{6}$$

The fuzzy integral model can be used in a nonlinear situation since it does not need to assume the independence of each criterion.

3.4 Fuzzy integral multi-criteria assessment methodology

The fuzzy integral is used in this study to combine assessments primarily because this model does not need to assume independence among the criteria. The fuzzy integral proposed by Sugeno [24] and Sugeno and Kwon [25] is then applied to combine the efficiency value of those related criteria to produce a new combined performance value. A brief overview of the fuzzy integral is presented here: Assume under general conditions, $h(x_1^k) \geq \cdots \geq h(x_i^k) \geq \cdots \geq h(x_n^k)$ where $h(x_i^k)$ is the performance value of the k-th alternative for the i-th criterion, the fuzzy integral of the fuzzy measure $g_\lambda(X_n^k)$ with respect to $h(x_n^k)$ on $\aleph$ $(g{:}\aleph \rightarrow [0,1])$ can be defined as follows [6, 7, 15].

$$(c)\int^k hdg = h(x_n^k)g_\lambda(X_n^k) + [h(x_{n-1}^k) - h(x_n^k)]g_\lambda(X_{n-1}^k) + \cdots + [h(x_1^k) - h(x_2^k)]g_\lambda(X_1^k) \quad (7)$$

where $g_\lambda(X_1^k) = g_\lambda(\{x_1^k\})$, $g_\lambda(X_2^k) = g_\lambda(\{x_1^k, x_2^k\})$, …, $g_\lambda(X_n^k) = g_\lambda(\{x_1^k, x_2^k, \cdots, x_n^k\})$

The fuzzy measure of each individual criterion group $g_\lambda(X_n^k)$ can be expressed $\sum_{i=1}^{n} g_\lambda(x_i^k) + \lambda\sum\sum g_\lambda(\{x_i\})g_\lambda(\{x_j\}) + \cdots \lambda^{n-1} g_\lambda(\{x_1\})\cdots g_\lambda(\{x_n\})$ as follows:

$$g_\lambda(X_n^k) = g_\lambda(\{x_1^k, x_2^k \cdots x_n^k\}) = \sum_{i=1}^{n} g_\lambda(x_i^k) + \lambda\sum\sum g_\lambda(\{x_i\})g_\lambda(\{x_j\}) + \cdots \lambda^{n-1} g_\lambda(\{x_1\})\cdots g_\lambda(\{x_n\})$$

$$= \frac{1}{\lambda}\left[\prod_{i=1}^{n}(1+\lambda g_\lambda(x_i^k)) - 1\right] \qquad \text{for } -1 < \lambda < +\infty \quad (8)$$

λ is the parameter that indicates the relationship among related criteria (if $\lambda = 0$, equation (7) is an additive form, if $\lambda \neq 0$, equation (7) is a non-additive form). The fuzzy integral defined by equation $(c)\int f dg$ is called the Choquet integral.

4. Evaluation model for prioritizing the mutual funds strategy

We build up a hierarchical system [29] for evaluating mutual funds strategies. Its analytical procedures stem from three steps: (i) factor (ii) criteria (iii) investment style. We employ factor analysis to extract four independent common factors from those criteria which is (1) Market timing (2) Stock selection ability (3) Fund size (4) Team work. And, we construct the evaluation frame using AHP composed of the above four common factors with sixteen evaluated criteria, then derive the relative weights with respect to considered criteria. According to the risk of investment, the mutual funds with investment style classified as S1: Asset Allocation style; S2: Aggressive Growth style; S3: Equity Income style; S4: Growth style; S5: Growth Income style. Based on a review of the literature, personal experience, and interviews with senior mutual fund managers, relevance trees are used to create hierarchical strategies for developing the optimal selection strategy of mutual funds. The elements (nodes) of relevance trees are defined and identified in hierarchical strategies, the combination of which consists of an evaluating mechanism for selecting a mutual fund strategy, as shown in Fig. 1.

