Valuation and Analysis of Basket Credit Linked Notes with Issuer Default Risk

Po-Cheng Wu*

*Department of Banking and Finance, Kainan University, Address: No.1 Kainan Rd., Luchu Shiang, Taoyuan 33857, Taiwan, R.O.C., E-mail: pcwu@mail.knu.edu.tw, Tel.: 886-3-3412500 ext. 6171, Fax: 886-3-3412228.

Valuation and Analysis of Basket Credit Linked Notes with Issuer Default Risk

Abstract

This paper explores a reasonable coupon rate for basket credit linked notes (CLN) with issuer default risk. Based on the one factor Gaussian copula model, this paper proposes three methods for incorporating issuer default into basket CLN pricing. Numerical results indicate that issuer default risk impacts basket CLN coupon rate. Furthermore, the coupon rate differs with changes in correlation structure among the three methods. Finally, one of the three methods is identified as the most suitable.

Keywords: Basket credit linked notes, issuer default risk, default correlation, factor Gaussian copula, Monte Carlo simulation.

1. Introduction

A credit linked note (CLN) is a note for which the price or coupon is linked to the credit event of the reference entity (obligation) (Anson *et al.*, 2004; Das, 2000; Fabozzi *et al.*, 2007). A CLN linked to multiple reference entities is called a basket CLN. Such a CLN can be structured by a note and a basket default swap (BDS). The conventional form of basket CLN is the kth-to-default CLN. The CLN holder (the protection seller) pays the notional principal to the CLN issuer (the protection buyer) at the start of the contract and receives the coupon payments until either the kth default or the contract matures, whichever occurs earlier. If the kth default occurs before contract maturity, the CLN holder receives the recovered value of the reference entity from the CLN issuer. Otherwise, the CLN holder receives the notional principal back on contract maturity.

In derivative markets, the issuer default risk is attracting considerable attention because of the recent financial turmoil and collapses of large financial institutions. If the CLN issuer defaults, the CLN holder will not receive the recovered value of the reference entity as the credit event happens, nor the notional amount at the contract maturity. The coupon payments also ceases due to the issuer default. The issuer default results in a large loss. Thus it is important to incorporate issuer default risk in basket CLN pricing to obtain a reasonable coupon rate.

Two main approaches exist to modeling the default risk in the literature: the structural and reduced form models. The structural model was developed by Merton (1974), and defined credit events as occurring when firm asset value falls below firm debt. The reduced form model, also known as the intensity model, was developed by Jarrow and Turnbull (1995). This model views the credit event as an unexpected exogenous stochastic event and uses market data to estimate the default risk.

Hull and White (2000) provided a methodology for valuing credit default swap (CDS) without counterparty default risk when the payoff is contingent on the default of a single reference entity. Hull and White (2001) developed a model of default correlations between different corporate or sovereign entities. The model of Hull and White is an extension of the structural model, sets a credit index variable for each reference entity, and selects correlated diffusion processes for the credit indexes. Their model defines default as the credit index falling below the predetermined default barriers. Monte Carlo simulation is used to calculate the vanilla CDS and BDS spread given the possibility of issuer default. Hui and Lo (2002) developed a model to price the single-name CLN with issuer default risk using the framework of Merton’s model. They demonstrated that the credit spreads of a CLN increase non-linearly with decreasing correlation between the reference entity and the issuer.

Pricing multi-name credit derivatives requires a joint distribution model of the default times. However, whether using the structural or reduced form models, valuing the multi-name credit derivative is computationally complex. Thus the copula function (Sklar, 1959), also known as the dependence function, which simplifies the estimation of the joint distribution, recently has been widely used to price the multi-name credit derivatives. Li (1999, 2000) first introduced the copula function to deal with the dependence structure in multi-name credit derivative pricing. He assumed the default times of reference entities to be Poisson processed, and set the dependence structure as a Gaussian copula function. Finally, Li performed Monte Carlo simulation to obtain the default times. Mashal and Naldi (2003) applied Li’s method to analyze how the default probabilities of the protection sellers and buyers affect BDS spread.

Calculating default times using the copula method is fairly easy. However, the computational complexity of the Monte Carlo simulation with Gaussian copula increases with number of reference entities. The factor copula method, which makes the credit event conditional on independent state variables, was introduced to deal with these problems. Hull and White (2004) employed a multi-factor copula model to price the kth-to-default swap and collateralized debt obligation (CDO). Moreover, Laurent and Gregory (2005) proposed one factor Gaussian copula to simplify the dependence structure of reference entities, and applied this approach to price BDS and CDO.

