Regime Switching Correlation Hedging

Hsiang-Tai Lee
Associate Professor
Department of Banking and Finance,
1, University Rd., Puli, Nantou Hsien,
National Chi Nan University, Taiwan 54561
(886) 49-2910960 # 4648
sagerlee@ncnu.edu.tw

Abstract

The article investigates the hedging effectiveness of commodity futures when the correlations of spot and futures return series are subject to multi-state regime shifts. An independent switching dynamic conditional correlation GARCH (IS-DCC) which is free from the path-dependency and recombining problems is proposed to model multi-regime switching correlations. Results of hedging exercises show that in general, IS-DCC outperforms state-independent DCC GARCH and three-state IS-DCC exhibits superior hedging effectiveness when full sample period is applied, illustrating importance of modeling higher-state switching correlations for futures hedging.

Keywords: Dynamic Futures Hedging, Hedge ratio, GARCH model, Markov regime switching

Regime Switching Correlation Hedging

Abstract

The article investigates the hedging effectiveness of commodity futures when the correlations of spot and futures return series are subject to multi-state regime shifts. An independent switching dynamic conditional correlation GARCH (IS-DCC) which is free from the path-dependency and recombining problems is proposed to model multi-regime switching correlations. Results of hedging exercises show that in general, IS-DCC outperforms state-independent DCC GARCH and three-state IS-DCC exhibits superior hedging effectiveness when full sample period is applied, illustrating importance of modeling higher-state switching correlations for futures hedging.

Keywords: Dynamic Futures Hedging, Hedge ratio, GARCH model, Markov regime switching

I. Introduction

It is widely known that when hedging a spot position with a position in the futures market, the minimum variance hedge ratio (MVHR) is equal to the ratio of the covariance of spot and futures returns to the variance of futures returns. A common approach to conditional minimum-variance hedging is to model the time-varying conditional variance-covariance matrix of returns using a multivariate GARCH model, and use forecasts from this model to construct a forecast of the conditional MVHR (Baillie and Myers, 1991; Kroner and Sultan, 1993; Park and Switzer, 1995; Gagnon and Lypny, 1995; Brooks et al., 2002; and Byström, 2003).

Recent studies recognize that the relationship between spot and futures returns may be characterized by regime shifts (Sarno and Valente, 2000, 2005a, 2005b). The implication is that to improve the futures hedging effectiveness, the state-dependent property between spot and futures series should also be taken account in developing dynamic hedging strategies. Alizadeh and Nomikos (2004), Lee et al. (2006), Lee and Yoder (2007a), Lee and Yoder (2007b) and Alizadeh et al. (2008) respectively, propose regime switching least square model, regime switching state space model, regime switching Varying Correlation GARCH (VC-GARCH) model, regime switching BEKK-GARCH model and regime switching vector error correction model for futures hedging and find that the hedging effectiveness are improved compared to state-independent strategies. To further incorporate the effects of unanticipated news events in determining of optimal hedge ratio, Lee (2009a) develops a Markov regime switching Generalized Orthogonal GARCH model with conditional jump dynamics for estimating the optimal hedge ratio. Further extension is to release the assumption of joint normality between spot and futures return series and using regime switching copula GARCH model for futures hedging (Lee, 2009b). All these elaborations are found to improve futures hedging effectiveness.

Although these regime switching GARCH models have captured much of the observed behavior in the spot and futures return series, they possess some limitations. Firstly, these models allow mean, volatility, and correlation equations to be state-dependent simultaneously and as a consequence, discussion of the number of regimes is limited to two due to the potential problems of overparameterization and convergence for higher regimes. As pointed out by Caporin and Billio (2005), a full Markov switching model is highly unstable given the huge number of switching parameters. To the author's knowledge, no multi-regime multivariate GARCH model has been applied for futures hedging. Secondly, all these models are subject to the well-known path-dependency problem, (Cai, 1994; Hamilton and Susmel, 1994; Gray, 1996; and Lee et al., 2007a, 2007b). Recombining procedures are required to approximate the residuals, variances and correlation at each time point and these procedures inevitably create computational burden and as pointed out by Hass, et. al. (2004), the analytical tractability of the dynamic process is problematic.

This study attempts to investigate if allowing the correlation of spot and futures return series to be subject to multi-state switching improves the futures hedging effectiveness by proposing an independent switching dynamic conditional correlation GARCH (IS-DCC) model. There are several reasons that we argue for an independent switching model for the correlation. Firstly, time-varying correlations risks are widely noticed in recent finance literature. For instance, Krishnan et al. (2009) find that the correlation of returns between assets has varied substantially over time and investors would pay a premium for securities that perform well in regimes in which the correlation is high. Ang and Chen (2002) find that Correlations between U.S. stocks and the aggregate U.S. market are much greater for downside moves than for upside moves. Ledoit et al. (2003) find that the level of correlation for international stock markets depends on the phase of the business cycle. All these findings suggest a state-dependent time-varying correlation for modeling financial time series. Secondly, limiting the switching only for the correlation mitigates the problems of overparameterization and convergence and the discussion of regime switching effect on futures hedging with more than two states is possible. Lee et al. (2007a) model explicitly the state-dependent time-varying correlation process. Their model, however, limits the number of states to two. Finally, the proposed IS-DCC avoids the problem of path-dependency and is free from the requirement of recombining procedure. This reduces the burden of computation and avoids the analytical intractability problem.

The remainder of the article is organized as follows. The proposed IS-DCC is presented in section II. Section III addresses the estimation issue encountered for the IS-DCC. The minimum variance hedge ratio under regime switching and measuring hedging performance are discussed in section IV. This is followed by data description and empirical results. A conclusion ends the article.