4.1 Evaluating the mutual funds strategy hierarchy system

Minimum risk or maximum return is usually used as the measurement index in traditional financial evaluation methods. According to the risk of investment, the mutual funds are classified as five investment style and we evaluate the performance by the rate of return. Within a dynamic and diversified decision-making environment, the traditional quantity method does not solve the non-quantity problems of mutual funds selection. Therefore, what is needed is a useful and applicable strategy that addresses the issues of selection mutual funds. We propose a FMCDM method to evaluate the hierarchy system for selecting mutual funds strategies.

The performance of mutual funds architecture includes four components as market timing, stock selection ability, fund size and team work. An empirical investigation discusses conceptual and econometric issues associated with identifying four components of mutual funds performance. In addition, the criteria in the investment process are sometimes vague. When this occurs, the investment process becomes ambiguous and subjective for the investor. The evaluation is conducted in an uncertain, fuzzy situation and to what extent vague criteria are realized by research is unknown [7, 27]. Evaluation in an uncertain, fuzzy situation applies to the formulation of mutual funds strategies as well. We have chosen a fuzzy multiple criteria evaluation method for selecting and prioritizing the mutual funds strategies to optimize the real scenarios faced by managers or investors.

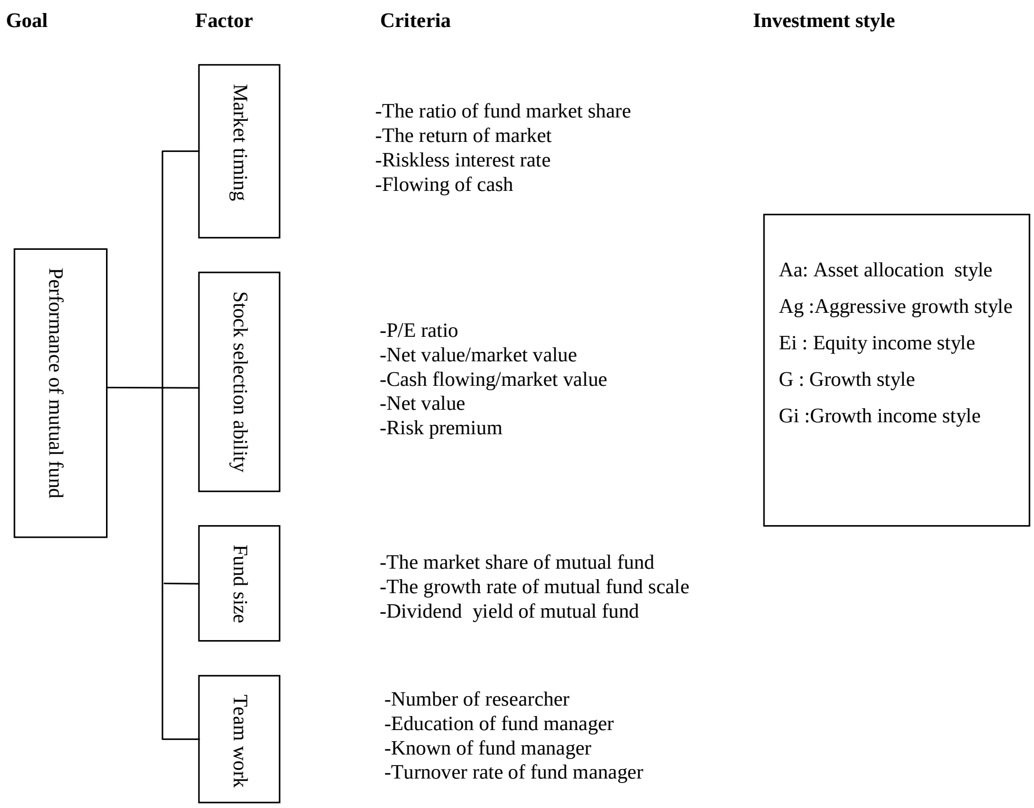

Fig.1 Relevance system of hierarchy tree for evaluating mutual funds strategy

4.2 The process for evaluating and prioritizing mutual funds strategies

Bellman and Zadeh [1] were the first to study the decision-making problem in a fuzzy environment and initiated FMCDM. In this study, we use this method to evaluate various mutual funds strategies and rank them by their performance. The following subsection describes the method of FMCDM.

