Interest Rates Term Structure Forecasting and Bond Portfolio Risk Management

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ABSTRACT

The dynamic Nelson-Siegel-style models, which are popular in the literature of interest rates term structure forecasting, may be unstable because of the potential existence of unit roots in the parameter series. In this paper, the dynamic Nelson-Siegel-style models are modified by modelling the first-order differenced instead of original parameter series. Empirical study shows that the modified models yield significantly smaller RMSE than the original ones when forecasting, meanwhile the forecasting RMSE of the modified models is much more stable even when the forecasting horizon increases. Besides, we interpret that the traditional duration of one bond is proportional to its expected return. Based on that, the information concerned with the forecasted bonds' yields, which can be calculated by discounting their future cash flows after the term structure of interest rates is forecasted, are introduced to the constraint equations of the traditional duration matching model in interest rates risk hedging. Empirical study shows that incorporation of the forecasted yields information of bonds can improve the performance of interest rates risk hedging when applied to mid-term or long-term target bonds.

Key Words: dynamic Nelson-Siegel-style models; interest rates term structure forecasting; risk management

JEL classification: C53; E43; G11

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1. Introduction

Forecasting the term structure of interest rates is crucial for bond portfolio risk management. Despite great advances in modeling the dynamic behavior of yield curve (Vasicek 1977; Cox et al. 1985; Duffie and Kan 1996; Ho and Lee 1986; Hull and White 1990; Heath et al. 1992) in recent years, Duffee (2002) concludes that these models perform poorly in forecasting.

To provide an enhanced forecasting model for the yield curve, Diebold and Li (2006) extend the parsimonious Nelson-Siegel model (Nelson and Siegel, 1987) to the dynamic Nelson-Siegel model, which performs particularly well. Besides, some other works focusing on interest rates term structure forecasting (Almeida et al. 2009; Laurini and Hotta, 2010; Pooter et al. 2010; Yu and Zivot, 2011) are all based on the parsimonious Nelson-Siegel model or its extended versions. These models are named as dynamic Nelson-Siegel-style models by Pooter et al. (2010). Augmented Dickey-Fuller tests suggest that some parameter series of the dynamic Nelson-Siegel model have a unit roots (Diebold and Li, 2006), and the largest eigenvalue of the state transition equation estimated by Diebold et al. (2006) is 0.98, slightly less than 1, which implies that the dynamic Nelson-Siegel-style models may be unstable.

Nawalkha and Soto (2009) classify the interest rates risk hedging models into four categories given as M-absolute (Nawalkha and Chambers, 1996)/M-square (Fong and Vasicek, 1984) models, duration vector (Chambers et al. 1988; Crack and Nawalkha, 2000)/M-vector (Nawalkha and Chambers, 1997) models, key rate duration models (Ho, 1992), and principal component duration models (Litterman and Scheinkman, 1991). However, none of these models considers the problem of incorporating the interest rates forecasting information. Wang and Kang (2010) suggest introducing the forecasted price information to the target function of the traditional duration matching model, but the hedging portfolio under this model may be undiversified because of converging to certain bonds.

In this paper, we suggest modifying the dynamic Nelson-Siegel-style models by modeling the first-order differenced parameter series to solve the unstable problem. Besides, we propose to incorporate the forecasted price (yield) to the constraint equations instead of the target function of the traditional duration matching model to make sure the hedging portfolio diversified thoroughly.

The remainder of this paper is organized as follows. In Section 2 we provide the modified dynamic Nelson-Siegel-style models and the risk management model incorporating the forecasted yields of bonds. Then in Section 3, we proceed to the empirical study of forecasting the term structure of interest rates under both the original dynamic Nelson-Siegel-style models and the first-order difference modified models, and hedging the interest rates risk under the traditional duration model and our model with forecasted yield incorporated. And at last in Section 4, we draw the conclusion.