II. Independent Switching Dynamic Conditional Correlation GARCH Model (IS-DCC)

The independent switching dynamic conditional correlation GARCH (IS-DCC) is a modification of the Markov regime switching dynamic conditional correlation GARCH

(MS-DCC; Caporin and Billio, 2005) such that no problems of path-dependency will occur and recombining procedure is not required. The specification of IS-DCC is given below:

Suppose that the observed 2-dimensioned economic process $\{\mathbf{R}_t\}$ is given by

$$\mathbf{R}_t = \boldsymbol{\mu} + \mathbf{e}_t, \tag{1}$$

$$= \boldsymbol{\mu} + \mathbf{D_t}\boldsymbol{\varepsilon}_\mathbf{t}, \tag{2}$$

where $\boldsymbol{\mu} = [\mu_c \quad \mu_f]^T$ is a 2×1 vector of conditional means, "T" stands for transpose, $\mathbf{e}_t = [e_{c,t} \quad e_{f,t}]^T = \mathbf{D_t}\boldsymbol{\varepsilon}_\mathbf{t}$ is assumed to be normally distributed

$$\mathbf{e_t} \mid \psi_{t-1} \quad \sim N(0, \quad \mathbf{H_t}), \tag{3}$$

with time-dependent variance-covariance matrix $\mathbf{H_t}$. $\boldsymbol{\varepsilon}_t = \mathbf{D}_t^{-1}(\mathbf{R_t} - \boldsymbol{\mu})$ is the normalized residual vector, N stands for normal distribution and ψ_{t-1} is the information set up to time $t-1$. The time-varying variance-covariance matrix $\mathbf{H_t}$ is given by

$$\mathbf{H_t} = \mathbf{D_t}\boldsymbol{\Gamma}_\mathbf{t}\mathbf{D_t}, \tag{4}$$

where $\mathbf{D_t} = diag\left(\sqrt{h_{i,t}}\right)$, $i \in \{c, f\}$ is a diagonal matrix with the volatilities of spot and futures returns on the i^th element. The conditional variances dynamic are assumed to follow a state-independent GARCH(1,1) process

$$\mathbf{D_t^2} = \mathbf{diag}\{\boldsymbol{\gamma}_\mathbf{i}\} + \mathbf{diag}\{\boldsymbol{\alpha}_\mathbf{i}\} \circ \mathbf{e_{t-1}}\mathbf{e_{t-1}^T} + \mathbf{diag}\{\boldsymbol{\beta}_\mathbf{i}\} \circ \mathbf{D_{t-1}^2}, \tag{5}$$

where $\circ$ is Hadamard product and $\boldsymbol{\gamma}_\mathbf{i}$, $\boldsymbol{\alpha}_\mathbf{i}$, and $\boldsymbol{\beta}_\mathbf{i}$, $i \in \{c, f\}$ are GARCH coefficients.

In the state-independent dynamic conditional correlation GARCH model (Engle, 2002), $\boldsymbol{\Gamma}_\mathbf{t}$ is the correlation matrix and is defined as

$$\boldsymbol{\Gamma}_\mathbf{t} = diag\{\mathbf{Q_t}\}^{-1/2}\mathbf{Q_t}\,diag\{\mathbf{Q_t}\}^{-1/2}, \tag{6}$$

where $\mathbf{Q_t}$ is the conditional standardized residual covariance matrix and for a restrictive case, is given by

$$\mathbf{Q_t} = (1-\theta_1-\theta_2)\ \overline{\mathbf{Q}} + \theta_1\ \boldsymbol{\varepsilon}_{\mathbf{t-1}}\boldsymbol{\varepsilon}'_{\mathbf{t-1}} + \theta_2\mathbf{Q}_{\mathbf{t}-1}, \tag{7}$$

where $\overline{\mathbf{Q}}$ is the unconditional covariance matrix of the standardized residuals and can be replaced by the sample covariance matrix $\overline{\mathbf{Q}} = \frac{1}{T}\sum_{i=1}^{T}\boldsymbol{\varepsilon}_{\mathbf{t-i}}\boldsymbol{\varepsilon}'_{\mathbf{t-i}}$ to simplify the estimation (Bauwens, Laurent, and Rombouts, 2006).

To incorporate regime shift into Engel's DCC model, Caporin and Billio (2005) introduce a Markov regime switching dynamic conditional correlation (MS-DCC) model with the conditional standardized residual covariance matrix specified as

$$\mathbf{Q_t} = (1-\theta_1(s_t)-\theta_2(s_t))\ \overline{\mathbf{Q}} + \theta_1(s_t)\ \boldsymbol{\varepsilon}_{\mathbf{t-1}}\boldsymbol{\varepsilon}'_{\mathbf{t-1}} + \theta_2(s_t)\ \mathbf{Q}_{\mathbf{t}-1}, \tag{8}$$

where $s_t = \{1, 2\}$ is the state variable following a first-order, two-state Markov process. Compared to equation (7), parameters driving the system dynamics are state-dependent. The regime dependent structure is restricted to the correlation excluding any effect on variance. As pointed out by Caporin and Billio (2005), a full Markov switching model is highly unstable given the huge number of switching parameters.

Given the joint presence of regime switching and time-varying correlation in each regime in equation (8), recombining procedure is required to solve the well-known path-dependency problem (Cai, 1994; Hamilton and Susmel, 1994; Gray, 1996; and Lee et al., 2007a and 2007b). Analog to Kim's filter (1994), Caporin and Billio (2005) propose a modified Hamilton filter for estimating MS-DCC. In their proposed filtering algorithm, the conditional standardized residual covariance matrix $\mathbf{Q}^i$ evolves according to the following dynamic

$$\mathbf{Q}_t^{i,j} = \left(1-\theta_{1,j}-\theta_{2,j}\right)\ \overline{\mathbf{Q}} + \theta_{1,j}\boldsymbol{\varepsilon}_{\mathbf{t-1}}\boldsymbol{\varepsilon}'_{\mathbf{t-1}} + \theta_{2,j}\mathbf{Q}_{t-1}^{i}, \tag{9}$$

given S possible values of $\mathbf{Q}$ at time $t-1$, there will be S^2 possible values for $\mathbf{Q}$ at time t, $i, j \in \{1,2,\cdots,S\}$, S is the number of states. The recursive nature of the regime switching process produces an S-fold increase in the number of cases to consider in each iteration of the filter and make the model intractable. To make the evolution of the process tractable, the correlation matrixes are collapsed based on the following conditional expectations:

$$\mathbf{Q}_t^{j} = \frac{\sum_{i=1}^{S} P\left(s_t = j, s_{t-1} = i \mid \psi_t\right)\ \mathbf{Q}_t^{i,j}}{P\left(s_t = j \mid \psi_t\right)}, \tag{10}$$

where $P\left(s_t = j, s_{t-1} = i \mid \psi_t\right)$ is the conditional regime probability of being in state i at time $t-1$ and in state j at time t.