This paper focuses on how to incorporate issuer default risk into the basket CLN pricing methods with the one factor Gaussian copula model. Three different methods are proposed and numerical analysis is performed to compare them. The remainder of this paper is organized as follows. Section 2 briefly reviews the one factor Gaussian copula model. Section 3 then describes the proposed methods for incorporating issuer credit event into basket CLN pricing. Subsequently, Section 4 presents the results of numerical analysis and compares the methods. Conclusions are finally drawn in Section 5.

2. One Factor Gaussian Copula

Copula is a function which links the univariate marginal distributions to their full multivariate distribution and can be expressed as follows:

$$C(u_1, u_2, \ldots, u_N) = \Pr(U_1 \le u_1, U_2 \le u_2, \ldots, U_N \le u_N) \qquad (1)$$

where $U_i \sim U(0,1)$, $i = 1, 2, \ldots, N$. Sklar (1959) proved that if $F(x_1, x_2, \ldots, x_N)$ is a joint multivariate distribution with univariate marginal distributions

$F_i(x_i)$, $i = 1, 2, \ldots, N$, there exists a copula function such that:

$$F(x_1, x_2, \ldots, x_N) = C(F_1(x_1), F_2(x_2), \ldots, F_N(x_N)) \tag{2}$$

If each $F_i(x_i)$ is continuous, then the copula function is unique. The definition of a Gaussian copula is as follows:

$$C^{Ga}(u_1, u_2, \ldots, u_n) = \Phi_R(\phi^{-1}(u_1), \phi^{-1}(u_2), \ldots, \phi^{-1}(u_n)) \tag{3}$$

where Φ_R denotes a multivariate cumulative normal (Gaussian) distribution, R represents the correlation coefficient matrix, and ϕ^{-1} is the inverse function of one dimensional cumulative normal distribution.

Using the reduced form model, each reference entity default follows a Poisson process. Suppose the credit portfolio contains N reference entities, and the default times are $\tau_1, \tau_2, \ldots, \tau_N$, respectively. τ_i is a positive random variable with distribution:

$$P(\tau_i > t) = 1 - P(\tau_i \le t) = e^{-\lambda_i t}, \quad i = 1, 2, \ldots, N \tag{4}$$

where λ_i is the hazard rate of the reference entity i . The cumulative default probability before time t is:

$$F_i(t) = P(\tau_i \le t) = 1 - e^{-\lambda_i t}, \quad i = 1, 2, \ldots, N \tag{5}$$

Because $F_i(t) \sim U(0,1)$, applying the Gaussian copula obtains the multivariate joint distribution of default times, as follows:

$$F(\tau_1, \tau_2, \ldots, \tau_N) = \Phi_R\left(\phi^{-1}(F_1(\tau_1)), \phi^{-1}(F_2(\tau_2)), \ldots, \phi^{-1}(F_N(\tau_N))\right) \qquad (6)$$

In the one factor model, the default times of all reference entities depend on a common factor Y, and firm specific risk factors ε_i, $i = 1, 2, \ldots, N$. Y and ε_i are independent standard normal variables. Based on the above setting, a new Gaussian vector $(X_1, X_2, \ldots, X_N)$ can be created via Cholesky decomposition, as follows:

$$X_i = \rho_i Y + \sqrt{1 - \rho_i^{\,2}}\,\varepsilon_i, \quad i = 1, 2, \ldots, N \qquad (7)$$

where ρ_i denotes the correlation coefficient between the new Gaussian variable X_i and the common factor Y. One factor Gaussian copula model with constant pairwise correlations has become the standard market model. Let $\rho_i = \rho$ in Eq. (7), then the constant pairwise correlations $\rho_{i,\,j}$ will be ρ^2.

Let $X_1 = \phi^{-1}(F_1(\tau_1))$, $X_2 = \phi^{-1}(F_2(\tau_2))$, $\ldots$, $X_N = \phi^{-1}(F_N(\tau_N))$, in which case $F_1(\tau_1) = \phi(X_1)$, $F_2(\tau_2) = \phi(X_2)$, $\ldots$, $F_N(\tau_N) = \phi(X_N)$. By mapping the cumulative normal distribution between default time τ_i and Gaussian variable X_i, we can simulate the default time of the reference entity i using the following equation:

$$\tau_i = F_i^{-1}(\phi(X_i)) = \frac{-\ln(1 - \phi(X_i))}{\lambda_i}, \quad i = 1, 2, \ldots, N \qquad (8)$$

3. Proposed Methods

This paper proposes three possible methods of incorporating issuer default risk into basket CLN pricing using the one factor Gaussian copula model. For comparison, the method without issuer default risk is named method A. Meanwhile, the three proposed methods with issuer default risk are named methods B, C and D, respectively. Assuming that the normal random variable corresponding to the default time of each underlying reference entity (abbreviated here as the underlying variable) is X_i, the normal random variable corresponding to the issuer default time (abbreviated as the issuer variable) is Z, and the common factor is Y. The different structures of methods A to D are listed below and shown in Figure 1. Table 1 summarizes the parameters of these four methods.

