

# **A Comprehensive Analysis of Bank Consolidation Values: A Real Option Approach**

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## **Abstract**

In this paper, we modify Schwartz and Moon (2000) model to value bank consolidation. From the case study (the first case of Taiwanese bank merge), we find that the consolidation value from the ex-ante viewpoint is average about 30% of the original total values of the associated banks. Further, we find that the probability of bankruptcy after merge will much lower than that of before merge. Hence it is worthwhile to merge for each bank in our case study. We also find that the changes in the growth rate in the integrated loan, the changes in the growth rate in the integrated deposits and the saving factors of cost functions play most important roles in the gratitude of increased consolidation value of bank merge.

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## **1. Introduction**

Despite the growth of domestic financial institutions in the 1990s, the return on equity (ROE) and the return on assets (ROA) of domestic banks in Taiwan have been dramatically declined. This is one of the warnings of an over-banking problem. To solve this problem, the Taiwanese government had passed a Financial Institutions Merger Law in Nov. 2000, allowing banks to acquire other financial institutions or diversify scope of business. For increasing the reformation of financial institutions, the Financial Holding Company Law had been legislated in June 2001, which made the cross-managing and cross-selling through horizontal and vertical integration easier. After the passage of the Law it started a wave of financial institution consolidations in Taiwan search for capital efficiency and cost saving.

There are numerous papers that discuss about the efficiency effects of bank mergers. One of consolidation efficiency is capital efficiency. Berger, Demestz, and Strahan (1999) review over 250 references and find the evidence is consistent with increases in market power from some types of consolidation through improvement in profit efficiency and diversification of risks. The other is cost saving which comes from eliminating redundant facilities, staff or even department, Rhoades (1998) summarize nine case studies and find that the combined firms achieved their cost cutting objectives in all of the studies.

The literatures about consolidation efficiency studies the consequences for individual bank performance of consolidation including event studies of stock price responses, as well as studies of post-merger performance based on income statement and balance sheet information. This is a kind of after-consolidation analysis. There are few paper that explore the merger gain from the ex ante viewpoint. Under a viewpoint of merger, discussing participating banks' transaction values before a merger plays an

important role. Moreover to evaluate the merger gain in advance is also a key point of a merger decision, as it can tell the decision maker whether to implement the project.

Panayi and Trigeorgis (1998) examine multi-stage real options applications to analyze the international expansion option by a bank. They divide the expansion strategy into two stages, the first-stage investment by the bank that is being considered to take place in year 1, while the second-stage investment option is contemplated for year 10. They still use the traditional NPV method to estimate the investment value, including two stages of NPV<sup>1</sup>. For the growth option value, they use the Black and Scholes formulae to measure the option value. They first use the option model to apply a valuation, and measure the possible gain from bank consolidation ex ante. However using the B&S formulae still imposes lots of restrictions such as limited parameters in measuring a bank's consolidation value. This paper modifies the Schwartz and Moon (2000) real-option concept and least square Monte-carlo simulation approach to value an Internet company, which is more general and easier method to apply in valuing various types of investments. We apply this method in evaluation financial institutions and formulate a model that using for ex ante valuation.

We implement our model to value the first bank consolidation case in Taiwan - Taishin International Bank and Dah An Commercial Bank. We calculate the pre-merger individual and consolidation bank value that could help to find the conversion ratio in advance. On the other hand, our research also finds the increased value ratio of bank consolidation, which helps to estimate the merger decision in advance. In Section 2 we set up the model in continuous time, form a discrete time approximation for bank valuation. Section 3 uses a case to show how to implement our model. Section 4 carries out some simulations to investigate what are the key factors determining the bank

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<sup>1</sup> They define the expanded NPV as the sum of base-case expanded NPV and the second-stage opportunity NPV is valued as an expansion option.

consolidation value. We draw the conclusions in Section 5.

## 2. The Model

In developing the simple model to evaluate bank value, we generally consider that interest revenue earned from loans is the major revenue for bank. And we identify the scale of credits as a bank's total assets and deposits as total liabilities. We also consider that the cost function is non-interest operating expenses, which combine fixed and variable components. With revenue and cost functions, we could find the net after-tax rate of cash flow and determine the cash available at each time interval. We could discount all cash available before time horizon to find the present value of bank.

To develop the model precisely, we initially describe the model in continuous time. For implementation, we would use the quarterly accounting data available from each bank and practice the model in discrete time. In this section, we focus on developing continuous models of individual and consolidation banks:

### 2.1 Per-Merger Bank Model

The major of bank's revenue come from interest spreads between loans and deposits. Assume a bank's loan at time  $t$  given by  $L_t$  and the dynamics of loans are given by the stochastic differential process:

$$\frac{dL_t}{L_t} = \mu_t^L dt + \sigma_t^L dz_1 \quad (1)$$

where  $\mu_t^L$  (the drift) is the expected rate of growth in loans and is assumed to follow a mean-reverting process with a long-term average drift  $\bar{\mu}_L$ ;  $\sigma^L$  is volatility in the rate of loan growth; and  $z_1$  follows a standard Wiener process. We assume that the company can earn an abnormal return in an observed period. It will converge stochastically to the more reasonable rate of growth for the industry. The mean-reverting process is:

$$d\mu_t^L = k^L (\bar{\mu}_L - \mu_t^L) dt + \eta_t^L dz_2, \quad (2)$$

where  $\eta_t^L$  is the volatility of expected rates of growth in loans. The mean-reversion coefficient  $k^L$  describes the rate at which the growth is expected to converge to its long-term average. Term  $z_2$  also follows a standard Wiener process.

We also assume that unanticipated changes in loans converge to a normal level  $\bar{\sigma}_L$ , and the unanticipated changes in the drift converge to zero.

Similarly, the bank's deposits at time  $t$  are given by  $D_t$  and the dynamics of loans are given by the stochastic differential process:

$$\frac{dD_t}{D_t} = \mu_t^D dt + \sigma_t^D dz_3, \quad (3)$$

where  $\mu_t^D$  is the expected rate of growth in deposits and is also assumed to follow a mean-reverting process with a long-term average drift  $\bar{\mu}_D$ ;  $\sigma^D$  is volatility in the rate of deposit growth; and  $z_3$  is a standard Wiener process.