Although this recombining method solves the problem of estimation difficulties, it creates computational burden and its analytical intractability is a serious drawback. Consider the following correlation dynamic

$$\mathbf{Q}_{\mathbf{t}} = \left(1-\theta_1-\theta_2\right)\ \overline{\mathbf{Q}} + \theta_1\mathbf{E}_{\mathbf{t-1}} + \theta_2\mathbf{Q}_{\mathbf{t}-1}, \tag{11}$$

where $\mathbf{E}_{\mathbf{t-1}} = \boldsymbol{\varepsilon}_{\mathbf{t-1}}\boldsymbol{\varepsilon}'_{\mathbf{t-1}}$. If $\theta_2 < 1$, $\mathbf{Q}_{\mathbf{t}}$ can be expressed as

$$\mathbf{Q}_{\mathbf{t}} = \left(1-\theta_1-\theta_2\right)\ \left(1-\theta_2\right)^{-1}\overline{\mathbf{Q}} + \theta_1\sum_{i=1}^{\infty}\theta_2^{i-1}\mathbf{E}_{\mathbf{t}-i}, \tag{12}$$

where θ_1 reflects the magnitude of a unit shock's immediate impact on the next period's $\mathbf{Q}$, θ_2 is a parameter of inertia and indicates the memory in $\mathbf{Q}$, and the total impact of a unit shock to future $\mathbf{Q}$ is $\theta_1\left(1-\theta_2\right)^{-1}$. In the regime switching GARCH model, the relationship between the pattern with which $\mathbf{Q}$ responds to shocks and the parameters θ_1

and θ_2 is far from obvious if recombining method is used because the lagged $\mathbf{Q}$ is replaced with the recombined variances. Moreover, it is possible that the covariance of one regime will still be affected by shocks even if θ_1 in that regime is zero. A more detail discussion of this problem is given in the appendix A.

Analog to the independent switching idea proposed by Hass et al. (2004) that is aimed to solve the problem of univaraite path-dependency problem in the variance process, an independent switching covariance process is suggested below:

$$\mathbf{Q}_t(s_t) = (1-\theta_1(s_t)-\theta_2(s_t))\ \overline{\mathbf{Q}} + \theta_1(s_t)\ \boldsymbol{\varepsilon}_{\mathbf{t-1}}\boldsymbol{\varepsilon}'_{\mathbf{t-1}} + \theta_2(s_t)\mathbf{Q}_{t-1}(s_t), \tag{13}$$

and the corresponded correlation dynamic is

$$\boldsymbol{\Gamma}_t(s_t) = diag\{\mathbf{Q}_t(s_t)\}^{-1/2}\mathbf{Q}_t(s_t) diag\{\mathbf{Q}_t(s_t)\}^{-1/2}. \tag{14}$$

Compared to equation (8), this specification allows the covariance process in each regime to evolving independently and avoid the path-dependency problem. Furthermore, because we have S covariance process to evolve in parallel according to different set of parameters, the specification preserves the economic significant of the covariance dynamics in each regime and we refer this model as the independent switching dynamic conditional correlation GARCH (IS-DCC). Equations (1)-(5) and (13)-(14) constitute the specification of the IS-DCC model and the $i-state$ IS-DCC model is denoted as $IS-DCC(i)$ in this article. Under this notation, $IS-DCC(1)$ will be the state-independent DCC GARCH proposed by Engle.

III. Estimation and Hamilton Filter for the IS-DCC model

The estimation of parameters is performed with maximum likelihood approach. To maximize the likelihood one has to evaluate

$$L(\Theta)=\sum_{t=1}^{T}\log f(\mathbf{R_t}\mid\psi_{t-1}), \tag{15}$$

where Θ is the vector of unknown parameters to be estimated, T is the total number of observations, and $f(\mathbf{R_t}\mid\psi_{t-1})$ is the mixture distribution weighted by regime probability. To do this we have to use Hamilton filter (Hamilton, 1989, 1994) to evaluate the regime probability because the state variable is unobserved. The Hamilton filtering procedure for the IS-DCC is depicted below:

(i) Given the filtered probabilities $\hat{\boldsymbol{\xi}}_{\mathbf{t-1|t-1}}$ projects the state probabilities

$$\hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}}=\mathbf{P}\hat{\boldsymbol{\xi}}_{\mathbf{t-1|t-1}}\ , \tag{16}$$

where

$$\hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}}=\begin{bmatrix} p(s_t=1\mid\psi_{t-1})\\ p(s_t=2\mid\psi_{t-1})\\ \vdots\\ p(s_t=S\mid\psi_{t-1})\end{bmatrix},\quad \hat{\boldsymbol{\xi}}_{\mathbf{t-1|t-1}}=\begin{bmatrix} p(s_{t-1}=1\mid\psi_{t-1})\\ p(s_{t-1}=2\mid\psi_{t-1})\\ \vdots\\ p(s_{t-1}=S\mid\psi_{t-1})\end{bmatrix}, \tag{17}$$

and $\mathbf{P}$ is the transition probability matrix with the (i,j) element $p(s_t=j\mid s_{t-1}=i)$ defined as

$$p(s_t=j\mid s_{t-1}=i)=\frac{\exp(\tau_{i,j})}{1+\exp(\tau_{i,1})+\exp(\tau_{i,2})+\cdots+\exp(\tau_{i,S-1})},\quad j=1,2,\cdots,S-1$$

$$=\frac{1}{1+\exp(\tau_{i,1})+\exp(\tau_{i,2})+\cdots+\exp(\tau_{i,S-1})},\quad j=S \tag{18}$$

where $\tau's$ are unrestricted parameters to be estimated.