4.2.1 Fuzzy weights for the hierarchy process

An evaluator always perceives the weight of a hierarchy subjectively. Therefore, to consider the uncertain, interactive effects coming from other criteria when calculating the weight of a specified criterion, we have used fuzzy weights of criteria.

Buckley [5] was the first to investigate fuzzy weights and fuzzy utility for AHP techniques, extending AHP by geometric mean method to derive the fuzzy weights.

The fuzzy weights $\tilde{w}_j$ corresponding to each criterion is as follows:

$$\tilde{w}_j = \tilde{r}_j \otimes (\tilde{r}_1 \oplus \cdots \oplus \tilde{r}_m)^{-1} \tag{9}$$

where $\tilde{r}_j$ is the geometric mean of each row of AHP reciprocal matrix

$$\tilde{r}_j = (\tilde{a}_{j1} \otimes \cdots \otimes \tilde{a}_{jm})^{1/m} \tag{10}$$

4.2.2 Measuring criteria

The evaluators were asked to make subjective judgments using linguistic variable measurement to demonstrate the criteria performance with expressions of effectiveness ranging from “very high”, “high”, “fair”, “low”, to “very low”. Each linguistic variable was indicated using a Triangular Fuzzy Number (TFN) with a range from 0 to 100. Let $\tilde{E}_{ik}^{j}$ indicate the fuzzy performance value in terms of estimator *j* toward strategy *k* under criteria *i* and the performance of the criteria is represented by the *S*, then,

$$\tilde{E}_{ik}^{j} = (L\tilde{E}_{ik}^{j}, M\tilde{E}_{ik}^{j}, U\tilde{E}_{ik}^{j}), \quad i \in S \tag{11}$$

In this study, we used the notion of average value to consolidate the fuzzy judgment value of *m* estimators, i.e.,

$$\tilde{E}_{ik}^{j} = (1/m) \otimes (\tilde{E}_{ik}^{1} \oplus \tilde{E}_{ik}^{2} \oplus \cdots \oplus \tilde{E}_{ik}^{m}), \quad j = 1, 2, ..., m. \tag{12}$$

The sign $\otimes$ denotes fuzzy multiplication and the sign $\oplus$ denotes fuzzy addition. $\tilde{E}_{ij}^{k}$ is the average fuzzy number from the judgment of the decision-maker. It can be represented using a triangular fuzzy number as follows:

$$\tilde{E}_{ik}^{j} = (L\tilde{E}_{ik}^{j}, M\tilde{E}_{ik}^{j}, U\tilde{E}_{ik}^{j}) \tag{13}$$

where

$$L\tilde{E}_{ik}^{j} = (1/m) \otimes \left(\sum_{j=1}^{m} L\tilde{E}_{ik}^{j} \right)$$

$$M\tilde{E}_{ik}^{j} = (1/m) \otimes \left(\sum_{j=1}^{m} M\tilde{E}_{ik}^{j} \right)$$

$$U\tilde{E}_{ik}^{j} = (1/m) \otimes \left(\sum_{j=1}^{m} U\tilde{E}_{ik}^{j} \right)$$

The preceding end point value may be solved using the method introduced by Buckly [5] or by Chiou and Tzeng [7].

4.2.3 Fuzzy synthetic decision

The weight of the different criteria and the fuzzy performance value needs to be operated using fuzzy integral techniques to generate the synthetic performance of each strategy within the same dimension.

Furthermore, we have calculated the synthetic performance of each alternative strategy using different λ values. Additionally, the fuzzy synthetic performance is conducted by a simple additive weight method assuming the criteria are independent in a fuzzy environment. Since each individual criterion is not completely independent from the others, we use the non-additive fuzzy integral technique to find the synthetic performance of each alternative, and to investigate the order of the synthetic performance of different λ values.