Method A: Issuer default risk is not considered. The correlation between each X_i and Y is ρ, as shown in Figure 1 (A).

Method B: Issuer default risk is considered. The correlation between each X_i and Y is ρ, but Z is independent of X_i and Y, as shown in Figure 1 (B).

Method C: Issuer default risk is considered. The correlation between each X_i, Z and Y is ρ, as shown in Figure 1 (C).

Method D: Issuer default risk is considered, but the common factor Y is replaced by the issuer variable Z. The correlation between each X_i and Z is ρ, as shown in Figure 1 (D).

[Insert Figure 1 about here]

[Insert Table 1 about here]

3.1 Pricing basket CLN without Issuer Default Risk

Assume a k th-to-default CLN involving N reference entities which the notional principal of each reference entity is one dollar. The coupon rate is c. The coupon (the notional principal multiplied by the coupon rate) is paid annually, and the payment dates are $t_i, i = 1, 2, \ldots, T$. The maturity date of the basket CLN is t_T. Furthermore, τ_k is the k th default time, and $\tau_1 < \tau_2 < \cdots < \tau_N$. Moreover, δ_k is the recovery rate of the k th default reference entity. Thus δ_k denotes the redemption proceeds (the notional principal multiplied by the recovery rate) which the issuer pays to the basket CLN holder on the k th default. The discount rate is $r\%$. Finally, Q denotes the risk-neutral probability measure, and $I(\cdot)$ represents an indicator function. The value of a k th-to-default CLN can be represented as follows:

$$CLN = E^Q\left[c \times \sum_{i=1}^{T} e^{-rt_i} I(t_i < \tau_k) + \delta_k \times e^{-r\tau_k} \times I(\tau_k \le t_T) + e^{-rt_T} \times I(\tau_k > t_T)\right] \quad (9)$$

Let the above equation equals one, the equation can be rewritten as:

$$\begin{aligned} & c \times E^Q\left[\sum_{i=1}^{T} e^{-rt_i} I(t_i < \tau_k)\right] \\ & = E^Q\left[1 - \delta_k \times e^{-r\tau_k} \times I(\tau_k \le t_T) - e^{-rt_T} \times I(\tau_k > t_T)\right] \end{aligned} \quad (10)$$

Rearranging Eq. (10) can yield the fair coupon rate at the start of the CLN as follows:

$$c = \frac{E^Q\left[1-\delta_k \times e^{-r\tau_k} \times I(\tau_k \le t_T) - e^{-rt_T} \times I(\tau_k > t_T)\right]}{E^Q\left[\sum_{i=1}^{T} e^{-rt_i} I(t_i < \tau_k)\right]} \tag{11}$$

By using W runs of Monte Carlo simulation to price the basket CLN, the numerator of Eq. (11) is:

$$\frac{1}{W}\sum_{s=1}^{W}\left[1-\delta_k^s \times e^{-r\tau_k^s} \times I(\tau_k^s \le t_T) - e^{-rt_T} \times I(\tau_k^s > t_T)\right] \tag{12}$$

where δ_k^s denotes the recovery rate of the k th default reference entity at the s th simulation, and τ_k^s represents the k th default time at the s th simulation. The denominator of Eq. (11) is:

$$\frac{1}{W}\sum_{s=1}^{W}\left[\sum_{i=1}^{T} e^{-rt_i} I(t_i < \tau_k^s)\right] \tag{13}$$

Therefore, the fair value of the coupon rate c is:

$$c = \frac{\sum_{s=1}^{W}\left[1-\delta_k^s \times e^{-r\tau_k^s} \times I(\tau_k^s \le t_T) - e^{-rt_T} \times I(\tau_k^s > t_T)\right]}{\sum_{s=1}^{W}\left[\sum_{i=1}^{T} e^{-rt_i} I(t_i < \tau_k^s)\right]} \tag{14}$$

