**Chapter 19 Solutions**

1. The delta (∆) of a call options is represented the sensitivity of the call option price to the current price of its underlying asset. ∆=0.7 implies that the purchase of 0.7 times the number of underlying shares in the contract of the call option to hedge the loss from value of call option when the underlying asset’s price changes.
2. Theta (Θ) is represented the sensitivity of the option price to the time to maturity. The negative Theta value implies that the time value of the option decreases when time passes.

Gamma (Γ) is represented the sensitivity of the change of delta (∆) to the rate of change of the underlying asset price. The positive Gamma implies that the value of option increases when the changes in the underlying stock price increase.

1. The vega (ν) is represented the sensitivity of the option price to the volatility of the underlying asset. The positive vega implies that the option price is positively related to the volatility of stock price. Increase 1% in volatility of underlying stock price will contribute to the increase of vega % in option price.
2. When we utilize a Taylor series expansion to evaluate the option price on the base of the current price of the underlying asset (S0) , then the option price (V) can be written as



If we ignore the higher terms of error, then the change of option price can be arranged as



On the right side of equation above, the first term is Delta multiplied by the change of underlying asset price and the second term is the Gamma times the square of the change of underlying asset price.

1. For an European call option on a non-dividend stock, the formula to calculate Delta is Equation (20.5) as follows:

, 

When *r* = 5% per year,  = 15%, and the maturity  = 2, the Delta of an at-the-money call option is





1. For an European call option on a non-dividend stock,











, 

Given the information as follows:

S= $110, *X* = $100, *r* = 0.05,  = 15%, and  = 7 months (  = 7/12 year)













1. If the stock pays dividend with 1% dividend yield, then the Greek letters of an European call option on the stock are











 

Therefore,













1. The put-call parity for European options on non-dividend stock is shown as

*P* = *C* + *Xe-rτ – S*

Then the Delta of a put option can be represented as the formula below in terms of the Delta of a call option:



Therefore, the delta of put option is equal to the delta of call option minus one.

For European options on stock with dividend yield q%, the put-call parity as follows:

*P* = *C* + *Xe-rτ - Se-qτ*

Then the Delta of a put option on stock with dividend can be represented as the formula below in terms of the Delta of a call option with dividend:



1. Use the relationship between the delta of put option and delta of call option from previous question (#8), we can get



The Gamma is defined as  , then the gamma of put option can be derived as follows:



Therefore, for European options, the Gamma of put option is the same as the Gamma of call option whether the underlying asset pays dividends or not.

1. The traded option with Delta=0.8, Gamma = 2.0, Vega=0.8, we want to compute
	1. Delta neutral portfolio,

If we set x as the number of put options with delta, -0.5, to make a delta neutral portfolio, then

(-0.5)x+0.8=0, x=0.8/0.5=1.6

Therefore, we can buy 1.6 put options with delta (-0.5) and the traded option with delta (0.8) to make a delta neutral portfolio.

* 1. Gamma neutral portfolio,

If we choose x call options with gamma 0.4 to make a gamma neutral portfolio, then

(0.4)x+2.0=0, x=-5

Therefore, we can sell 5 call options with gamma (0.4) and buy the traded option with gamma (2.0) to make a gamma neutral portfolio.

* 1. Both Delta and Gamma neutral portfolio,

If we buy 3 call options with delta (0.3) and gamma (0.4), sell 1.5 call options with delta (0.9) and gamma (1.3), sell 0.5 call potions with delta (0.7) and gamma (2.5), and buy the traded option with delta (0.8) and gamma (2.0), then

(0.3)(3)-(1.5)(0.9)-(0.5)(0.7)+0.8=0 for delta of the portfolio

(3)(0.4)-(1.5)(1.3)-(0.5)(2.5)+2.0=0 for gamma of the portfolio

Therefore, we can obtain a delta and gamma neutral portfolio by the combination of buying 3 call options with delta (0.3) and gamma (0.4) and buying a traded option with the sale of 1.5 call options with delta (0.9) and gamma (1.3) and 0.5 call potions with delta (0.7) and gamma (2.5).