The expected rate of growth in deposits also converges to the long-term rate as follow:

$$d\mu_t^D = k^D (\bar{\mu}_D - \mu_t^D) dt + \eta_t^D dz_4, \quad (4)$$

where  $\eta_t^D$  is the volatility of expected rates of growth in deposits. The mean-reversion coefficient  $k^D$  describes the rate at which the growth is expected to converge to its long-term average. Term  $z_4$  follows a standard Wiener process.

The unanticipated changes in deposits are also assumed to converge to  $\bar{\sigma}_D$ , and the unanticipated changes in the expected rates of growth in deposits are assumed to converge to zero.

The interest spreads relate to changes in the interest rate on loans and on deposits. Although there are different kinds of deposits rate, we use the average for a proxy. The

loans and deposits rates follow two stochastic processes. We define loans interest as a spread  $S_t$  above the average deposits rate  $r_t$ , and they both follow the square-root process of CIR(1985):

$$dr_t = a(b - r_t)dt + \sigma_r \sqrt{r_t} dz_5 \quad (5)$$

$$dS_t = a^S (b^S - S_t)dt + \sigma_t^S \sqrt{S_t} dz_6, \quad (6)$$

where  $a$  and  $a^S$  are the reversion speed parameters;  $b$  and  $b^S$  are the values toward which the interest rates revert over time;  $\sigma_r$  and  $\sigma_t^S$  are the standard deviation;  $z_5$  and  $z_6$  follow standard Wiener processes.

We define the net interest income  $R_t$ , is total amount of interest spreads:

$$R_t = L_t \times (r_t + S_t) - D_t \times r_t \quad (7)$$

In defining all the parameters about net interest income, there are six standard Wiener process,  $z_1, z_2, z_3, z_4, z_5$  and  $z_6$ , and each Wiener process is instantaneously correlated with each other (see **Appendix A**).

Further, we define that the non-interest operating expenses  $O_t$  with fixed and variable components. The fixed cost is the building and operating equipment cost, which assume to be always unchanged in the estimation horizon; the variable cost is defined as the non-performing loan (NPL), which is a percentage of total amount of loans:

$$\frac{dO_t}{O_t} = F_t + \alpha L_t, \quad (8)$$

where  $\alpha$  is a percentage of loans.

With revenue and cost of the bank, we define the net after-tax rate of the cash flow

$Y_t$  given by:

$$Y_t = (R_t - O_t) \times (1 - \tau_t), \quad (9)$$

where  $\tau_t$  is the corporate tax rate.

Finally, the bank is assumed to have an amount of cash available  $X_t$ , that evolves according to

$$dX_t = Y_t dt \quad (10)$$

Schwartz and Moon (2000) assume the company goes bankrupt when the amount of its available cash reaches zero. We adopt Schwartz and Moon's definition as base valuation. If a bank runs out of cash, we do not take into account the possibility of additional financing, even the bank have good enough prospects to be able to raise cash. To simplify the model, we assume that the cash flow remains in the banks, earns risk free interest rate, and will be available for distribution to the shareholders at time horizon.

Finally, the objective of the model is to determine the values of individual bank Kaplan and Ruback (1995) provide empirical evidence that shows that discounted cash flows provide a reliable estimate of the market value. We estimate the value of bank by calculating the discounted value at risk-free measure:

$$V_0 = E_Q \left[ \left[ X_T + M \times (R_T - O_T) \right] \cdot e^{-rT} \right] \quad (11)$$

Term  $E_Q$  is equivalent martingale measure, and  $e^{-rT}$  is the continuously-compounded discount factor. In Equation (11), we assume bank value is similar to a European option. At time horizon, the bank has two amounts of cash available: the cash available accumulated to time horizon and the terminal value for the bank. Schwartz and Moon (2000) set the terminal value at the horizon to be a multiple  $M$ , (e.g., 10 times) of net after-tax rate of cash flow. According to Brennan and Schwartz (1982), we should

convert the variable measured in the real world to be risk-neutral, because we use the risk-neutral measure (see **Appendix B**). In this model the value of bank is a function of the state variables (loans, expected growth in loans, deposits, expected growth in deposits, interest rate, interest spread and cash balances) and time. It can be written as:

$$V \equiv V(L, \mu^L, D, \mu^D, r, S, X, t). \quad (12)$$

## 2.2 Consolidation Bank Model

Based on Berger, Demsetz, and Strahan's (1999) study, we assume that the primary motive for consolidation is to maximize shareholder value. All actions taken by the banks would be toward maximizing the value of shares owned by existing shareholders. We design the consolidation model to focus on increasing efficiency. First, we discuss if increasing the scale of loans and deposits could help reach economics of scale and improve the value of the existing bank. Then, we discuss the change of cost structure through consolidation.

Assuming there is an existing bank after consolidation; bank  $M$ 's loans at time  $t$  are given by  $L_t^M$ . The dynamics of the integrated loans are given by the stochastic differential equation similar to what we discuss above. But the growth rate of integrated loans processes should be redefined. Because each bank's source of loans might be overlap or have a correlation. For example bank A would merger bank B, in order to consider the correlation between two banks; we assume that there is an instantaneous correlation  $\rho_L$  between the two Wiener processes. It is implemented by transforming to new variables  $\psi_t^A$  and  $\psi_t^B$  as follows:

$$d\psi_t^A = u_t^{LA} \mu_t^{LA} dt + \eta_t^{LA} \mu_t^{LA} dz_1^M \quad (13)$$

$$d\psi_t^B = u_t^{LB} \mu_t^{LB} dt + \eta_t^{LB} \mu_t^{LB} dz_2^M \quad (14)$$