3.2 Pricing basket CLN with Issuer Default Risk

In situations involving issuer default risk, it is necessary to consider whether the issuer defaults before or after the k th default. This paper defines $\hat{\tau}^s$ as the issuer default time at the sth simulation and $\hat{\delta}$ as the issuer recovery rate. The CLN holder gets back the recovered value of the reference obligation if the kth default occurs before both the issuer default time $\hat{\tau}^s$ and the maturity date t_T. If the issuer defaults before the kth default and the maturity date, the issuer will not provide the CLN holder with the redemption proceeds and stop the coupon payments. In this situation, the notional principal multiplied by the issuer recovery rate is returned to the CLN holder. To obtain all of the notional principal back, both the kth default time and the issuer default time must be later than the contract maturity date. Thus, the value of a kth-to-default CLN with issuer default risk must be modified as follows:

$$CLN = E^Q\left[\begin{array}{l} c \times \sum_{i=1}^{T} e^{-rt_i} I\left(t_i < \min(\tau_k^s, \hat{\tau}^s)\right) + \delta_k^s \times e^{-r\tau_k^s} \times I\left(\tau_k^s < \min(\hat{\tau}^s, t_T)\right) \\ + \hat{\delta} \times e^{-r\hat{\tau}^s} \times I\left(\hat{\tau}^s < \min(\tau_k^s, t_T)\right) + e^{-rt_T} \times I\left(t_T < \min(\tau_k^s, \hat{\tau}^s)\right) \end{array}\right] \quad (15)$$

The definitions of δ_k^s and τ_k^s are as in Eq. (12). The fair value of the coupon rate c with issuer default risk is:

$$c = \frac{\sum_{s=1}^{W}\left[\begin{array}{l} 1 - \delta_k^s \times e^{-r\tau_k^s} \times I\left(\tau_k^s < \min(\hat{\tau}^s, t_T)\right) \\ - \hat{\delta} \times e^{-r\hat{\tau}^s} \times I\left(\hat{\tau}^s < \min(\tau_k^s, t_T)\right) - e^{-rt_T} \times I\left(t_T < \min(\tau_k^s, \hat{\tau}^s)\right) \end{array}\right]}{\sum_{s=1}^{W}\left[\sum_{i=1}^{T} e^{-rt_i} I\left(t_i < \min(\tau_k^s, \hat{\tau}^s)\right)\right]} \quad (16)$$

4. Numerical Analysis

This paper adopts a five-year basket CLN with three reference entities as an example of numerical analysis. Assume all three reference entities have notional principal one dollar, hazard rate 5% and recovery rate 30%. Furthermore, assume the coupon is paid annually, the hazard rate and recovery rate of the issuer is 1% and 30%, respectively. Discount rates are obtained by bootstrapping from the government bond data. Sixty-thousand runs of Monte Carlo simulation are executed to calculate the coupon rates for methods A to D, and the results are shown in Table 2 to Table 5 and Figure 2.

[Insert Table 2 about here]

[Insert Table 3 about here]

[Insert Table 4 about here]

[Insert Table 5 about here]

[Insert Figure 2 about here]

The correlation coefficient ρ represents the correlation between each reference entity and the common factor (or the issuer default risk). The correlation between the reference entities X_i and X_j, which equals ρ^2, is positively correlated to $|\rho|$. As shown in Figure 2 (A), the coupon rate of the first-to-default (k=1) CLN is negatively correlated with $|\rho|$, because the probability of the first-to-default occurring increases as $|\rho|$ decreases. Conversely, as shown in Figure 2 (B) and Figure 2 (C), the coupon rate of the second (k=2) and third-to-default (k=3) CLN is positively correlated with $|\rho|$, because the probability of the joint default increases as $|\rho|$ increases.

Figure 2 also shows that the coupon rates with issuer default risk in methods B to D are higher than those without issuer default risk in method A. Furthermore, when the correlation coefficient is positive, the coupon rates in method D are lower than those in methods B and C. Meanwhile, when the correlation coefficient is negative, the coupon rates in method D are higher than those in methods B and C. Thus the curves of coupon rates in method D are asymmetric.

The following explores the above phenomenon. When the reference entity defaults, the CLN holder receives the recovered value of the reference entity and loses the coupon incomes after the reference entity credit event. The issuer default exhibits a similar effect. When the issuer defaults, the CLN holder receives the recovered value of the CLN and loses the coupon incomes after the issuer default event. The issuer default may occur before or after the reference entity default. When the issuer default occurs after the reference entity default, the contract is terminated at the default time of the reference entity and the CLN holder receives the recovered value and loses the following coupon income. Thus the loss of a CLN with issuer default risk will be identical to that without issuer default risk. On the other hand, when the issuer default is earlier than the reference entity default, the contract is terminated simultaneously with the issuer default. Suppose the recovery rates of the issuer and the reference entity are identical. The CLN holder then receives the same recovered value as when the reference entity defaults first. However, the CLN holder not only loses the coupon incomes after the reference entity default time, but also those between the issuer and reference entity default time. Thus, the loss of a CLN with issuer default risk exceeds that without issuer default risk. This situation causes the total risk of the CLN with issuer default risk to be higher than that without issuer default risk. Therefore the coupon rate of the CLN with issuer default risk is higher than that without issuer default risk.