The result of fuzzy synthetic decisions reached by each alternative is a fuzzy number. It is therefore the non-fuzzy ranking method for fuzzy numbers that must be employed in order to compare the various strategies. In previous works the procedure of de-fuzzification had involved the mutual funds of the best non-fuzzy performance (BNP) value. The methods for defuzzified fuzzy ranking generally include the mean of maximum, center of area (COA), and α-cut [23, 33].

We utilize the center of area (COA) method in this paper to rank the order of importance of each strategy. The BNP value for the fuzzy number $\tilde{R}_i$ can be found using the following equation:

$$BNP_i = [(U\tilde{R}_i - L\tilde{R}_i) + (M\tilde{R}_i - L\tilde{R}_i)]/3 + L\tilde{R}_i \qquad \forall i \tag{14}$$

5. Empirical study and discussions

In order to demonstrate the practicality of our proposed method of evaluating the strategy of mutual funds, we conducted an empirical study based on a total of 30 valid samples from 12 Taiwanese mutual fund companies and 8 research institutes and universities.

The majority of the respondents were fund managers who are responsible for financial or general management. The mutual funds strategy selection process is examined below.

5.1 Evaluating the weights of criteria

By using the fuzzy AHP method the weights of the factors and criteria were found and are shown in Table 1.

5.2 Estimating the performance matrix

In this study, the estimators define their individual range for the linguistic variables employed in this study based on their judgments within the range from 0-100. The fuzzy judgment values of different estimators regarding the same evaluation criteria are averaged. In general, fuzzy addition and multiplication were used to retrieve the average fuzzy numbers for the performance values under each criterion indicated by the estimators for mutual funds strategy.

Table 1 The weights of criteria for evaluating the mutual funds

Criteria	fuzzy weight $(L\tilde{E}_{ik}^{j}, M\tilde{E}_{ik}^{j}, U\tilde{E}_{ik}^{j})$	BNP of overall weight
Market timing	(0.236 0.427 0.719)	**0.461**
The ratio of fund market share	(0.114 0.197 0.359)	0.223
The return of market	(0.263 0.437 0.729)	0.476
Riskless interest rate	(0.130 0.242 0.432)	0.268
Flowing of cash	(0.066 0.124 0.226)	0.139
Stock selection ability	(0.218 0.353 0.592)	**0.388**
P/E ratio	(0.119 0.211 0.368)	0.232
Net value/market value	(0.081 0.143 0.257)	0.160
Cash flowing/market value	(0.039 0.062 0.110)	0.070
Net value	(0.097 0.172 0.323)	0.197
Risk premium	(0.235 0.412 0.694)	0.571
Fund size	(0.090 0.143 0.244)	**0.159**
The market share of mutual fund	(0.207 0.323 0.522)	0.351
The growth rate of mutual fund scale	(0.087 0.129 0.218)	0.145
Dividend yield of mutual fund	(0.335 0.548 0.851)	0.578
Team work	(0.049 0.076 0.133)	**0.086**
Number of researcher	(0.130 0.269 0.452)	0.284
Education of fund manager	(0.081 0.138 0.270)	0.163
Known of fund manager	(0.253 0.439 0.776)	0.489
Turnover rate of fund manager	(0.095 0.154 0.292)	0.180

Table 2 The evaluation results of mutual funds strategy

Mutual funds strategy ranking	
λ =-1, -0.5;	S2 $\succ$ S4 $\succ$ S3 $\succ$ S1 $\succ$ S5
λ = 0, 1, 3;	S4 $\succ$ S3 $\succ$ S2 $\succ$ S5 $\succ$ S1
λ = 5;	S4 $\succ$ S3 $\succ$ S5 $\succ$ S2 $\succ$ S1
λ = 10, 20;	S4 $\succ$ S5 $\succ$ S3 $\succ$ S2 $\succ$ S1
λ = 40, 100;	S5 $\succ$ S4 $\succ$ S3 $\succ$ S2 $\succ$ S1
λ = 150, 200;	S5 $\succ$ S4 $\succ$ S2 $\succ$ S3 $\succ$ S1

where: S1: Asset Allocation style; S2: Aggressive Growth style; S3: Equity Income style; S4: Growth style; S5: Growth Income style.