$$dz_1^M dz_2^M = \rho_L dt, \quad (15)$$

where  $dz_1^M$  and  $dz_2^M$  are standard Wiener processes of the new variables  $\psi_t^A$  and  $\psi_t^B$ , and they have a correlation  $\rho_L$ . We define the loan growth rate of the consolidation bank  $\mu_t^{LM}$  as the value-weighted average of each bank's new growth rate processes:

$$d\mu_t^{LM} = k_t^A d\psi_t^A + k_t^B d\psi_t^B \quad k_t^A = \frac{L_0^A}{L_0^A + L_0^B} \quad k_t^B = \frac{L_0^B}{L_0^A + L_0^B}, \quad (16)$$

where  $k_t^A$  and  $k_t^B$  are the value-weighted of each bank's initial amount of loans. The dynamics of the integrated loans stochastic differential equation as is given as follow:

$$\frac{dL_t^M}{L_t^M} = \mu_t^{LM} dt + \sigma_t^{LM} dz_3^M, \quad (17)$$

where  $\mu_t^{LM}$  is the growth rate of integrated loans, which is defined above.  $\sigma_t^{LM}$  is the volatility of integrated loans growth rate.

We define the integrated deposits as  $D_t^M$ . Similarly, we should redefine each bank's expected rate of growth in deposits. And the dynamics of the integrated deposits stochastic differential equation is as follows:

$$\frac{dD_t^M}{D_t^M} = \mu_t^{DM} dt + \sigma_t^{DM} dz_4^M, \quad (18)$$

where  $\mu_t^{DM}$  is integrated deposits' growth rate;  $\sigma_t^{DM}$  is the volatility of integrated deposits growth rate.

The interest rate models still follow the square-root process of CIR (1985), and we use the existing bank's average deposit rate and interest spread for the integrated bank's average deposit rate  $r_t^M$  and interest spread  $S_t^M$ .

The net interest income of consolidation bank  $R_t^M$  is the difference between

interest earned from loans and interest paid for depositors:

$$R_t^M = L_t^M \times (r_t^M + S_t^M) - D_t^M \times r_t^M \quad (19)$$

Similarly, The six Wiener processes, the integrated bank's loans, deposits, growth of loans and deposits, deposits rate and interest spread, might be relative to each other as we discuss at per-merger model (see **Appendix A**).

Rhoades (1998) find that there exists cost efficiency in some cases of bank consolidation. In our study, we assume that there is cost reductions opportunity in consolidation bank cost function:

$$O_t^M = F_t^M + \alpha_t^{MO} \times L_t^M = \alpha^{MF} \times (F_t^A + F_t^B) + \alpha_t^{MO} \times L_t^M, \quad (20)$$

where  $\alpha_t^{MF}$  is the fixed cost saving factors or the economics scale factor and  $\alpha_t^{MO}$  are the variable cost saving factors. Basically, there is a cost saving motive or economy of scale for bank consolidation if the magnitude of  $\alpha_t^{MF}$  is less than unity. The variable cost might produce a cost-saving opportunity by increasing scale of loans or dispelling NPL. We measure the effect of the variable cost-saving by comparing the variable cost-saving factor with the rate of the value-weighted variable cost of each participating bank.

The net after-tax rate of cash flow to the integrated bank  $Y_t^M$ , an amount of cash available  $X_t^M$ , and finally the estimation of integrated bank  $V_0^M$  is the same as per-merger bank model.

To sum up, the integrated bank value is a function of the state variables (integrated loans, loans growth rate, integrated deposits, deposits growth rate, interest rate, interest spread, and cash balances) and time. It can be showed as:

$$V^M \equiv V^M(L^M, \mu^{LM}, D^M, \mu^{DM}, r^M, S^M, X^M, t). \quad (21)$$

The objective of our study is to find whether the bank merger has a positive effect and achieves the goal of maximizing the value of shares or not. If the consolidation value is greater than the summed-up value of the individual banks, then the bank merger has synergy. We measure the increased value ratio  $\gamma$ , as:

$$\text{The increased bank value ratio } (\gamma) = \frac{V_0^M - V_0^A - V_0^B}{V_0^A + V_0^B} . \quad (22)$$

The model developed in continuous time model above is path dependent. These path dependencies could easily be taken into account by using Monte Carlo simulation to solve for the valuation. To implement the simulation, the discrete version of the risk-adjusted process is used (see **Appendix C**).

### **3. A Case Study**

In order to show how to implement our model, we use the first bank merger case of Taishin International Bank and Dah An Bank as an example.

#### **3.1 The introduction for the case**

Since the Financial Holding Company (FHC) Law had been legislated in June 2001, it brought a merger agitation. Beginning with the merger of Hua Nan bank, Taiwan bank and first bank at December 19, 2001, there are 14 financial holding companies established in the period of 2002 and first quarter of 2003. Most of those FHC lead by commercial banks and composed security and insurance companies, there are no commercial bank's merger before first quarter of 2002. In this section, we introduce the first case of commercial bank's merger in Taiwan and discuss the value created according to our model.

Taishin International Bank and Dah An Bank held temporary shareholder meetings at December 2001; they agreed to establish Taishin Financial Holding Company that which would include Dah An Bank, Taiwan Securities Company, and Taishin Bills Finance Corporation. For the planned FHC, Taishin International Bank offered shares to take over Dah An Bank. After the merger Taishin International Bank was to be the existing bank, and the agreement of the share-exchange ratio was one share in Taishin Bank exchanged for 2 shares in Dah An Bank.

After the merger, Taishin Financial increased its assets to NT\$42 billion from the current NT\$30 billion. Estimating the effect of the merger, Taishin was expected to provide one-stop financial services to over 3.5 million customers at its 133 branch offices (up from the current 88) and have 4,500 staff members, adding that the consolidation would save an estimated NT\$600 million to NT\$700 million per year in costs from 2002 to 2004. The market share of loans would reach 2.5%, and the new company's non-performing loan (NPL) ratio was expected to be reduced to 2.24 % in 2002 year and the new company would have an after-tax income of nearly NT\$5 billion. Taishin International Bank Chairman Thomas Wu said that he estimated that the merger between his bank and Dah An Commercial Bank would create earnings per share of NT\$1.6 by the end of 2002, which is an increase of 201%, compared to earnings per share last year of NT\$0.53.