(ii) Evaluate the regime dependent likelihood

$$\mathbf{Q}_t(i)=(1-\theta_1(i)-\theta_2(i))\ \overline{\mathbf{Q}}+\theta_1(i)\ \boldsymbol{\varepsilon}_{\mathbf{t-1}}\boldsymbol{\varepsilon}'_{\mathbf{t-1}}+\theta_2(i)\mathbf{Q}_{t-1}(i), \tag{19}$$

$$\boldsymbol{\Gamma}_t(i)=diag\{\mathbf{Q}_t(i)\}^{-1/2}\mathbf{Q}_t(i)diag\{\mathbf{Q}_t(i)\}^{-1/2},\ i=1,2,\cdots,S, \tag{20}$$

$$\mathbf{H_t}(i) = \mathbf{D_t}\mathbf{\Gamma_t}(i)\mathbf{D_t}\,, \tag{21}$$

$$f\left(\mathbf{R_t} \mid s_t = i, \psi_{t-1}\right) = \frac{1}{(2\pi)^{m/2}\left|\mathbf{H}_t(i)\right|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{R_t} - \boldsymbol{\mu})' \mathbf{H_t^{-1}}(i)(\mathbf{R_t} - \boldsymbol{\mu})\right\}, \tag{22}$$

where m is the number of dimension and is equal to two for our hedging application. Define

$$\boldsymbol{\eta}_\mathbf{t} = \begin{bmatrix} f\left(\mathbf{R_t} \mid s_t = 1\right) \\ f\left(\mathbf{R_t} \mid s_t = 2\right) \\ \vdots \\ f\left(\mathbf{R_t} \mid s_t = S\right) \end{bmatrix}, \tag{23}$$

the density of $\mathbf{R_t}$ conditional on past observations and being in regime $i = 1,2,\cdots,S$ at time t.

(iii) Evaluate the mixture likelihood

$$f\left(\mathbf{R_t} \mid \psi_{t-1}\right) = \mathbf{1}'\left(\hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}} \circ \boldsymbol{\eta}_\mathbf{t}\right), \tag{24}$$

where $\mathbf{1}$ is an $m \times 1$ vector of ones and $\circ$ denotes elements-by-elements multiplication.

(iv) Update the joint probabilities

The state-probability is updated with the following equation

$$\hat{\boldsymbol{\xi}}_{\mathbf{t|t}} = \frac{\hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}} \circ \boldsymbol{\eta}_\mathbf{t}}{\mathbf{1}'\left(\hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}} \circ \boldsymbol{\eta}_\mathbf{t}\right)} \tag{25}$$

(v) Iterate (i) to (iv) until the end of the sample and the likelihood is obtained as a by-product of this filter

$$L(\Theta) = \sum_{t=1}^{T} \log\left(\mathbf{1}'\left(\hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}} \circ \boldsymbol{\eta}_\mathbf{t}\right)\right) \tag{26}$$

Different from Billio and Caporin's filter, the step of approximation for the covariance matrix is not required in this filtering algorithm since IS-DCC is path-independent. To initialize the filter, the regime probabilities are set equal to the unconditional probabilities. Define the steady state probabilities vector as

$$\boldsymbol{\pi} = \left[p\left(s_t = 1\right) \quad p\left(s_t = 2\right) \quad \cdots \quad p\left(s_t = S\right) \right]^T . \tag{27}$$

These probabilities are the solution of the system of equations $\mathbf{P}\boldsymbol{\pi} = \boldsymbol{\pi}$ and $\mathbf{1}'\boldsymbol{\pi} = \mathbf{1}$, which can be shown as $\boldsymbol{\pi} = \left(\mathbf{A}'\mathbf{A}\right)^{-1}\mathbf{A}'\mathbf{v}_{\mathbf{S+1}}$, where $\mathbf{A} = \begin{bmatrix} \mathbf{I_S} - \mathbf{P} \\ \mathbf{1}' \end{bmatrix}$, and

$\mathbf{v}_{\mathbf{S+1}} = \begin{bmatrix} \mathbf{0_S} \\ \mathbf{1} \end{bmatrix}$, $\mathbf{I_S}$ is an $S \times S$ identity matrix and $\mathbf{0_S}$ is an $S \times 1$ zero vector.

IV. State-dependent MVHR and Measuring Hedging Performance

It is well known that the estimated time-varying minimum variance hedge ratio denoted as χ_t for state-independent hedging is given by

$$\chi_t = \frac{Cov\left(r_{c,t}, r_{f,t} \mid \psi_{t-1}\right)}{Var\left(r_{f,t} \mid \psi_{t-1}\right)} . \tag{28}$$

Lee (2009b) derives a formula for two-state regime switching hedge ratio which is given by

$$\chi_t = \frac{p_{1,t}^2 Cov\left(r_{c,1,t}, r_{f,1,t} \mid \psi_{t-1}\right) + \left(1 - p_{1,t}\right)^2 Cov\left(r_{c,2,t}, r_{f,2,t} \mid \psi_{t-1}\right)}{p_{1,t}^2 Var\left(r_{f,1,t} \mid \psi_{t-1}\right) + \left(1 - p_{1,t}\right)^2 Var\left(r_{f,2,t} \mid \psi_{t-1}\right)}, \tag{29}$$

where $p_{1,t}$ is the regime probability of being in state one at time t. Because the focus of this article has been investigating the effects of multi-regime switching in correlation on futures hedging, a formula for multi-state regime switching hedge ratio is required. The $S-state$ regime switching hedge ratio can be generalized as [1]

$$\chi_t = \frac{\sum_{i=1}^{S} p_{i,t}^2 Cov\left(r_{c,i,t}, r_{f,i,t} \mid \psi_{t-1}\right)}{\sum_{i=1}^{S} p_{i,t}^2 Var\left(r_{f,i,t} \mid \psi_{t-1}\right)}, \tag{30}$$

where $p_{i,t}$, $i \in \{1,2,\cdots,S\}$ are the state probabilities of being in state i and $Cov\left(r_{c,i,t}, r_{f,i,t} \mid \psi_{t-1}\right)$ and $Var\left(r_{f,i,t} \mid \psi_{t-1}\right)$ are respectively the conditional covariance of spot and futures returns and conditional variance of futures returns in state *i*. Notice that, when there is no regime shifts, $S=1$ and equation (30) collapses to the conventional state-independent hedge ratio given in equation (28).