Table 3 The synthetic performance of mutual funds strategy

λ	-1.00	-0.50	0.00	1.00	3.00	5.00	10.00	20.00	40.00	100.00	150.00	200.00
Aa	385.25	527.97	299.72	298.89	297.82	297.13	296.07	294.90	293.73	292.47	291.55	291.08
Ag	606.77	971.22	310.88	309.76	307.63	306.00	303.26	300.20	297.12	294.00	291.88	290.87
Ei	459.00	672.52	312.89	311.82	309.50	307.67	304.55	301.07	297.57	294.42	291.70	290.58
G	553.23	856.70	314.68	313.72	311.79	310.25	307.59	304.50	301.30	297.94	295.69	294.59
Gi	353.09	383.85	310.72	309.25	307.46	306.37	304.80	303.30	302.00	300.95	300.18	299.89

5.3 Evaluation and prioritization of the mutual funds strategy

The empirical evidence in this paper indicates that the weight of criteria such as market timing (0.461), stock selection ability (0.388), fund size (0.159) and team work (0.086). So the market timing was the most important factor to influence the performance of mutual funds, next is the stock selection ability. Some econometric methodology is developed to simultaneously estimate the magnitudes of these portfolio performance evaluation measures. Those results show that mutual fund managers are on average with positive security selection and negative market timing ability. It means that mutual fund managers are on average better with selectivity ability than with market timing ability. Therefore, the mutual fund managers should enhance the ability of market timing, the performance of mutual funds can be better.

By ranking fuzzy weights and fuzzy synthetic performance values, we can determine the relative importance of criteria and decide the best strategies. We apply what is called a λ fuzzy measure and non-additive fuzzy integral technique to evaluate investment strategies. The fuzzy synthetic performance of each alternative using different λ is as shown in Table 2 and Table 3. In table 2, our empirical results show when $\lambda < 0$, the aggressive growth style is the most important strategy and growth style is selected as the second important strategy. When $\lambda \geq 0 \sim 5$, the results show that growth style is the most important strategy, and equity income style is selected as the second important strategy. When $\lambda = 10 \sim 30$, the results show that growth style is the most important strategy, then is the growth income style strategy. When $\lambda \geq 40$, the results show that growth income style replaces growth style becoming the second ranking. In the other hand, when $\lambda \geq 0$, the results show that asset allocation style is the worst strategy which has the smallest synthetic performance, we can infer that the less risk the funds are, the less performance of the funds will be.

From Table 3, when $\lambda \geq 0$, implying that mutual funds have a multiplicative effect. But λ is bigger, the synthetic performance is smaller. So we should not adopt several strategies at the same time. On the other hand, when $\lambda < 0$, implying that mutual funds have a substitutive effect. Unexpectedly the synthetic performance is bigger, so we adopt independent strategy to be better. From investment style shows that the aggressive growth style has the largest deviation in performance. In other word, the more aggressive the funds are, the more volatility of the fund performance will be.

5.4 Comparing with the empirical data

To detect the performance of mutual funds, monthly returns from January 1980 to September 1996 (201 months) for a sample of 65 U.S. mutual funds are used in this study. The random sample of mutual funds is provided by the MorningStar Company. The MorningStar Company segregates mutual funds into four basic investment styles on the basis of manager's portfolio characteristics. Our sample consists of 8 Asset Allocation (AA), 14 Aggressive Growth (AG), 10 Equity Income (EI), 16 Growth (G), and 17 Growth Income (GI) mutual funds. The monthly returns on the S&P 500 Index were used for market return. Monthly observations of the 30-day Treasury bill rate were used as a proxy for the risk-free rate.

Appendix 2 Panel A is the abbreviation of investment style index. Appendix 2 Panel B contains summary statistics for returns of the mutual funds. All values are computed in excess of the return on the U.S. T-bill closest to 30 days to maturity. Data contains mean, standard deviation, maximum, and minimum. Each investment style average shows that the asset allocation style has the smallest expected return and it also has the smallest standard deviation. However, the aggressive growth style has the largest maximum return but it also has the smallest minimum return and the largest standard deviation. In other word, the more aggressive the funds are, the more volatility of the fund returns will be.