The first stage of the FHC is the merger of Taishin International Bank with Dah An Commercial Bank, which was handed to the Ministry of Finance, under plans to be completed in February 2002. The second stage is Taiwan Securities Company and Taishin Bills Finance Corp scheduled to join the planned financial holding company in the second quarter of 2002 once the share swap ratio is negotiated at their June share holder meetings.

This merger is the first example of domestic commercial banks consolidating in

Taiwan; it involves a complex process of shares being exchanged, the elimination of the acquired bank, and the offering up of the existing bank. Hence, it is important to estimate the value of each bank in order to handle the problem above. In our study we also want to know the value, created by the merger from the ex ante viewpoint.

### **3.2 Estimating the Parameters**

The model described in the previous section requires many parameters for implementing the calculation of bank value. Because Taishin International Bank applied for a merger in January 2001 and they expected to complete it before February 2002, we estimate the bank value based on the third quarter of 2001 and use the financial reports from the first quarter of 1997 to the third quarter of 2001 as the historical data in this study.

#### **3.2.1 The parameters of pre-merger**

The first part of the model requires more than 50 parameters for its implementation. Some of these parameters are easily observed from each bank's quarterly financial reports. However, some parameters, requiring the use of judgment, can be determined by management's forecasted estimates or analysts' investigation. Finally, those parameters, which are difficult to observe or estimate, are consulted directly from previous reference.

Tables 1 and 2 are the basic data of each bank including quarterly loans, deposits, interest income, and interest expenses for the last 19 quarters. In Table 3, we use the basic data to determine the initial loans, deposits, fixed component of expenses and cash balance available of each bank. For the initial expected growth rates in loans and deposits we take the average growth rate in loans and deposits over the last two quarters (March 2000 and June 2000), and for the rate of growth over the next four quarters, we

used analyst's expectations from Morgan Stanley.

For 2002, an average loan growth of 3% is expected in financial industry of Taiwan. When estimating Taishin Bank's loan growth rate, it has a high growth rate in non-mortgage consumer loans (24.5% of total loans) and it is expected to continue growing in 2002. We forecast its loan growth rate to be 7% a quarter. In the other hand, Dah An Bank did not have any additional information about its loan growth rate, so we estimate it from the analyst's expectation of 3%.

The deposit growth rate has a high correlation with the currency rate and foreign interest rate, especially the U.S interest rate. Hence, the currency remained under pressure and the U.S interest rate continued its fall in 2001. We forecast the deposit growth rate in 2002 to remain unchanged as before. Taishin Bank's deposit growth rate is set at 6%, which is the same as its historical growth rate record, and Dah An Bank's is 4%.

The initial volatility of loans and deposits are the standard deviations of past changes in loans and deposits. The initial volatility of the expected rate of growth in loans and deposits is inferred from the observed stock price volatility. Other parameters generated from the stochastic processes of loans and deposits, such as the long-term rates of growth in loans and deposits and the long-term volatility of loans, which we use average growth rate(2% in loans and deposits) and standard deviations(3% in loans and deposits) in industry to be proxies. Finally, the mean-reversion coefficients are 7%<sup>2</sup>.

For the cost parameters, we classify cost as a fixed cost and a variable cost, and it is observed from the income statement of each bank. We found the expenses of each bank contain two main items - financial activity expenses and operating expenses. The financial activity expense includes interest expenses, non-performing loan expenses, and procedure fees; in our study we focus on discussing the net cash flow from the

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<sup>2</sup> All the parameters are defined as Schwartz and Moon (2000).

interest spread, so we do not take account of procedure fees, which result from other activities. The interest expense is considered as before.

We define the other two items of cost: one is the non-performing loan expense, which comes from bad loans, and so we define non-performing loans as the variable cost and it is a constant percentage of the loans. Taishin Bank's non-performing loan (NPL) ratio is expected to fall to about 2% in 2002, and we assume its variable component of expenses to be 2% of loans. Because the scale of loans at Dah An Bank is smaller and economic conditions look to be better next year, we assume its variable component of expenses to be 1% of loans. The other is the operating expense, which includes selling, general, administrative (SG&A), and other expenses; for example, R&D expenses, training expense, and rental expense, and we define it as the fixed component of expense.

The net interest income is the difference between interest earned from loans and interest paid to depositors. The change in interest rates on loans and on deposits is very important for estimating the bank value. We use the average deposit rate as the initial interest rate for deposits and take the difference between average lending rate and deposit rate as the interest spread. They are acquired from Taiwan Economic Journal (TEJ). For setting up other interest rate parameters, we refer to Chen and Scott (1994).

Other parameters, which should be identified in discrete model, the tax rate is the average tax rate from the first quarter of 1997 to the third quarter of 2001, which is available from TEJ. We take 10 years as the horizon of the estimation and one quarter as the time increment. For the correlation parameters of each Wiener process, we took the correlation of each parameter for the estimation, which is obtained from historical data first quarter of 1997 to third quarter of 2001.

### **3.2.2 The parameters of consolidation**

The second part of the model also requires 39 parameters for implementation. In Table 4, bank consolidation increases the scale of assets, if we assume all things are equal and the initial integrated loans are the sum of the two bank's loans. The same as the integrated loans, the initial deposits of consolidation are the sum of the two bank's deposits. The drift and volatility terms of stochastic processes of the loans and deposits are very important for simulating consolidation value, and we take the initial expected rate of growth in loans and deposits as the value-weighted average of each bank's initial expected rates of growth in loans and deposits.

As the previous section, to make our model more realistic we consider the correlation between each bank, which concerning the expected rate of growth in loans and in deposits. There exists correlation between each bank's expected loan growth rate, and its initial value is assumed to be the correlation between two bank's historical loans' growth rate. Similar to the correlation of each bank's expected deposit growth rate.