Hedging performance is evaluated from both a risk-minimization and a utility standpoint. From a risk-minimization standpoint, a hedger chooses a hedging strategy to minimize the variance of the hedged portfolio return or equivalently to maximize the variance reduction of a hedging strategy compared to the unhedged position. The variance of the hedged portfolio return is

$$Var\left(r_{c,t} - \chi_t r_{f,t}\right), \tag{31}$$

where χ_t is defined in equation (30) and estimated from the proposed $IS-DCC$ model.

---

1 This can be proved as follows. Let r_p be the hedging portfolio return which is given by

$$\begin{aligned} r_p &= \begin{bmatrix} p_1 & p_2 & \cdots & p_S \end{bmatrix} \begin{bmatrix} portfolio\ return\ in\ state\ 1 \\ portfolio\ return\ in\ state\ 2 \\ \vdots \\ portfolio\ return\ in\ state\ S \end{bmatrix} \\ &= p_1\left[r_c(s_t=1) - \chi\, r_f(s_t=1)\right] + \cdots + p_S\left[r_c(s_t=S) - \chi\, r_f(s_t=S)\right], \end{aligned}$$

Deriving the variance of this state-dependent hedging portfolio return r_p with respect to χ and using the assumption of independent switching gives equation (30).

Because dynamic hedging strategies are potentially more costly implement than static models since frequent rebalancing of the hedged portfolio is required, hedging effectiveness is more appropriately assessed by considering the economic benefits measured with utility functions. Consider a hedger with a mean-variance expected utility function (Kroner and Sultan, 1993; Gagnon et al., 1998; Lafuente and Novales, 2003; Alizadeh and Nomikos, 2004; and Lee et al., 2006):

$$E\left[U\left(r_{p,t}\right) | \psi_{t-1}\right] = E\left[r_{p,t} | \psi_{t-1}\right] - \kappa \, Var\left(r_{p,t} | \psi_{t-1}\right), \tag{32}$$

where κ is the coefficient of absolute risk aversion, E stands for expectation operator and $r_{p,t}$ is the return from the hedged portfolio. A dynamic hedging strategy is considered to be superior to a static ordinary least square (OLS) method if it has higher expected utility net of transaction costs.

In addition to measuring the economic significance of dynamic hedging strategies with utility function, it is also interesting to test if the best $IS - DCC$ model statistically significantly outperforms OLS. According to Sullivan et al. (1999) and White (2000), data snooping occurs when a given set of data is used more than once for purposes of inference or model selection. To avoid data snooping problem, White's reality check (Sullivan et al., 1999 and White, 2000) is also performed to test the hypothesis that the best performing $IS - DCC$ model has no predictive superiority over the benchmark, static OLS model. White's reality check is based on the following $l \times 1$ performance statistic:

$$\bar{\mathbf{f}} = N^{-1} \sum_{t=R+1}^{T} \widehat{\mathbf{f}}_{\mathbf{t+1}} , \tag{33}$$

where l is the number of alternative models considered and $\widehat{\mathbf{f}}_{\mathbf{t+1}}$ is the observed performance measure for period $t+1$. The k^{th} element of $\hat{\mathbf{f}}_{t+1}$ is defined as:

$$\hat{f}_{k,t+1} = -\left(r_{c,t} - \hat{\chi}_{Best\ IS-DCC,t}\ r_{f,t}\right)^2 + \left(r_{c,t} - \hat{\chi}_{OLS}\ r_{f,t}\right)^2, \tag{34}$$

where $\hat{\chi}_{Best\ IS-DCC,t}$ and $\hat{\chi}_{OLS}$ are the estimates of hedge ratios from the best $IS-DCC$ model and static OLS, respectively.

The null hypothesis that the best performing $IS-DCC$ has no predictive superiority over the static OLS is given by

$$H_0: \quad \max_{k=1,2,\cdots,l} \left[E\left(f_k^*\right)\right] \le 0, \tag{35}$$

where f_k^* is the true performance value for each model applied to the data.[2]

Because $IS-DCC(i)$, $i \in \{1,2,\cdots,S\}$ are nested models, to investigate if $IS-DCC(i)$ outperforms $IS-DCC(i-1)$, $i \in \{1,2,\cdots,S\}$, the Diebold-Mariano (1995) and West (1996) (DMW) test is performed. To construct the DMW statistic, let $\hat{d}_t = f\left(v_{i,t+1}\right) - f\left(v_{i+1,t+1}\right)$, $i \in \{1,2,\cdots,S\}$, and $\bar{d} = N^{-1}\sum_{t=R+1}^{T}\hat{d}_{t+1}$, then the DMW test statistic is computed as follows,

$$DMW = \frac{\bar{d}}{\sqrt{N^{-1}\,\hat{V}}}, \tag{36}$$

where $\hat{V} = N^{-1}\sum_{t=R+1}^{T}\left(\hat{d}_{t+1} - \bar{d}\right)^2$, R denotes the length of estimation period, N is the length of the prediction period, T is the sample size, f is the square error loss function, and

[2] Politis and Romano's (1994) stationary bootstrap resampling method is used for implementing the White's reality check with 1000 bootstrap simulations and a smoothing parameters of q=0.5 (Lee et al., 2006; Lee and Yoder, 2007a).

$v_{i,t} = r_{c,t} - \hat{\chi}_{i,t}\ r_{f,t}$ and $v_{i+1,t} = r_{c,t} - \hat{\chi}_{i+1,t}\ r_{f,t}$ with $\hat{\chi}_{i,t}$ the hedge ratios estimated from the $IS - DCC(i)$ model.