When the issuer and reference entitiy default are highly positively correlated, their movements are almost identical, making the defaults times of the issuer and reference entities close together. As the correlation approaches one, the joint default of the issuer and the reference entity become more likely and the default times become closer together. In extreme cases, for example when the correlation coefficient is one, the issuer will default simultaneously with the reference entity default. Thus the issuer default does not influence the total risk of the CLN. On the other hand, as the correlation approaches negative one, the default times become more dispersive. Other things being equal, the impact of issuer default risk decreases with increasing closeness of default times. Therefore, the differences of the coupon rate between the with and without issuer default risk situations reduce as the correlation approaches one. The default correlation between the reference entities and the issuer is zero in method B, ρ^2 in method C, and ρ in method D, respectively. When the correlation is positive, i.e. when $0 \le \rho \le 1$, then $0 \le \rho^2 \le \rho \le 1$. Thus, the coupon rates follow the ranking method B > method C > method D as shown in Figure 2. When the correlation is negative, i.e. when $-1 \le \rho \le 0$, then $\rho \le 0 \le \rho^2 \le 1$. Thus, the coupon rates follow the ranking method D > method B > method C. Moreover, in methods B and C the coupon rate curves are symmetric. However, in method D the coupon rate curves are asymmetric, depending on whether the default correlation is positive or negative. Therefore, using the issuer variable as the common factor in the factor copula framework will include more information about the default correlation between the reference entities and the issuer.

5. Conclusions

To obtain the most reasonable coupon rate, issuer default risk in basket CLN pricing must be considered. This paper proposes three methods for incorporating the issuer default risk into basket CLN pricing using one factor Gaussian copula model. The analytical results show that the coupon rates obtained by all three proposed methods with issuer default risk are higher than the method without issuer default risk. Thus issuer default risk increases the basket CLN coupon rate. Furthermore, among the proposed methods, because method D directly takes account of the default correlation between the reference entities and the issuer, the positive or negative effect of the default correlation is fully reflected in the coupon rates. Therefore, method D is the most preferable model for pricing basket CLN with issuer default risk.

References

Anson, M. J., Fabozzi, F. J., Choudhry, M., and Chen, R. (2004), *Credit derivatives: instruments, applications and pricing*, John Wiley and Sons.

Das, S. (2000), *Credit Derivatives and Credit Linked Notes*, John Wiley and Sons.

Fabozzi, F. J., Davis, H. A., and Choudhry, M. (2007), "Credit-Linked Notes A Product Primer," *The Journal of Structured Finance*, 12, 67-77.

Hui, C. H. and Lo, C. F. (2002), "Effect of Asset Value Correlation on Credit-Linked Note Values," *International Journal of Theoretical and Applied Finance*, 5, 455-478.

Hull, J. and White, A. (2000), "Valuing Credit Default Swaps I: No Counterparty Default Risk," *Journal of Derivatives*, 8, 29-40.

Hull, J. and White, A. (2001), "Valuing Credit Default Swaps II: Modeling Default Correlation," *Journal of Derivatives*, 8, 12-22.

Hull, J. and White, A. (2004), "Valuation of a CDO and an nth to Default CDS without Monte Carlo Simulation," *Journal of Derivatives*, 12, 8-23.

Jarrow, R. and Turnbull, S. (1995), "Pricing Derivatives on Financial Securities

Subject to Credit Risk," *Journal of Finance*, 1, 53-85.

Laurent, J. P. and Gregory, J. (2005), "Basket Default Swaps, CDOs and Factor Copulas," *Journal of Risk*, 7, 103-122.

Li, D. X. (1999), "The Valuation of Basket Credit Derivatives," *CreditMetrics Monitor*, April, 34-50.

Li, D. X. (2000), "On Default Correlation: A Copula Function Approach," *Journal of Fixed Income*, 9, 43-54.

Mashal, R. and Naldi, M. (2003), "Pricing Multi-name Default Swaps with Counterparty Risk," *Quantitative Credit Research*, November, 1-17.

Merton, R. C. (1974), "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, 449-470.