The other main issue about the gain from a merger is efficiency improvement. The efficiency gain comes from cost reductions. In our study we assume two opportunities of cost savings: the fixed cost saving factors and the variable cost saving factors. The fixed cost saving factor is the reduction of the integrated bank's fixed cost, compared to the sum of both. For the initial fixed cost saving factor, we used analyst's expectations, which estimate that the savings of operating expense after merger are NT\$3 to 4 billion, and we set the initial fixed cost saving factor at 0.09 per quarter. As we definite in pervious section, the variable cost saving factor is non-performing loans expenses, Taishin Financial Holding Company's non-performing loan ratio is expected to fall to 2.24 % from 2.7% in 2002; hence, we assume the initial variable cost saving factor to be 0.01 per quarter.

If we do not consider an integrated bank that increases market concentration may



increase market power from setting prices on retail services<sup>3</sup>, then we assume initial deposits rate and interest spread after merger had been the same. The amount of cash available for the integrated bank is NT\$ 380 million, the sum of the two bank's initial cash available.

## 4. Results

After estimating the parameters, we use 100,000 Monte Carlo simulations to value the pre-merger and consolidation values. The simulations results are as follow.

### 4.1 The fair transaction values of banks

To simulate the value of each bank, we should determine the assumption of bankrupt. First, we use Schwartz and Moon (2000) definition that a company goes bankrupt when the amount of its available cash reaches to zero. Then we take Longstaff (2001) Least-Square Method (LSM) to find the optimal stop point of each simulation path and calculate bank value. According to LSM approach, it is more reasonable that the definition of a stopping point of valuation is when the conditional expected value is *less than* the cash available at  $T-\Delta t$ . We could regard the conditional expected value fitting by LSM as the expected continuing value of bank at  $T-\Delta t$ ., and compare to the actually cash available at the same time.

Table 5 summarizes the simulation results of each method. In the base method of Schwartz and Moon, Taishin Bank's value is NT\$338.358 billion; Dah An Bank is NT\$315.542 billion. To be a pre-merger view, the value increased ratio is what we are concerned about. The consolidation value increases 26.38% in this method. On the other

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<sup>3</sup> Berger, Demsetz, and Strahan (1999) find that M&As among institutions that have significant local market overlap ex ante and may increase local market concentration and allow the consolidated firm to raise profits by setting less favorable prices to customers. This may affect rates and fees on retail deposits and small business loans.

hand, LSM approach shows that Taishin Bank's value is NT\$339.003 billion; Dah An Bank is NT\$297.714 billion. Although each bank's value in LSM approach is like the base method, the increased value is more significant. Because we relax the restriction that the cash available stochastic process might reach to zero by simulation and it could not be seen as bankrupt if it's expected continuing value is still larger than zero. Table 5 also shows the value increased ratio is conspicuous by LSM approach.

## **4.2 The bankrupt and stopping points**

Table 6 shows the base valuation method follow Schwartz and Moon (2000), in which the bankrupt condition is defined as the cash available at each discrete time reaches to zero. Note that the bankruptcy per year of Taishin International Bank is higher than Dah An Commercial Bank at beginning, especially in Year 2. But the bankrupt probability of Taishin International Bank decreases rapidly after Year 4; it becomes lower than Dah An Commercial Bank. The consolidation bankruptcy start in Year 3 and it is much lower before Year 8 than individual bank's bankrupt probability.

Table 7 shows that the LSM approach simulates bank's stopping points' distributions. We develop the LSM approach to determine the stop points of each simulation paths as we have discussed above, and note that the optimal stopping points of each year of both banks is closer than base valuation. And we find the total stopping probabilities of Dah An Commercial Bank is higer than Taishin International Bank in this simulation results, it is more reasonable according to the real situation.

### 4.3 The sensitivity analysis

In this section we perform the sensitivity of the total value of the consolidation bank to the most critical parameters. We obtain the bank values by using a perturbation (a 10 percent higher value) for the indicated parameter while leaving all the other parameters the same as the base valuation of the consolidation bank value. Table 8 reports the sensitive analysis results by base valuation and we find three sets of parameters that have a significant effect on the value of the bank. First is the parameter for the fixed and variable components of cost, even in the equation two components are the same effect on the cost function, but in simulation result the variable component increases by 1 percent, as in Table 8, the consolidation bank value decreases by 1.73%, which is more significant than the fixed component increases by 1 percent.

Second, an increase in the initial integrated loans and deposits growth rate, from 5 percent to 5.5 percent per quarter (a 10 percent increase) are also significant for consolidation bank valuations. And we find that the increase in loans initial growth rate has positive relation with consolidation bank valuation. On the other hand, there is a negative relation between deposits growth rate and consolidation bank, because increase in deposits growth rate would increase the interest expenditure, if the loans growth rate remain unchanged. And the variance the distribution of loans and deposits growth rate is important in the valuation because it determines the option value of growth, implies a higher probability of both growth opportunities, as in Table 8, we find the increase in both standard deviation of loans and deposits would raise consolidation bank value.

Third and not so obvious, the set of parameters that generate from the mean-reverting process have effect on consolidation bank value. An increase in the mean-reversion coefficient in integrated loans and deposits, as in Table 8,  $k_1$  and  $k_2$  would decrease the value of the consolidation bank. An increase in the long-term level of growth rate in integrated loans would increase the value of the consolidation bank,

and it is contrary to increase in long-term level of growth rate in integrated deposits.

Table 9 shows the sensitivity of the total value of the consolidation bank using the LSM approach. We find similar results as in the previous discussion.

## **5. Conclusions**

In this paper, we modify Schwartz and Moon (2000) model to value bank consolidation. From the case study (the first case of Taiwanese bank merge), we find that the consolidation value from the ex-ante viewpoint is average about 30% of the original total values of the associated banks. Further, we also find that the probability of bankruptcy after merge will much lower than that of before merge. Hence it is worthwhile to merge for each bank in our case study.

We also find that the changes in the growth rate in the integrated loan, the changes in the growth rate in the integrated deposits and the saving factors of cost functions play most important roles in the gratitude of increased consolidation value of bank merge. However we have to emphasize that the gratitude of increased consolidation value of bank merge will be significantly affected by the initial setting of parameters. Hence a more thorough analysis has to use cross-sectional data from a sample of the financial service institutions to estimate the parameters. We leave this issue to future research.