2. The models

2.1 Term structure forecasting models

2.1.1 Dynamic Nelson-Siegel model

Diebold and Li (2006) modify the parsimonious Nelson-Siegel model and assume that the term structure evolves as equation (1):

$$y_t(\tau) = \beta_{0t} + \beta_{1t}\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} + \beta_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) \tag{1}$$

where $y_t(\tau)$ denotes the time t yield of zero coupon bond whose maturity is τ. The parameter λ_t which is fixed to a constant determines the decay rate of the loadings on β_{1t} and β_{2t}. The loading on β_{0t} for all maturities is always 1, so it can be viewed as the level factor of interest rates term structure. The loading on β_{1t} is $(1-e^{-\lambda_t\tau})/\lambda_t\tau$, which initiates at 1 and decrease to 0 monotonically, so β_{1t} represents the slope factor. The loading on β_{2t} is $((1-e^{-\lambda_t\tau})/\lambda_t\tau) - e^{-\lambda_t\tau}$, which initiates at 0, increase and then decrease to 0, so β_{2t} represents the curvature factor.

Diebold and Li (2006) propose two models for the evolution of $\beta_{it}, i = 0,1,2$. Either assume the dynamics of the vector of these parameters follows VAR(1) process:

$$\beta_t = u + A\beta_{t\text{-}1} \tag{2}$$

or assume the evolution of these parameters follow univariate AR(1) process:

$$\beta_{it} = u_i + a_i\beta_{it\text{-}1} \qquad i = 0,1,2 \tag{3}$$

where $\beta_t = (\beta_{0t}, \beta_{1t}, \beta_{2t})'$, u is a (3×1) vector, A is a (3×3) matrix, u_i and a_i are constants. It is obvious that β_t can be fitted and forecasted through the model constructed by equation (1) and (2), or the model constructed by equation (1) and (3). For simplicity, we denote the former model by NS-VAR and the latter one by NS-AR.

2.1.2 Incorporating macroeconomic variables

There is some link between macro factors and interest rates term structure. Theoretically, yield spread of long-term and short-term interest rates is often used to measure inflation; Interest rates level can influence consuming and investing behavior so as to influence the growth of real economy; Short-term interest rates is the target of monetary policy, et al. Empirically, Pooter et al. (2010), Yu and Zivot (2011) find that the incorporation of macro factors such as real economy, inflation and monetary policy can enhance the models' forecasting performance. Denote the time t (3×1) vector of macro factors as M_t, which is consist of the real economy factor, inflation factor and monetary policy factor, Diebold et al. (2006) incorporate it as follows:

$$f_t = \bar{u} + \bar{A} f_{t-1} \tag{4}$$

where $f_t = (\beta_t', M_t')'$, $\bar{u}$ represents the (6×1) constant vector and $\bar{A}$ is the (6×6) coefficient matrix. In fact, equation (4) is similar to equation (2), the only difference is the dimension of these state vector. We denote the model consist of equation (1) and equation (4) by NS-VAR-Macro.

Meanwhile, M_t can be introduced as an exogenous vector (Pooter et al. 2010) as in equation (5):

$$\beta_t = u + A\beta_{t-1} + BM_t \tag{5}$$

where u and A keep the same as in equation (2), and B is another coefficient matrix. We denote the model consist of equation (1) and equation (5) by NS-VAR-X.

2.1.3 First-order difference modified dynamic Nelson-Siegel-style models

Diebold and Li (2006) find that some parameter series of the dynamic Nelson-Siegel model have a unit roots. Besides, the largest eigenvalue of A in equation (2) estimated through the US interest rates term structure data (Diebold et al. 2006) is 0.98, slightly less than 1. These imply that the dynamic Nelson-Siegel-style models may be unstable. So, we argue that the first-order differenced series of β_t (or f_t) should be used when modeling. In fact, the Augmented Dickey-Fuller tests on β_t presented in Section 3.1.3 suggest doing so.

Take the NS-VAR-Macro model as an example, equation (6) and equation (7) gives the modified model:

$$\Delta y_t = \bar{\Lambda} \Delta f_t \tag{6}$$

$$\Delta f_t = \bar{u} + \bar{A}\Delta f_{t-1} \tag{7}$$

where Δ is the first-difference operator. Obviously, equation (6) and equation (7) are the modified version of equation (1) and equation (4) respectively. We denote the model consist of equation (6) and equation (7) by D-NS-VAR-Macro.

The modified models of NS-AR, NS-VAR and NS-VAR-X are similar to equation (6) and equation (7) in form, so the specific equations are omitted here. For simplicity, we denote the modified version of NS-AR, NS-VAR and NS-VAR-X by D-NS-AR, D-NS-VAR and D-NS-VAR-X respectively.