The critical values of DMW test for nested models have to be adjusted to produce correct tests (McCracken, 2007). McCracken's critical values depend on the N/R ratio and the number of additional estimated parameters in the unrestricted model. The test is one-sided with the null hypothesis that the predictive ability of an unrestricted model is not superior to its nested model which is given by

$$H_0 = E\left[f\left(v_{i,t+1}\right) - f\left(v_{i+1,t+1}\right)\right] \leq 0\,, \tag{37}$$

while the alternative is

$$H_A = E\left[f\left(v_{i,t+1}\right) - f\left(v_{i+1,t+1}\right)\right] > 0\,. \tag{38}$$

Rejection of the null hypothesis implies that the predictive ability of an unrestricted model is superior to its nested model. Although the $MS - DCC$ model is not of interest in this study, its hedging performance is also compared with $IS - DCC$. Since $MS - DCC$ is not nested within the $IS - DCC$ model, regular critical values for DMW statistics are applied.

V. Data Description and Empirical Results

The proposed $IS - DCC$ is applied to nearby futures contracts of wheat and corn traded in the Chicago Board of Trade (CBOT), cocoa and coffee traded in the New York Board of Trade (NYBOT), and crude oil, natural gas, heating oil, and platinum traded in the New York Mercantile Exchange (NYMEX) for the period January 1991 to December 2008. Spot and Futures prices are Wednesday prices obtained from Datastream and the Energy Information Administration (US Department of Energy). Tuesday's closing price is used when a holiday occurs on Wednesday. The returns of each price series are computed as the changes in the natural logarithms of prices multiplied by 100. Estimation of all models was conducted using data from January 1991 to December 2007; the remaining data are used for out-of-sample analysis. The sub-period hedging effectiveness is also investigated in this study. The sample is further split into two periods: pre-2000 (from January 1991 to December 1999) and post-2000 (from January 2000 to December 2008). The last year data in each sub-period are used for out-of-sample analysis.

Table I provides summary statistics of the returns series for each commodity over the full sample period and two sub-sample periods. For the full sample period, all returns are positive and small. The largest mean returns are 0.114% and 0.09% for spot and futures data, respectively and the smallest mean returns are 0.001% for both spot and futures data, respectively. The unconditional volatilities indicate that in general, the post-2000 period is more volatile than pre-2000 period. According to the Skewness, leptokurtosis, and significant Jarque-Bera statistics, the unconditional distributions of spot and futures returns for all commodities are asymmetric, fat-tailed, and non-Gaussian.

Parameter estimates from alternative models are presented in table II. The parameters are estimated by maximizing the log-likelihood functions in equation (15) using numerical constrained optimization procedure in GAUSS. Shown in the last row of table II, LRT reports the statistics of likelihood ratio test of $IS-DCC(i)$ and its nested model $IS-DCC(i-1)$. The number of state i is increased until that the $IS-DCC(i)$ does not show significant increase in likelihood value compared to $IS-DCC(i-1)$ and the critical values at 1% for $i=2,3,4$ and 5 are 13.28, 16.81, 20.09 and 23.21, respectively. The number of parameters in $IS-DCC(i)$ is equal to $8+2i+i(i-1)$.

Namely, the number of parameters to be estimated for DCC, $IS-DCC(2)$ to $IS-DCC(5)$ are 10, 14, 20, 28, and 38, respectively.[3] The LRT of crude oil is still significant when the number of states is increased to five. However, we do not proceed to $IS-DCC(6)$ since there are fifty parameters to be estimated ($8+2\times 6+6(6-1)=50$) and empirically, increasing the number of states to five no longer create significant gains compared to four states.

As shown in table II, all conditional mean $\mu's$ estimated are small which is consistent with the small average return reported in the summary statistics table. For the volatility equation, heating oil data has the largest volatility persistence and wheat data has the smallest volatility persistence among all commodities investigated in this article. Taking DCC model for instance, heating oil data has the largest $\alpha+\beta$ which is equal to 1 and 0.993 for spot and futures returns, respectively and wheat data has the smallest $\alpha+\beta$ which is equal to 0.767 and 0.435 for spot and futures returns, respectively.

In the correlation equation, θ_2 reflects the memory in correlation. In state-independent DCC, coffee and corn have the largest and smallest memory in correlation with θ_2 equal to 0.924 and 0.113, respectively. For the state-dependent DCC models, the memory is not a constant but regime-dependent. For example, θ_2 in the $IS-DCC(5)$ for corn is decomposed into five possible memory strengths, 0.002, 0.013, 0.379, 0.4886 and 0.728 in five different regimes. Most of the parameters in the correlation equation are significant implying the importance of modeling the regime-switching time-varying correlation of spot and futures returns. The total impact of a unit shock to future

3 To save space, estimation results of MS-DCC and parameters $\tau's$ for the transition probabilities are not reported here but are available from the authors upon request.

correlation is $\theta_1(1-\theta_2)^{-1}$. For the state-dependent DCC models, the total impact of a unit shock of natural gas and corn have the largest and smallest shock to the future correlations with $\theta_1(1-\theta_2)^{-1}$ equal to 0.94 and 0.206, respectively. The total impact in the regime-independent model is somewhere in between the largest impact and the smallest impact in the regime-dependent model. Taking natural gas for instance, the state-dependent impact strengths are 0, 0.169, 0.958, and 0.997 and the regime-independent impact 0.94 ($= 0.068(1-0.928)^{-1}$) is somewhere in between the largest impact 0.997 and the smallest impact 0.

Table III reports the out-of-sample hedging effectiveness of alternative hedging strategies. Out-of-sample hedging effectiveness is considered because for the hedger, what matters most is the hedging performance in the future not in the past. It is found that in general $IS-DCC(2)$ outperforms DCC. The only exception is coffee. The percentage variance reduction of $IS-DCC(2)$ is 64.13% which is lower than that of DCC with a 64.31% variance reduction.[4] This is consistent with most findings in the previous regime switching hedging studies that allowing the hedge ratio to be state-dependent increases the hedging effectiveness. This article investigates if allowing the number of regime to be more than two can further improve the hedging effectiveness. Empirical results reveal that when the number of regimes is increased from two to three, $IS-DCC(3)$ outperforms $IS-DCC(2)$ for all commodities considered in this study. Compared to $IS-DCC(2)$, the largest and smallest improvements of $IS-DCC(3)$ are 2.55% and 0.03% for cocoa and natural gas, respectively. The results, however, are not

[4] Percentage variance reductions are calculated as the differences of variance of unhedged position and estimated variance of alterative models over variance of unhedged position multiplied by 100.

promising when the number of states is increased from three to four. Only $IS-DCC(4)$ of corn creates a 0.26% significantly improvement compared to $IS-DCC(3)$. It is also found that increasing the number of states from four to five as appeared in the corn, crude oil and heating oil data does not always provide further hedging benefit. Only $IS-DCC(5)$ for crude oil provides a 0.02% improvement compared to $IS-DCC(4)$ and $IS-DCC(5)$ is inferior to $IS-DCC(3)$ with a 0.07% less in variance reduction. Generally speaking, allowing the correlation to be subject to regime switching improves the hedging effectiveness compared to a model with state-independent correlation and three-state correlation hedging exhibits superior performance when full sample data is investigated.