Sklar, A. (1959), "Fonctions de repartition a n dimensions et leurs marges," Publ. Inst. Statist. Univ. Paris, 8, 229-231.

Table 1 Summary of the parameters of the four methods.

Method	Is issuer default risk considered?	Parameters Setting (Correlation with underlying variable X_i)		
		Common Factor Y	Issuer Variable Z	Underlying Variable X_j
A	N	ρ	Not considered	ρ^2
B	Y	ρ	Independent	ρ^2
C	Y	ρ	ρ^2	ρ^2
D	Y	Not considered	ρ	ρ^2

Table 2 Coupon rates under various correlation coefficients in method A (k=1: first-to-default, 2: second-to-default, 3: third-to-default).

Method A			
ρ	k=1	k=2	k=3
-0.9	8.0195%	5.2169%	3.5790%
-0.8	9.2625%	4.9714%	3.0115%
-0.7	10.2417%	4.7203%	2.6429%
-0.6	11.0469%	4.4753%	2.3993%
-0.5	11.7604%	4.2476%	2.2319%
-0.4	12.3336%	4.0568%	2.1107%
-0.3	12.7890%	3.8888%	2.0280%
-0.2	13.1304%	3.7766%	1.9809%
-0.1	13.3336%	3.6917%	1.9499%
0	13.3875%	3.6646%	1.9359%
0.1	13.3547%	3.6833%	1.9470%
0.2	13.1236%	3.7480%	1.9719%
0.3	12.7744%	3.8817%	2.0171%
0.4	12.2933%	4.0242%	2.0883%
0.5	11.7091%	4.2155%	2.2105%
0.6	10.9854%	4.4092%	2.3793%
0.7	10.1941%	4.6447%	2.6279%
0.8	9.2061%	4.9138%	2.9783%
0.9	7.9788%	5.1952%	3.5583%

Table 3 Coupon rates under various correlation coefficients in method B (k =1: first-to-default, 2: second-to-default, 3: third-to-default).

Method B			
ρ	k =1	k =2	k =3
-0.9	8.7558%	5.9139%	4.2528%
-0.8	10.0115%	5.6654%	3.6828%
-0.7	10.9992%	5.4107%	3.3111%
-0.6	11.8112%	5.1587%	3.0658%
-0.5	12.5292%	4.9317%	2.8982%
-0.4	13.1138%	4.7378%	2.7792%
-0.3	13.5773%	4.5695%	2.6958%
-0.2	13.9227%	4.4569%	2.6480%
-0.1	14.1276%	4.3697%	2.6164%
0	14.1785%	4.3412%	2.6020%
0.1	14.1455%	4.3624%	2.6133%
0.2	13.9126%	4.4263%	2.6392%
0.3	13.5598%	4.5618%	2.6843%
0.4	13.0772%	4.7071%	2.7558%
0.5	12.4855%	4.8983%	2.8783%
0.6	11.7559%	5.0926%	3.0476%
0.7	10.9577%	5.3317%	3.2985%
0.8	9.9551%	5.6050%	3.6515%
0.9	8.7173%	5.8949%	4.2371%

Table 4 Coupon rates under various correlation coefficients in method C (k =1: first-to-default, 2: second-to-default, 3: third-to-default).

	Method C		
ρ	k =1	k =2	k =3
-0.9	8.0346%	5.2585%	3.7167%
-0.8	9.3471%	5.1490%	3.3632%
-0.7	10.4279%	5.0406%	3.1441%
-0.6	11.3501%	4.9077%	2.9853%
-0.5	12.1928%	4.7844%	2.8735%
-0.4	12.8846%	4.6587%	2.7835%
-0.3	13.4509%	4.5440%	2.7177%
-0.2	13.8645%	4.4490%	2.6597%
-0.1	14.1181%	4.3717%	2.6251%
0	14.1785%	4.3412%	2.6020%
0.1	14.1215%	4.3559%	2.6101%
0.2	13.8336%	4.4025%	2.6320%
0.3	13.4073%	4.5070%	2.6668%
0.4	12.8209%	4.5995%	2.7214%
0.5	12.1252%	4.7249%	2.8231%
0.6	11.2806%	4.8338%	2.9471%
0.7	10.3750%	4.9559%	3.1099%
0.8	9.2802%	5.0873%	3.3144%
0.9	7.9892%	5.2336%	3.6858%

Table 5 Coupon rates under various correlation coefficients in method D (k =1: first-to-default, 2: second-to-default, 3: third-to-default).