In fact, NS-AR, NS-VAR, NS-VAR-X and NS-VAR-Macro are the so-called dynamic Nelson-Siegel-style models (Pooter et al. 2010), so, we name the modified models (D-NS-AR, D-NS-VAR, D-NS-VAR-X and D-NS-VAR-Macro) as first-order difference modified dynamic Nelson-Siegel-style models.

2.2 Risk management models

After forecasting the interest rates term structure, we can predict the prices and yields of bonds, which can be incorporated to hedge the risk of bonds. In this section, we introduce the constraint condition of the predicted yields matching to the traditional duration matching model.

2.2.1 Duration and bonds' yield

Duration is the classic tool for measuring and hedging interest rates risk. Duration D_i of bond i is defined as equation (8):

$$D_i = -\frac{1}{\Delta y}\frac{\Delta P_i}{P_i} \tag{8}$$

where P_i is the price of bond i, Δy and ΔP_i represent the change of yield and price separately. Because of Δy is the same for all bonds, duration D_i measures the expected yield of bond i. So we can assume that the risk hedging performance can be improved by incorporating the forecasted yields of bonds to the traditional duration matching model.

2.2.2 Risk management model incorporating forecasted yields

The traditional duration matching model can be formulated as follows:

$$\min \quad \sum \omega_i^2 \tag{9}$$

$$\text{s.t. } D_T = \sum \omega_i D_i \tag{10}$$

where D_T is the target bond's duration, ω_i and D_i are the weight and duration of the ith bond in the hedging portfolio respectively. Obviously, the sum of ω_i should equal to 1. The target function defined by equation (9) can diversify the portfolio thoroughly (Nawalkha and Soto, 2009).

Suppose at time t, we intend to build a portfolio to hedge the interest rates risk of the target bond over horizon h. In fact, if we can forecast the interest rates term structure at time $t+h$, then we can discount the cash flows of a bond to calculate its theoretical price at time $t+h$. Denote the forecasted price of target bond over horizon h as $\tilde{B}_{t+h,T}$, then we can forecast that its yield from time t to $t+h$ is:

$$\tilde{y}_{t,t+h,T} = \ln(\tilde{B}_{t+h,T} / B_{t,T}) \tag{11}$$

where $B_{t,T}$ is its price at time t. Similarly, we can forecast the yield of the ith bond in the hedging portfolio from time t to $t+h$ follows:

$$\tilde{y}_{t,t+h,i} = \ln(\tilde{B}_{t+h,i} / B_{t,i}) \tag{12}$$

where $B_{t,i}$ and $\tilde{B}_{t+h,i}$ are the ith bond's forecasted price at time $t+h$ and real price at time t respectively.

If the portfolio can perfectly hedge interest rates risk of the target bond, their forecasted yields should be equal:

$$\tilde{y}_{t,t+h,T} = \sum \omega_i \tilde{y}_{t,t+h,i} \tag{13}$$

Then we can expect to enhance the hedging performance by incorporating the forecasted yields matching constraint into the risk management model. That is the enhanced model follows:

$$\min \quad \sum \omega_i^2 \tag{14}$$

$$\text{s.t.} \quad \begin{cases} D_{Target} = \sum \omega_i D_i \\ \tilde{y}_{t,t+h,T} = \sum \omega_i \tilde{y}_{t,t+h,i} \end{cases} \tag{15}$$

where the sum of ω_i should be also equal to 1.

3. Empirical study

3.1 Term structure forecasting

3.1.1 Data

Limited by the availability, the end-of-month T-bonds' price data of Shanghai Security

Exchange (SSE) in China range from March 2003 to March 2011 is employed. We use the unsmoothed Fama-Bliss method (Fama and Bliss, 1987) as Diebold and Li (2006) demonstrated to estimate the monthly interest rates term structure. Those maturities employed to compare the forecasting performance under all the original and modified dynamic Nelson-Siegel-style models are 3, 6, 9, 12, 24, 36, 48, 60, 84, and 120 months.

