**DIFFERENTIATION USING THE CHAIN RULE**

The following problems require the use of the chain rule. The chain rule is a rule for differentiating compositions of functions. In the following discussion and solutions the derivative of a function *h*(*x*) will be denoted by or *h*'(*x*) . Most problems are average. A few are somewhat challenging. The chain rule states formally that

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However, we rarely use this formal approach when applying the chain rule to specific problems. Instead, we invoke an intuitive approach. For example, it is sometimes easier to think of the functions *f* and *g* as ``layers'' of a problem. Function *f* is the ``outer layer'' and function *g* is the ``inner layer.'' Thus, the chain rule tells us to first differentiate the outer layer, leaving the inner layer unchanged (the term *f*'( *g*(*x*) ) ) , then differentiate the inner layer (the term *g*'(*x*) ) . This process will become clearer as you do the problems. In most cases, final answers are given in the most simplified form.

* + *PROBLEM 1 :* Differentiate .

* + *PROBLEM 2 :* Differentiate .
	+ *PROBLEM 3 :* Differentiate .
	+ *PROBLEM 4 :* Differentiate .
	+ *PROBLEM 5 :* Differentiate .

**SOLUTIONS TO DIFFFERENTIATION OF FUNCTIONS USING THE CHAIN RULE**

*SOLUTION 1 :* Differentiate .

( The outer layer is ``the square'' and the inner layer is (3*x*+1) . Differentiate ``the square'' first, leaving (3*x*+1) unchanged. Then differentiate (3*x*+1). ) Thus,



= 2 (3*x*+1) (3)

= 6 (3*x*+1) .

*SOLUTION 2 :* Differentiate .

( The outer layer is ``the square root'' and the inner layer is . Differentiate ``the square root'' first, leaving unchanged. Then differentiate . ) Thus,











*SOLUTION 3 :* Differentiate .

( The outer layer is ``the 30th power'' and the inner layer is . Differentiate ``the 30th power'' first, leaving unchanged. Then differentiate . ) Thus,







*SOLUTION 4 :* Differentiate .

( The outer layer is ``the one-third power'' and the inner layer is . Differentiate ``the one-third power'' first, leaving unchanged. Then differentiate . ) Thus,





(At this point, we will continue to simplify the expression, leaving the final answer with no negative exponents.)























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*SOLUTION 5 :* Differentiate .

( First, begin by simplifying the expression before we differentiate it. ) Thus,



( The outer layer is ``the negative four-fifths power'' and the inner layer is . Differentiate ``the negative four-fifths power'' first, leaving unchanged. Then differentiate . )





(At this point, we will continue to simplify the expression, leaving the final answer with no negative exponents.)





















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