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## Appendix A: the Correlations between Standard Wiener Process Generated from Bank's Net Interest Income

The relationships are as follows:

$$\begin{aligned}
 dz_1 dz_2 &= \rho_{Lg^L} dt & dz_1 dz_3 &= \rho_{LD} dt & dz_1 dz_4 &= \rho_{Lg^D} dt \\
 dz_1 dz_5 &= \rho_{Lr} dt & dz_1 dz_6 &= \rho_{LS} dt & & \\
 dz_2 dz_3 &= \rho_{Dg^L} dt & dz_2 dz_4 &= \rho_{g^L g^D} dt & dz_2 dz_5 &= \rho_{g^L r} dt & dz_2 dz_6 &= \rho_{g^L S} dt \\
 dz_3 dz_4 &= \rho_{Dg^D} dt & dz_3 dz_5 &= \rho_{Dr} dt & dz_3 dz_6 &= \rho_{DS} dt & & \\
 dz_4 dz_5 &= \rho_{g^D r} dt & dz_4 dz_6 &= \rho_{g^D S} dt & & & & 
 \end{aligned}$$

where  $\rho_{Lg^L}$  is the correlation between loans and loan growth rate;  $\rho_{LD}$  is the correlation between loans and deposits;  $\rho_{Lg^D}$  is the correlation between loans growth rate and deposits;  $\rho_{Lr}$  is the correlation between loans and deposits rate;  $\rho_{LS}$  is the correlation between loans and interest spread;  $\rho_{Dg^L}$  is the correlation between deposits and loan growth rate;  $\rho_{g^L g^D}$  is the correlation between loan growth rate and deposit growth rate;  $\rho_{g^L r}$  is the correlation between loan growth rate and deposit rate;  $\rho_{g^L S}$  is the correlation between loan growth rate and interest spread;  $\rho_{Dg^D}$  is the correlation between deposits and deposits growth rate;  $\rho_{Dr}$  is the correlation between deposits and deposit rate;  $\rho_{DS}$  is the correlation between deposits and interest spread;  $\rho_{g^D r}$  is the correlation between deposit growth rate and deposits rate;  $\rho_{g^D S}$  is the correlation between deposit growth rate and interest spread.

## Appendix B: the Risk-Adjusted Processes

The model has six sources of uncertainty: The first is uncertainty about changes in loans, the second is uncertainty about the expected rate of growth in loans, the third is uncertainty about changes in deposits, the fourth is uncertainty about the expected rate of growth in deposits, the fifth is uncertainty about change in the average deposit rate, and sixth is uncertainty about the change in the interest spread. Under Brennan and Schwartz (1982) there are some simplifying assumptions - the risk-adjusted processes for the state variables can be obtained from the true processes, and the loans and deposits of the risk-adjusted processes are as follow:

$$dL_t = (\mu_t^L - \lambda_1 \sigma_t^L) L_t dt + \sigma_t^L L_t dz_1^*$$

$$d\mu_t^L = [k^L (\bar{\mu}_L - \mu_t^L) - \lambda_3 \eta_t^L] dt + \eta_t^L dz_2^*$$

$$dD_t = (\mu_t^D - \lambda_2 \sigma_t^D) D_t dt + \sigma_t^D D_t dz_3^*$$

$$d\mu_t^D = [k^D (\bar{\mu}_D - \mu_t^D) - \lambda_4 \eta_t^D] dt + \eta_t^D dz_4^*,$$

where the market prices of factor risks,  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$ , and  $\lambda_4$ , are constants.

Similarly, we have to adjust the interest rate to be risk-neutral:

$$dr_t = [a(b - r_t) + \lambda_5 \sigma_t \sqrt{r_t}] dt + \sigma_t \sqrt{r_t} dz_5^*$$

$$dS_t = [a^S (b^S - S_t) + \lambda_6 \sigma_t^S \sqrt{S_t}] dt + \sigma_t^S \sqrt{S_t} dz_6^*,$$

where  $\lambda_5$  and  $\lambda_6$  are the market prices of the factor risks.



## Appendix C: the Discrete Version of the Risk-Adjusted Process

To implement the model developed in previous section, we have to rewrite the model a discrete-time version. For example, each bank's loans and loans growth rate under the risk-adjusted process are used:

$$L_{t+\Delta t} = L_t \cdot e^{\left\{ \left[ \mu_t^L - \lambda_1 - \frac{(\sigma_t^L)^2}{2} \right] \Delta t + \sigma_t^L \cdot \sqrt{\Delta t} \cdot \varepsilon_1 \right\}}$$

$$\mu_{t+\Delta t}^L = e^{-k^L \cdot \Delta t} \cdot \mu_t^L + \left(1 - e^{-k^L \cdot \Delta t}\right) \cdot \left( \bar{\mu}_L - \frac{\lambda_3 \eta_t^L}{k^L} \right) + \sqrt{\frac{1 - e^{-2k^L \cdot \Delta t}}{2k^L}} \cdot \eta_t^L \cdot \sqrt{\Delta t} \cdot \varepsilon_2$$

$$\sigma_t^L = \sigma_0^L \cdot e^{-k_t^L \cdot t} + \bar{\sigma}_L \left(1 - e^{-k_t^L \cdot t}\right)$$

$$\eta_t^L = \eta_0^L \cdot e^{-k_2^L \cdot t}.$$

where  $\sigma_0^L$  and  $\eta_0^L$  are initial values of variance of loans and growth rate;  $\varepsilon_1$  and  $\varepsilon_2$  are draw from a standard normal distribution with correlation between loans and growth rate.

Each bank's deposits and deposits growth rate under the risk-adjusted process are similar to loans process.