When analyzing the impacts of macro factors on term structure, the factors usually are considered in three categories: real economy, inflation and monetary policy (Matsumura and Moreira, 2011). Limited by the availability of the data, Industrial Added Value, CPI and Monetary Base (M1) are chosen to measure real economy, inflation and monetary policy respectively. Obviously, we should also employ the monthly data. The bonds' price and macro data is collected from the CSMAR database and the NBSC's website http://*www.stats.gov.cn.*

3.1.2 Value of λ

Nelson and Siegel (Nelson and Siegel, 1987) insist that the fitting result would not be influenced significantly if λ is a pre-specified value. The main purpose of this section is to study the influence of first-order difference on the forecasting results, so we fix λ to 0.79 according to Kang and Wang (2010) for simplicity.

3.1.3 Two-steps method

The two-steps method is employed to model and forecast the parameters of original and modified dynamic Nelson-Siegel-style models. We exemplify the two-steps method under the NS-VAR model. The series of β_t can be estimated through equation (1) by ordinary least squares method period by period, then substituted into equation (2) to model and forecast β_t. Similar to the results of Diebold and Li (2006), Table 1 shows that the Augmented Dickey-Fuller tests suggest that β_{0t} and β_{1t} may have a unit roots, and that β_{2t} does not, while at the ten percent level.

Table 1. p value of the ADF tests on the parameter series

	level	1st differenced
β_{0t}	0.6505	0
β_{1t}	0.4361	0
β_{2t}	0	0

As a result, it's reasonable and necessary to modify the dynamic Nelson-Siegel-style models by modeling first-order differenced parameter series.

3.1.4 Forecasting results

We estimate and forecast the term structure of interest rates recursively. The data range from March 2003 to the time when the forecast is made is employed to fit the models, and the data range from April 2009 to March 2011 is employed to test the forecasting performance.

In Table 2-4, the forecasting results are compared when forecast horizon is 1 month, 6 months or 12 months respectively:

We define the gap between forecasted yields and real yields (yields estimated by the Fama-Bliss method) as the forecasting error. In Table 2-4, the first column shows the model code, the second column reports the RMSE of total 10 maturities, the third to seventh columns report the RMSE of some typical maturities.

Table 2. RMSE of 1-month-ahead forecasting

Models	Total RMSE	3 m	1y	3y	5y	10y
RW	0.003517	0.007232	0.003016	0.002165	0.001575	0.001186
NS-AR	0.003501	0.006937	0.00332	0.002041	0.001506	0.001347
D-NS-AR	0.003476	0.007118	0.003213	0.00229	0.001734	0.001368
NS-VAR	0.003144	0.006306	0.002779	0.002031	0.001507	0.00118
D-NS-VAR	0.003078	0.007049	0.003104	0.002081	0.001538	0.001297
NS-VAR-Macro	0.00368	0.00688	0.003358	0.00263	0.00219	0.001976
D-NS-VAR-Macro	0.003612	0.007392	0.003265	0.001939	0.001452	0.001365
NS-VAR-X	0.003461	0.006644	0.003087	0.002391	0.001953	0.00178
D-NS-VAR-X	0.003322	0.007171	0.003174	0.002048	0.001451	0.00129

Table 3. RMSE of 6-months-ahead forecasting

Models	Total RMSE	3 m	1y	3y	5y	10y
RW	0.006159	0.010167	0.006251	0.005088	0.004217	0.003367
NS-AR	0.004867	0.008049	0.005188	0.003795	0.003075	0.002445
D-NS-AR	0.003889	0.007921	0.00347	0.002291	0.001692	0.001342
NS-VAR	0.005469	0.009296	0.005817	0.003964	0.00326	0.002813
D-NS-VAR	0.003875	0.008069	0.003262	0.002251	0.001656	0.001397
NS-VAR-Macro	0.008284	0.009609	0.008275	0.008361	0.007883	0.007226
D-NS-VAR-Macro	0.004171	0.008425	0.003655	0.002638	0.00197	0.001558
NS-VAR-X	0.004857	0.006979	0.004042	0.004461	0.004543	0.004629
D-NS-VAR-X	0.003983	0.008086	0.003386	0.002557	0.001929	0.001628