	Method D		
ρ	k =1	k =2	k =3
-0.9	9.1615%	6.1338%	4.3829%
-0.8	10.5103%	5.8678%	3.7781%
-0.7	11.5696%	5.5968%	3.3869%
-0.6	12.4242%	5.3335%	3.1286%
-0.5	13.1511%	5.0877%	2.9515%
-0.4	13.6788%	4.8797%	2.8231%
-0.3	14.0447%	4.6906%	2.7359%
-0.2	14.2518%	4.5530%	2.6849%
-0.1	14.2996%	4.4321%	2.6497%
0	14.1802%	4.3584%	2.6277%
0.1	13.9761%	4.3244%	2.6298%
0.2	13.5867%	4.3172%	2.6411%
0.3	13.0893%	4.3634%	2.6595%
0.4	12.4911%	4.4004%	2.6870%
0.5	11.8164%	4.4850%	2.7424%
0.6	11.0371%	4.5868%	2.8192%
0.7	10.2099%	4.7318%	2.9414%
0.8	9.2093%	4.9376%	3.1492%
0.9	7.9788%	5.1959%	3.5901%

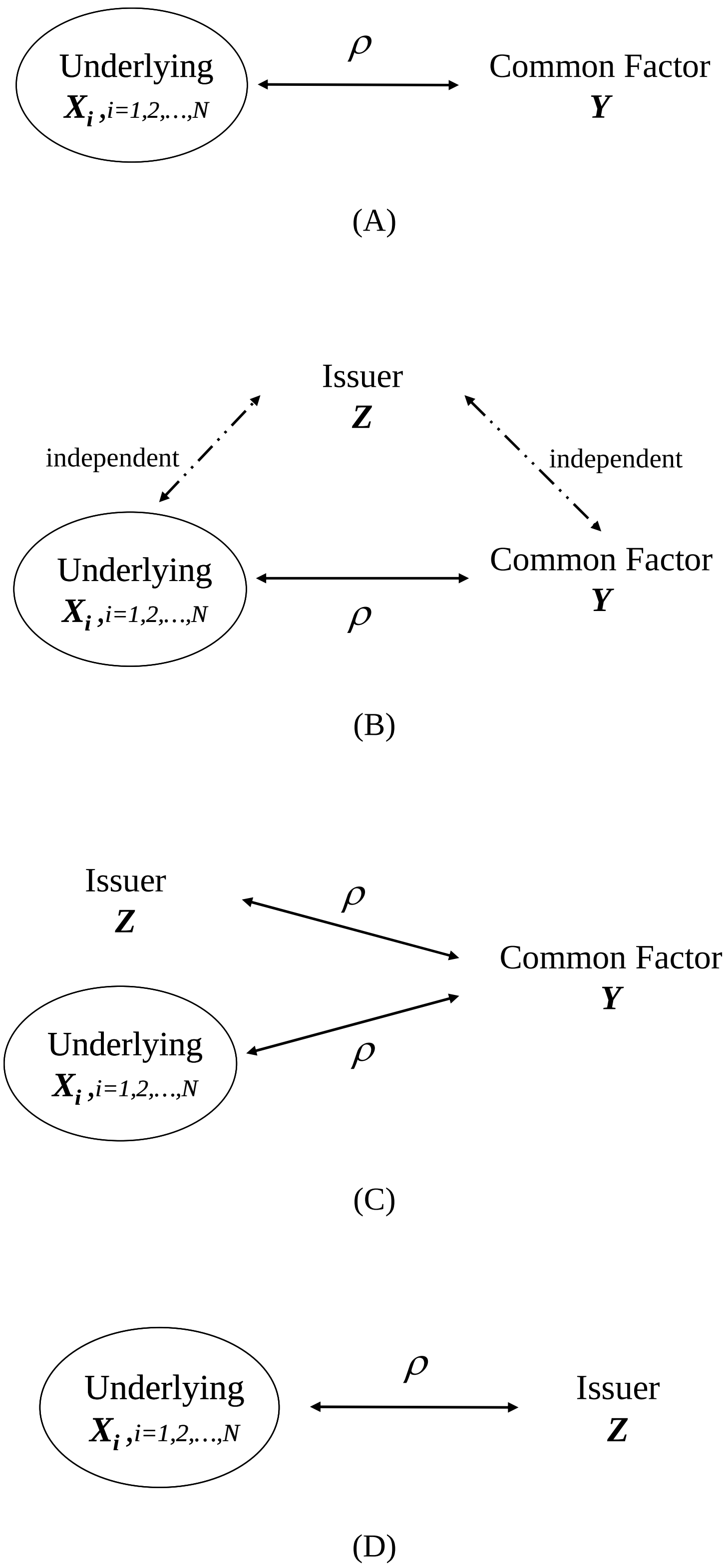
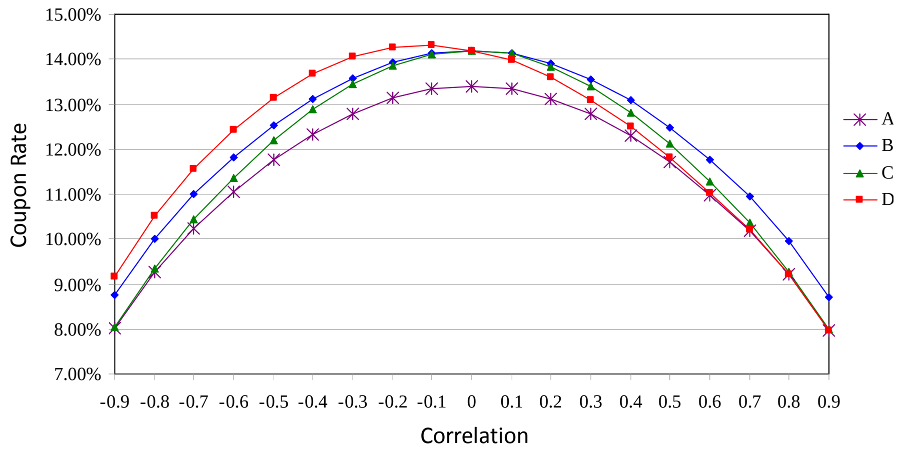
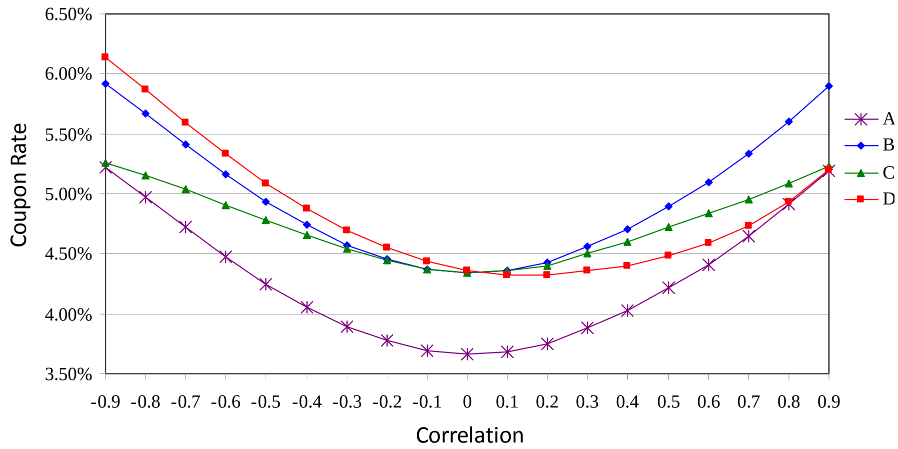