**Table 1: Quarterly Sales and Costs for Taishin International Bank, March 1997-September 2001 (NT\$ 10 mn)**

Date	Loans	Deposits	Interest Income	Interest Expenses	Non-Performance Loan Expenses	Operating Expenses	Gross Profit	Operating Profit Before Taxes (EBITDA)
1997								
March	92.453	95.657	2.508	1.595	0.145	0.710	0.375	0.406
June	103.432	96.726	4.750	3.214	0.400	1.440	0.825	0.887
September	115.105	112.576	8.217	5.100	0.620	2.253	1.150	1.241
December	130.221	148.140	10.841	7.359	0.863	3.081	1.505	1.621
1998								
March	145.511	166.797	4.043	2.703	0.146	0.959	0.547	0.588
June	161.354	175.910	7.648	5.620	0.255	2.019	0.910	1.001
September	161.134	186.273	12.996	8.694	0.479	3.175	1.357	1.469
December	163.441	197.166	16.245	11.799	0.816	4.374	1.672	1.810
1999								
March	161.889	198.380	4.820	3.045	0.463	1.185	0.467	0.500
June	162.930	200.010	8.756	5.868	1.084	2.428	1.041	1.105
September	165.610	195.101	14.465	8.545	1.766	3.645	1.556	1.633
December	176.137	197.314	17.722	11.074	2.673	4.903	2.175	2.252
2000								
March	175.961	210.578	5.008	2.678	1.023	1.271	0.779	0.834
June	176.166	214.840	9.372	5.396	1.584	2.665	1.738	1.853
September	180.174	219.554	15.507	8.237	2.468	4.142	2.201	2.374
December	185.694	224.619	19.226	11.169	2.990	5.687	2.299	2.507
2001								
March	183.183	231.224	5.420	2.907	1.014	1.562	0.715	0.820
June	182.630	232.996	9.792	5.611	2.249	3.163	1.169	1.421
September	180.272	229.595	15.941	8.173	4.141	4.867	0.849	1.143

**Table 2: Quarterly Sales and Costs for Dah An Commercial Bank, March  
1997-September 2001 (NT\$10 mn)**

Date	Loans	Deposits	Interest Income	Interest Expenses	Non-Performance Loan Expenses	Operating Expenses	Gross Profit	Operating Profit Before Taxes (EBITDA)
1997								
March	82.961	91.379	1.834	1.270	0.101	0.434	0.344	0.347
June	91.784	89.857	3.753	2.583	0.116	0.891	0.755	0.760
September	101.594	97.145	5.929	4.012	0.324	1.366	1.169	1.174
December	114.352	115.943	8.488	5.764	0.623	1.917	1.480	1.430
1998								
March	115.389	115.083	2.640	1.904	0.065	0.527	0.473	0.472
June	126.430	130.748	5.347	3.877	0.113	1.098	0.889	0.888
September	135.219	128.233	8.270	5.986	0.228	1.685	1.336	1.336
December	136.495	142.245	11.311	8.150	0.909	2.286	1.102	1.100
1999								
March	138.728	147.075	3.006	2.159	0.284	0.604	0.324	0.329
June	147.936	149.782	5.926	4.273	0.645	1.218	0.552	0.547
September	147.323	149.295	8.824	6.324	1.001	1.719	0.813	0.808
December	146.241	153.316	11.792	8.270	5.235	2.204	-2.572	-2.572
2000								
March	149.138	164.056	2.938	2.040	0.319	0.536	0.571	0.583
June	159.936	164.273	5.954	4.196	0.398	1.095	1.224	1.295
September	166.294	184.955	9.192	6.565	0.493	1.668	1.732	1.825
December	163.439	191.037	12.235	8.717	0.920	2.261	0.400	0.549
2001								
March	164.203	180.183	3.157	2.374	0.593	0.560	0.343	0.371
June	168.742	182.264	5.991	4.562	1.008	1.137	0.677	0.714
September	167.280	178.245	8.783	6.567	1.703	1.703	0.441	0.488

**Table 3: Parameters Used in the Base Valuation of Pre-merger**

Parameters	Notation	Initial Estimation of Taishin International bank	Initial Estimation of Dah An commercial bank
<b>Loan parameters</b>			
Initial Loans	$L_0$	NT\$180 ten million//quarter	NT\$167 ten million/quarter
Initial expected rate of growth in loans	$\mu_0^L$	0.07/quarter	0.03/quarter
Initial volatility of loans	$\sigma_0^L$	0.07/quarter	0.05/quarter
Initial volatility of expected growth rates in loans	$\eta_0^L$	0.2082/quarter	0.1882/quarter
Long-term growth rate in loans	$\bar{\mu}_L$	0.02/ quarter	0.02/ quarter
Long-term volatility of the growth rate in loans	$\bar{\sigma}_L$	0.03/ quarter	0.03/ quarter
<b>Deposits parameters</b>			
Initial Deposits	$D_0$	NT\$229 ten million/quarter	NT\$178 ten million /quarter
Initial expected growth rate in deposits	$\mu_0^D$	0.06/quarter	0.04/quarter
Initial volatility of deposits	$\sigma_0^D$	0.08/quarter	0.07/quarter
Initial volatility of expected growth rates in deposits	$\eta_0^D$	0.2082/quarter	0.1882/quarter
Long-term rate of growth in deposits	$\bar{\mu}_D$	0.02/ quarter	0.02/ quarter
Long-term volatility of the rate of growth in deposits	$\bar{\sigma}_D$	0.03/ quarter	0.03/ quarter
<b>Cost parameters</b>			
Variable component of expenses as a percentage of loans	$\alpha$	0.02/quarter	0.01/quarter
Fixed component of expenses	$F$	NT\$4.8 ten million/quarter	NT\$1.7 ten million/quarter
<b>Other parameters</b>			
Initial cash balance available	$X_0$	NT\$16 ten million/quarter	NT\$22 ten million/quarter
Time increment for the discrete version of the model	$\Delta t$	1 quarter	1 quarter
Horizon for the estimation	$T$	10 years	10 years