5.5 Discussions and managerial implications

This study focuses on providing a mutual funds strategy for the companies of mutual fund managers so that they may be successful in their decision-making. Our empirical study demonstrates the validity of this method. In this study, the mutual funds strategy stems from four aspects: market timing, stock selection ability, fund size as well as team work. The related issues, evaluation criteria are defined in this research (see Appendix 1).

Picking a mutual fund from among the thousands offered is not easy. Mutual fund managers have difficulty in selecting the proper strategy. The major reasons are the uncertain and dynamic environment and numerous criteria that they are facing. Managers are hence overwhelmed by this vague scenario and do not make proper decisions or allocate resources efficiently. The hierarchical method guides the manager how to select the investment style of mutual funds in the uncertainty environment. With the help of this model, managers can employ different experts to conduct the same proposed procedures and select the best investment alternative. The subjective judgment and risks of making wrong decisions is then minimized. In addition, this method can be applied to solve different kinds of problems by modifying the constructs of the hierarchy trees and finding the appropriate solution.

Few studies have addressed mutual funds related strategy planning. Providing that this is a first attempt to formally model the formulation process for a mutual funds strategy using FMCDM, we have the confidence that the analysis here is a significant contribution to the literature, and will help to establish groundwork for future research. Even though we are dedicated to setting up the model as completely as possible, there are additional criteria (for example, tax, expenses, dividend, etc.) and methods that could be adopted and added in future research. In the meantime, we should also begin to investigate how to execute several strategies simultaneously in order to achieve the optimal selecting of mutual funds under uncertainty environment.

6. Conclusion

The mutual fund is moving rapidly towards financial market development in response to increasing market demand. Therefore, what is needed is a useful and applicable method that addresses the selecting of mutual funds. We use a FMCDM method to achieve this goal. This study show that the less risk the funds are, the less performance of the funds will be, and the more aggressive the funds are, the more volatility of the fund performance will be.

We compare with empirical data and find that the model of FMCDM predict the rate of return very exactly when λ =10~30, hence the non-additive fuzzy integral technique is an effective method to predict the mutual fund performance, meanwhile it can help investor to make decision in different conditions (λ).

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Appendix 1

The description of evaluative criteria of mutual funds

Criteria	Description
Market timing	The ability of portfolio managers to time market cycles and takes advantage of this ability in trading securities
The ratio of fund market share	The ratio of fund invests in securities.
The return of market	The ups and downs of deep bid index current period divide by the deep bid index of last period.
Riskless interest rate	The risk-free interest rate is the interest rate that it is assumed can be obtained by investing in financial instruments with no default risk. In practice most professionals and academics use short-dated government bonds of the currency in question. For Taiwan investments, usually Taiwan bank one month deposit rate is used.
Flowing of cash	Cash flow refers to the amounts of cash being received and spent by a business during a defined period of time, sometimes tied to a specific project. Measurement of cash flow can be used to evaluate the state or performance of a business or project.
Stock selection ability	The ability of fund managers to identify the potential winning securities.
P/E ratio	The P/E ratio (price per share/earnings per share) of a mutual fund is used to measure how cheap or expensive its share price is. The lower the P/E, the less you have to pay for the mutual fund, relative to what you can expect to earn from it.
Net value/market value	The value of an entity's assets less the value of its liabilities divided by market value.
Cash flowing/market value	It equals cash receipts minus cash payments over a given period of time divided by market value; or equivalently, net profit plus amounts charged off for depreciation, depletion, and amortization (business) divided by market value.
Net value	Net value is a term used to describe the value of an entity's assets less the value of its liabilities. The term is commonly used in relation to collective investment schemes.
Risk premium	A risk premium is the minimum difference between the expected value of an uncertain bet that a person is willing to take and the certain value that he is indifferent to.
Fund size	The volume and scale of mutual funds.
The market share of mutual fund	It can be expressed as a company's sales revenue (from that market) divided by the total sales revenue available in that market. It can