Due to the frequently rebalancing requirement of dynamic hedging strategies, they are more costly than static OLS hedging. Following other empirical studies (Lafuente and Novales, 2003; Alizadeh and Nomikos, 2004; and Lee et al., 2006), the economics value of these dynamic hedging methods are also investigated by comparing the utility improvements of these methods relative to static OLS hedging. The hedger is assumed to have an expected utility function given by equation (32) with the coefficient of absolute risk aversion κ equal to 4. As shown in table III, taking wheat data for example, the average weekly variance of the returns from hedged portfolio for OLS and IS-DCC3 hedging are 11.753 and 8.039, respectively. Although not reported here, the hedged portfolio returns of OLS and $IS-DCC(3)$ hedging are -0.166% and 0.278%, respectively. Based on equation (32), if an investor adopts OLS hedging, the average weekly utility is $U_{OLS} = \text{-0.166\%} - 4(11.753) \approx -47.18$. With $IS-DCC(3)$, the average weekly utility is

$U_{IS-DCC3} = 0.278\% - 4(8.039) \approx -31.88$. The hedger's net benefit from using $IS-DCC(3)$ hedging over OLS hedging is equal to $U_{IS-DCC3} - U_{OLS} - C$ $= 15.3 - C$, where C stands for the transaction cost from dynamic rebalancing. This implies that if $C < 15.3$, the $IS-DCC(3)$ hedging is preferred to OLS hedging. Since the typical round trip transaction cost is around 0.02% to 0.04%, a mean-variance expected utility-maximizing hedger will benefit from hedging with $IS-DCC(3)$ even after taking account of these transaction costs. It is found that DCC does not create utility gain for cocoa, heating oil and natural gas data and all state-dependent $IS-DCC$ hedging generate utility gains compared to OLS hedging. To test the statistical significance of the hedging effectiveness of the best $IS-DCC$ over the benchmark, static OLS hedging, White's reality check is performed. As reported in table III, based on White's reality check p-values, the no improvement null hypothesis of the best $IS-DCC$ over OLS is rejected at least at 10% significant level for most of the commodities. Exceptions are cocoa and natural gas data with reality check p-values equal to 0.423 and 0.267, respectively.

Because $IS-DCC(i)$, $i \in \{1,2,\cdots,S\}$ are nested models, to investigate if $IS-DCC(i)$ significantly outperforms $IS-DCC(i-1)$, $i \in \{1,2,\cdots,S\}$, the Diebold-Mariano (1995) and West (1996) (DMW) test is performed with adjusted critical values reported by McCracken (2007). McCracken's critical values depend on the N/R ratio and the number of additional estimated parameters in the unrestricted model. When the full data sample is applied, the N/R ratio is equal to 0.06 and the number of additional estimated parameters for $IS-DCC(i)$, i=2,3,4 and 5 are four, six, eight, and ten, respectively. The critical values are tabulated for $N/R = 0$ and 0.1, and we construct the values for $N/R = 0.06$ by interpolation. As reported in table IV, It's found that $IS-DCC(2)$ is superior to DCC at 10% significant level for corn, crude oil and heating oil but not the rest of the commodities. $IS-DCC(3)$ is superior to $IS-DCC(2)$ at 5% level significant level for cocoa and coffee and at 10% level for wheat and crude oil. Although $IS-DCC(3)$ does not provide significant improvement over $IS-DCC(2)$ for corn, natural gas, heating oil and platinum, all DMW statistics are positive implying that $IS-DCC(3)$ is not inferior to $IS-DCC(2)$ and has a tendency to be superior to $IS-DCC(2)$. When $IS-DCC(3)$ is compared with DCC, again, all DMW statistics are positive and $IS-DCC(3)$ is superior to DCC at 5% level significant level for wheat and crude oil and at 10% level for corn, cocoa, coffee and heating oil. When the number of states is increased from three to four, $IS-DCC(4)$ significantly outperforms $IS-DCC(3)$ only for corn. The DMW statistics for coffee and natural gas are negative and significant at 10% level, implying that $IS-DCC(3)$ outperforms $IS-DCC(4)$ for these two commodities. The performances are not significantly different for crude oil, heating oil and platinum. When the number of states is further increased from four to five, all DMW statistics are not significant indicating that the performance of $IS-DCC(5)$ is statistically indifferent to $IS-DCC(4)$. Overall, $IS-DCC(3)$ exhibits superior performance when full sample data is investigated.

Although the $MS-DCC$ model is not of interest in this paper due to its requirement of recombining procedure and the problem of analytical intractability, a comparison of the proposed $IS-DCC$ and $MS-DCC$ is also performed and reported in table V. It's found that $IS-DCC(2)$ is superior to $MS-DCC$ at 10% significant level for wheat and at 5% level for natural gas and the performance of $IS-DCC(2)$ is not significantly different from $MS-DCC$ for the rest of the commodities. As for $IS-DCC(3)$, it is superior to $MS-DCC$ at 5% significant level for wheat and natural gas. All DMW statistics are positive implying that $IS-DCC(3)$ is not inferior to and has a tendency to be superior to $MS-DCC$.