**Table 4: Parameters Used in the Base Valuation of Consolidation**

<b>Parameters</b>	<b>Notation</b>	<b>Initial Estimation</b>
<b>Loans parameters</b>		
Initial Integrated Loans	$L_0^M$	NT\$347 ten million/quarter
Initial expected growth rate in integrated loans	$\mu_0^{LM}$	0.05/quarter
Initial volatility of integrated loans	$\sigma_0^{LM}$	0.07/quarter
Long-term growth rate in integrated loans	$\bar{\mu}_{LM}$	0.02/quarter
Long-term volatility of the growth rate in integrated loans	$\bar{\sigma}_{LM}$	0.03/quarter
Correlation of each bank's expected rate of growth in loans	$\rho_L$	0.58/quarter
<b>Deposits parameters</b>		
Initial Integrated Deposits	$D_0^M$	NT\$407/quarter
Initial expected growth rate in integrated deposits	$\mu_0^{DM}$	0.05/quarter
Initial volatility of integrated deposits	$\sigma_0^{DM}$	0.08/quarter
Long-term growth rate in integrated deposits	$\bar{\mu}_{DM}$	0.02/quarter
Long-term volatility of the growth rate in integrated deposits	$\bar{\sigma}_{DM}$	0.03/quarter
Correlation of each bank's expected rate of growth in deposits	$\rho_D$	0.62/quarter
<b>Cost parameters</b>		
Variable expense cost saving factors	$\alpha^{MO}$	0.01/quarter
Fixed expense cost saving factors	$\alpha^{MF}$	0.09/quarter
Integrated fixed component of expenses	$F^M$	NT\$6.5 ten million/quarter
<b>Other parameters</b>		
Initial cash balance available	$X_0^M$	NT\$38 ten million/quarter
Time increment for the discrete version of the model	$\Delta t$	1 quarter
Horizon for the estimation	$T$	10 years
Tax rate	$\tau_c$	0.18/year
Risk-free interest rate	$r_f$	0.05/year

**Table 5: Summary the simulation results**

Method	Pre-merger value		Consolidation value	Value increased ratio
	Taishin Bank	Dah An Bank		
<b>Schwartz and Moon (2000)</b>	NT\$369.747 bn	NT\$320.760 bn	NT\$872.658 bn	26.38%
<b>Longstaff (2001)</b>	NT\$339.003 bn	NT\$297.714 bn	NT\$821.418 bn	29.01%

**Table 6: Probability of Bankruptcy per Year for Base Valuation**

Year	Taishin Bank	Dah An Bank	Consolidation
1	1.17%	0.00%	0.00%
2	11.61%	1.01%	0.02%
3	9.66%	5.78%	0.11%
4	6.48%	6.53%	0.66%
5	4.68%	5.33%	1.40%
6	3.92%	4.31%	2.13%
7	3.05%	3.51%	2.22%
8	2.68%	3.04%	2.58%
9	2.25%	2.38%	2.41%
10	0.00%	0.00%	0.00%
Total	45.50%	31.89%	11.53%

**Table 7: Probability of Stop Points per Year (LSM approach)**

<b>Year</b>	<b>Taishin Bank</b>	<b>Dah An Bank</b>	<b>Consolidation</b>
1	2.51%	2.14%	0.45%
2	6.35%	5.81%	2.28%
3	9.87%	7.85%	4.59%
4	7.45%	6.58%	7.46%
5	5.91%	5.35%	8.57%
6	4.69%	5.72%	8.62%
7	5.06%	6.41%	9.32%
8	6.52%	8.56%	11.27%
9	8.38%	10.25%	10.83%
10	0.00%	0.00%	0.00%
<b>Total</b>	<b>56.74%</b>	<b>58.67%</b>	<b>63.39%</b>

**Table 8: Sensitivity of consolidation bank value to change parameters**

Parameter	Value of Perturbed Parameter	Total Consolidation Bank Value ( NT\$ thousands)	Increase/Decrease percentage to Base case
Base case		868,031,438	
$\mu_0^{LM}$	0.055 / quarter	913,308,472	5.22%
$\mu_0^{DM}$	0.055 / quarter	846,838,917	-2.44%
$\sigma_0^{LM}$	0.077 / quarter	875,041,757	0.81%
$\sigma_0^{DM}$	0.088 / quarter	868,454,744	0.05%
$\mu_{LM}$	0.022 / quarter	873,199,259	0.60%
$\mu_{DM}$	0.022 / quarter	865,663,870	-0.27%
$\sigma_{LM}$	0.033 / quarter	868,370,641	0.04%
$\sigma_{DM}$	0.033 / quarter	868,218,622	0.02%
$k_1$	0.077 / quarter	867,598,615	-0.05%
$k_2$	0.077 / quarter	867,760,803	-0.03%
$\alpha^{MO}$	0.011 / quarter	853,037,257	-1.73%
$\alpha^{MF}$	0.099 / quarter	866,134,239	-0.22%



**Table 9: Sensitivity of consolidation bank value to change parameters (LSM approach)**

Parameter	Value of Perturbed Parameter	Total Consolidation Bank Value ( NT\$ thousands)	Increase/Decrease percentage to Base case
Base case		817,088,184	
$\mu_0^{LM}$	0.055 / quarter	858,654,309	5.09%
$\mu_0^{DM}$	0.055 / quarter	798,235,081	-2.31%
$\sigma_0^{LM}$	0.077 / quarter	823,142,437	0.74%
$\sigma_0^{DM}$	0.088 / quarter	817,259,208	0.02%
$\bar{\mu}_{LM}$	0.022 / quarter	821,538,806	0.54%
$\bar{\mu}_{DM}$	0.022 / quarter	815,200,412	-0.23%
$\bar{\sigma}_{LM}$	0.033 / quarter	817,412,390	0.04%
$\bar{\sigma}_{DM}$	0.033 / quarter	817,321,305	0.03%
$k_1$	0.077 / quarter	817,060,177	0.00%
$k_2$	0.077 / quarter	816,829,042	-0.03%
$\alpha^{MO}$	0.011 / quarter	802,662,521	-1.77%
$\alpha^{MF}$	0.099 / quarter	815,223,433	-0.23%