	also be expressed as a company's unit sales volume (in a market) divided by the total volume of units sold in that market.
The growth rate of mutual fund scale	The scale of the fund of current period increases and decreased the amount of money divide by the scale of last fund.
Dividend yield of mutual fund	The dividend yield on a company mutual fund is the company's annual dividend payments divided by its market cap, or the dividend per share divided by the price per share.
Team work	The culture of mutual funds company.
Number of researcher	The number of researcher of each fund.
Education of fund manager	Fund manager’s seniority, quality and performance.
Known of fund manager	Fund manager’s rate of exposed in the medium and number of win a prize.
Turnover rate of fund manager	Fund manager leave his job temporarily.

Appendix 2

Summary Statistics for Returns of the Mutual Funds

The notations and definition of the investment style of mutual funds are in panel A.

Panel A

Classifications	Investment Style	Description
Aa	Asset allocation	A large part of financial planning is finding an asset allocation that is appropriate for a given person in terms of their appetite for and ability to shoulder risk. The designation of funds into various categories of assets.
Ag	Aggressive growth	Regardless of the investment style or the size of the companies purchased, funds vary widely in their risk and price behavior which is likely to have a high beta and high volatility.
Ei	Equity income	It will invest in common stock, but will have a portfolio beta closer to 1.0 than to 2.0. It likely favors stocks with comparatively high dividend yields so as to generate the income its name implied.
G	Growth	The pursuit of capital appreciation is the emphasis with growth funds. This class of funds includes those called aggressive growth funds and those concentrating on more stable and predictable growth.
Gi	Growth income	It pays steady dividends, and it is still predominately an investment in stocks, although some bonds may be included to increase the income yield of the fund.

Monthly mutual funds are from January 1980 to September 1996 for a sample of 65 U.S. mutual funds. The data are from Morningstar Company.

Panel B

Fund Name	Investment Style	Mean	Standard Deviation	Maximum	Minimum
GENERAL SECURITIES	Aa	0.477	5.084	15.389	-17.151
FRANKLIN ASSET ALLOCATION	Aa	0.407	3.743	10.424	-19.506
SELIGMAN INCOME A	Aa	0.394	2.414	8.474	-7.324
USAA INCOME	Aa	0.316	2.024	9.381	-5.362
VALLEY FORGE	Aa	0.293	1.803	9.980	-5.573
INCOME FUND OF AMERICA	Aa	0.566	2.552	9.166	-8.836
FBL GROWTH COMMON STOCK	Aa	0.273	3.599	10.466	-24.088