Table 4. RMSE of 12-months-ahead forecasting

Models	Total RMSE	3 m	1y	3y	5y	10y
RW	0.009475	0.013314	0.011055	0.008511	0.006545	0.004525
NS-AR	0.006708	0.009974	0.007587	0.005627	0.004562	0.003631
D-NS-AR	0.003548	0.006815	0.003208	0.002521	0.001967	0.001583
NS-VAR	0.005808	0.007926	0.005964	0.005391	0.004894	0.004425
D-NS-VAR	0.003442	0.006929	0.003009	0.002265	0.001665	0.001202
NS-VAR-Macro	0.007092	0.008195	0.006418	0.007047	0.007159	0.007151
D-NS-VAR-Macro	0.003581	0.007211	0.003157	0.00233	0.001685	0.001275
NS-VAR-X	0.008108	0.010549	0.008593	0.007453	0.006867	0.006378
D-NS-VAR-X	0.003477	0.006844	0.003105	0.002349	0.001772	0.001454

The second column of Table 2-4 show that the first-order difference modified models always perform better than the corresponding original model, no matter what forecasting horizon is. When exercising the Paired Sample Nonparametric Sign Test to compare the forecasting error under original and first-order difference modified dynamic Nelson-Siegel-style models, p-value is 0, which means that the first-order difference modified models perform significantly better than the corresponding models at the ten percent level.

Besides, when forecasting horizon increase, the volatilities of RMSE of first-order difference modified models are much smaller than that of the corresponding original models, which means that the forecasting result is more stable. So, we can conclude that the first-order difference modified models perform better in forecasting.

3.2 Risk management incorporating the forecasted yields

3.2.1 Data

As Section 3.1.1 mentioned, we also employ the end-of-month bonds' price data of SSE to exercise risk hedging study. Specifically, the data range from April 2009 to March 2011 is used to test the risk hedging performance.

3.2.2 Forecasting model specification

In Section 3.1.4, it is shown that first-order difference modified models perform better when forecasting. For simplicity, we employ the D-NS-VAR model to forecast the term structure and bonds' yields. Once the interest rates term structure between April 2009 and March 2011 is forecasted, bond price (and also yield) can be easily forecasted by discounting its cash flow, so as to substitute into the risk management model.

The forecasting and risk management horizon h is set to be 1 month, 6 months and 12 months as in Section 3.1.4.

3.2.3 Risk hedging result

We set the target bonds' maturities at 1 year, 5 years or 10 years. They represent the short-term, mid-term or long-term bond respectively. However, at each point of time, it's virtually impossible to find a bond whose maturity is exactly 1 year, 5 years or 10 years. So we choose the bond whose maturity is closest to 1 year, 5 years or 10 years as our target bond.

After forecasting the yields of all bonds, we can introduce them to the risk management model shown in section 2.2.2 to produce the optimal portfolio. So the yield gap between the optimal bond portfolio and the target bond after h months is the risk hedging error. Besides, the risk hedging error under the traditional duration model is also calculated to be the comparison benchmark. The hedging RMSE are shown in Table 4:

Table 4. RMSE of Risk Hedging

	$h=1$		$h=6$		$h=12$	
Maturity	Duration	Forecast	Duration	Forecast	Duration	Forecast
1y	0.007292	0.008047	0.007673	0.006527	0.02291	0.016946
5y	0.005017	0.005873	0.008608	0.006865	0.012984	0.009465
10y	0.004894	0.004564	0.005363	0.005299	0.012684	0.007974

The RMSE of risk hedging under all circumstance are shown in Table 4, in which the "Duration" columns show the RMSE under traditional duration matching model and the "Forecast" columns report the RMSE under our risk management model with forecasted yields incorporated.

Table 4 shows that when $h=6$ or $h=12$, the RMSE under our model is smaller, which means that when forecasting horizon is long enough, risk hedging performance can be improved by incorporating the forecasted yields matching constraint.

When $h=1$, our model performs better in hedging the interest rates risk of bond whose maturity is 10 years. That's because when horizon is only 1 month, the cash flow of short-term or mid-term bond is fewer, so its price is less volatile which may lead to bigger hedging error under our model.

4. Conclusion

In this paper, we suggest modifying the dynamic Nelson-Siegel-style models by modeling the first-order differenced parameter series. Empirical study shows that the first-order difference modified dynamic Nelson-Siegel-style models yields smaller error when forecasting. Besides, the RMSE of forecasting are stable under the modified models even if horizon increases.

Based on the forecasting of interest rates term structure, we introduce extra forecasted yields matching constraint to the traditional duration matching risk hedging model. Empirical tests show that when forecasting (hedging) horizon or target bond's maturity is long enough, incorporating the predicted yields matching constraint could reduce risk hedging error.

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