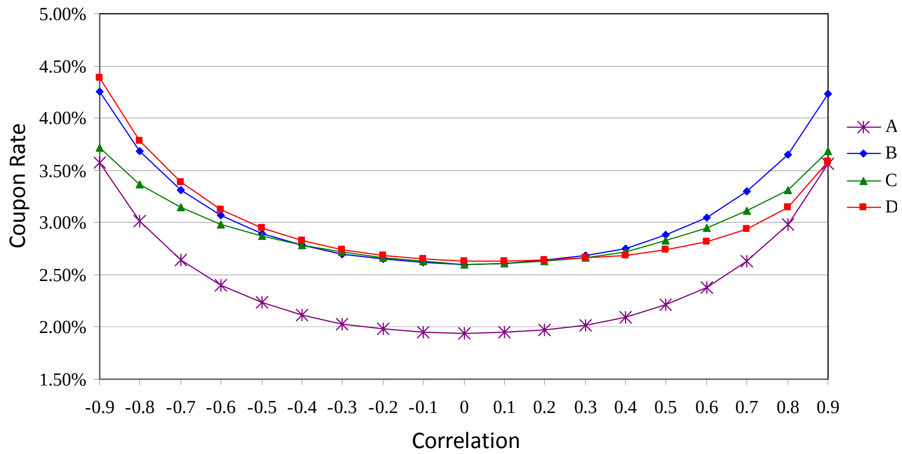
Figure 1 Relationships between the variables in methods A to D: (A) method A; (B) method B; (C) method C; (D) method D.



(A)



(B)



(C)

Figure 2 Coupon rates of the k th-to-default CLN in methods A to D: (A) $k=1$; (B) $k=2$; (C) $k=3$.