MATHERS	Aa	0.220	3.910	14.405	-14.750
Asset Allocation Average	Aa	0.391	2.550	8.962	-9.464
AMERICAN HERITAGE	Ag	-0.905	6.446	28.976	-33.101
ALLIANCE QUASAR A	Ag	0.644	6.547	15.747	-39.250
KEYSTONE SMALL CO GRTH (S-4)	Ag	0.433	7.053	19.250	-38.516
KEYSTONE OMEGA A	Ag	0.473	6.112	18.873	-33.240
INVESCO DYNAMICS	Ag	0.510	6.009	17.378	-37.496
SECURITY ULTRA A	Ag	0.222	6.940	16.297	-43.468
PUTNAM VOYAGER A	Ag	0.808	5.781	17.179	-29.425
STEIN ROE CAPITAL OPPORT	Ag	0.578	6.783	17.263	-32.135
VALUE LINE SPEC SITUATIONS	Ag	0.145	6.240	13.532	-37.496
VALUE LINE LEVERAGED GR INV	Ag	0.601	4.970	14.617	-29.025
WPG TUDOR	Ag	0.726	6.010	14.749	-33.658
WINTHROP AGGRESSIVE GROWTH A	Ag	0.476	5.596	17.012	-34.921
DELAWARE TREND A	Ag	0.787	6.536	14.571	-42.397
FOUNDERS SPECIAL	Ag	0.564	5.900	12.905	-31.861
Aggressive Growth Average	Ag	0.459	5.814	13.142	-35.335
SMITH BARNEY EQUITY INCOME A	Ei	0.601	3.270	7.813	-18.782
VAN KAMPEN AM CAP EQTY-INC A	Ei	0.510	3.530	12.292	-22.579
VALUE LINE INCOME	Ei	0.423	3.357	9.311	-18.242
UNITED INCOME A	Ei	0.714	4.037	11.852	-13.743
OPPENHEIMER EQUITY-INCOME A	Ei	0.555	3.422	10.071	-16.524
FIDELITY EQUITY-INCOME	Ei	0.706	3.612	10.608	-19.627
DELAWARE DECATUR INCOME A	Ei	0.547	3.615	10.269	-20.235
INVESCO INDUSTRIAL INCOME	Ei	0.601	3.705	9.349	-20.235
OLD DOMINION INVESTORS'	Ei	0.360	3.699	11.498	-21.092
EVERGREEN TOTAL RETURN Y	Ei	0.508	3.220	8.074	-13.857
Equity Income Average	Ei	0.527	3.238	9.094	-18.718
GUARDIAN PARK AVENUE A	G	0.740	4.391	11.321	-27.965
FOUNDERS GROWTH	G	0.718	4.986	13.055	-25.108
FORTIS GROWTH A	G	0.724	5.983	14.520	-30.771
FRANKLIN GROWTH I	G	0.570	4.050	12.907	-11.706
FORTIS CAPITAL A	G	0.682	4.791	12.818	-21.585
GROWTH FUND OF AMERICA	G	0.625	4.722	12.226	-23.962
HANCOCK GROWTH A	G	0.484	5.381	15.708	-25.236
FRANKLIN EQUITY I	G	0.469	5.156	12.818	-32.135

NATIONWIDE GROWTH	G	0.598	4.370	11.444	-27.570
NEUBERGER&BERMAN FOCUS	G	0.434	4.366	12.187	-25.108
MSB	G	0.517	4.665	13.452	-31.178
NEUBERGER&BERMAN PARTNERS	G	0.661	3.612	9.311	-19.385
NEUBERGER&BERMAN	G	0.606	5.095	11.574	-30.500
MANHATTAN	G	0.710	4.067	10.125	-19.385
NICHOLAS	G	0.225	5.234	11.321	-31.451
OPPENHEIMER A	G	0.727	5.802	19.120	-37.207
NEW ENGLAND GROWTH A	G	0.608	4.505	11.121	-26.081
Growth Average	G	0.594	4.775	12.649	-26.255
PIONEER II A	Gi	0.517	4.386	10.912	-29.693
PILGRIM AMERICA MAGNACAP A	Gi	0.611	3.949	10.843	-22.704
PIONEER	Gi	0.410	4.339	12.293	-28.361
PHILADELPHIA	Gi	0.244	4.004	11.074	-23.457
PENN SQUARE MUTUAL A	Gi	0.504	3.907	11.852	-20.724
OPPENHEIMER TOTAL RETURN A	Gi	0.507	4.451	13.861	-22.829
VANGUARD/WINDSOR	Gi	0.726	4.078	10.746	-18.542
VAN KAMPEN AM CAP GR & INC A	Gi	0.570	4.781	15.349	-32.135
VAN KAMPEN AM CAP COMSTOCK A	Gi	0.599	4.539	13.167	-34.921
WINTHROP GROWTH & INCOME A	Gi	0.430	3.987	10.717	-24.088
WASHINGTON MUTUAL INVESTORS	Gi	0.723	3.882	11.409	-20.113
SAFECO EQUITY	Gi	0.587	4.797	14.263	-31.042
SELIGMAN COMMON STOCK A	Gi	0.553	4.224	11.785	-23.331
SALOMON BROS INVESTORS O	Gi	0.583	4.194	11.785	-24.980
SECURITY GROWTH & INCOME A	Gi	0.233	3.825	10.161	-19.674
SELECTED AMERICAN	Gi	0.650	3.969	13.142	-19.385
PUTNAM FUND FOR GRTH & INC A	Gi	0.637	3.540	8.456	-22.081
Growth Income Average	Gi	0.544	3.940	10.380	-24.469