To check the consistency of the performance of $IS-DCC$ over different hedging periods, the data is further split into two sub-samples. In the first sub-sample (pre-2000), in- and out-of-sample periods are from January 1991 to December 1998 and from January 1999 to December 1999, respectively, and in the second sub-sample (post-2000), in- and out-of-sample periods are from January 2000 to December 2007 and from January 2008 to December 2008, respectively. Table VI and VII present the hedging performances of $IS-DCC$ over post-2000 and pre-2000 sub-periods, respectively. It is found that, most state-dependent $IS-DCC$ hedging outperform OLS in terms of percentage variance reduction and generate utility gains compared to OLS hedging. OLS occasionally outperforms all dynamic hedging methods. OLS has the best performance for coffee in the post-2000 period and for wheat and heating oil in the pre-2000 period.[5] For the post-2000 period, White's reality check p-values show that the no improvement null hypothesis of best $IS-DCC$ over OLS is rejected at the 10% significant level for corn, at the 5% significant level for platinum and at the 1% significant level for crude oil and natural gas. As for the pre-2000 period, the no improvement null hypothesis of best $IS-DCC$ over OLS is rejected only for corn at the 5% significant level.

[5] This is consistent with some previous findings that more elaborate dynamic hedging method might not improve the hedging effectiveness compared to the static hedging method (Byström, 2003; Lee et al., 2006; and Lee and Yoder, 2007a).

For the nested $IS-DCC(i)$, $i \in \{1,2,\cdots,S\}$, in the post-2000 period, $IS-DCC(2)$ outperforms other models for wheat, corn, cocoa, coffee and natural gas. $IS-DCC(3)$ has the best performance for heating oil and platinum and $IS-DCC(4)$ has the best performance for crude oil. As for the pre-2000 period, $IS-DCC(3)$ has the best performance for wheat, coffee, heating oil and platinum and $IS-DCC(4)$ has the best performance for corn and crude oil. $IS-DCC(2)$ outperforms other models only for natural gas and the state-independent DCC has the best performance for cocoa data. Overall, two-state $IS-DCC$ has better performance for majority of the commodities in the post-2000 period and more than two-state $IS-DCC$ has better performance for majority of the commodities in the pre-2000 period.

The Diebold-Mariano and West (DMW) test statistics for the sub-periods are reported in table VIII. The N/R ratio for the McCracken's critical values is equal to 0.125 for each sub-period and the number of additional estimated parameters for $IS-DCC(i)$, i=2,3,4 and 5 are four, six, eight, and ten, respectively. The critical values are tabulated for $N/R = 0.1$ and 0.2, and we construct the values for $N/R = 0.125$ by interpolation. In the post-2000 sub-period, $IS-DCC(2)$ is superior to DCC at the 5% significant level for wheat, crude oil, heating oil and platinum. $IS-DCC(3)$ provides further significant improvement over $IS-DCC(2)$ for heating oil and $IS-DCC(4)$ provides further significant improvement over $IS-DCC(3)$ for crude oil. As for the pre-2000 sub-period, $IS-DCC(2)$ is superior to DCC at the 10% significant level for platinum and at the 1% level for wheat and heating oil. $IS-DCC(3)$ provides further significant improvement over $IS-DCC(2)$ for heating oil and $IS-DCC(4)$ provides further significant improvement over $IS-DCC(3)$ for corn and crude oil. Overall, most of the DMW statistics of the best $IS-DCC$ are positive compared to state-independent DCC in both sub-periods reveals that allowing the correlation to be subject to regime shifts has a tendency to improve the hedging performances. These statistics are significant for wheat, crude oil, heating oil, and platinum in the post-2000 period and for wheat and heating oil in the pre-2000 period.

Figures 1 shows the hedge ratios estimated by using OLS, DCC, and $IS-DCC(3)$ for wheat.[6] The conditional hedge ratios are very volatile revealing that adjustment of the hedge portfolio using dynamic hedging strategies is highly required. Figure 2 shows the state-dependent time-varying correlations in each regime. The maximum number of states for wheat is three when full sample is considered. $IS-DCC(3)$ decomposes correlations into three different regimes with different volatilities in correlations. The volatilities of correlations are equal to 0.006, 0.162 and 0.301 in state three, one and two, respectively. State three is the regime that spot and futures return series have a nearly constant correlation. State two is the state with a highest volatility of correlation and the volatility of correlation in state one is somewhere in between. The regime probabilities of being in each regime are plotted in figures 3 to 5. As for the corn data, the hedge ratios estimated by using OLS, DCC, and $IS-DCC(5)$ are plotted in figure 6 and the state-dependent time-varying correlations in each regime are plotted in figure 7 and 8. $IS-DCC(5)$ decomposes correlations into five different states with different volatilities in correlations. The volatilities of correlations are equal to

[6] To save space, only those figures for three-state case of wheat and five-state case of corn are illustrated here and to make the correlation figures more clearly, only post-2000 period is plotted for the five-state case of corn.

0, 0.038, 0.175, 0.35 and 0.518 in state two, one, three, five and four, respectively. State two is the regime that spot and futures return series have a nearly constant correlation. State one and three have relatively smaller volatilities in correlations. Instead, as illustrated in figure 8, state four and five have relatively larger volatilities in correlations and volatility in correlation is larger in state four than in state five. The regime probabilities of being in each regime for the five-state case of corn are plotted in figures 9 to 13.

VI. CONCLUSIONS

The focus of this article has been investigating the effects of multi-regime switching in correlation on futures hedging via an independent switching dynamic conditional correlation GARCH ($IS-DCC$) model. $IS-DCC$ avoids the path-dependency and recombining problems inherent in the $MS-DCC$ which possess problems of computational intensive and analytical intractability. To author's knowledge, no existing paper investigates multi-regime correlation futures hedging. This might be the fact that previous regime switching hedging models allow a fully model switching and the potential overparameter and convergence problems limit the discussion of the possible number of state to two.

Empirical results from commodity futures hedging exercise show that $IS-DCC$ outperforms state-independent DCC and three-state $IS-DCC$ exhibits superior hedging effectiveness when full sample period is investigated. Sub-sample periods hedging results show that two-state $IS-DCC$ has better performance for majority of the commodities in the post-2000 period and $IS-DCC$ with more than two states have better performances for majority of the commodities in the pre-2000 period. Overall, the contribution of this paper is twofold. The proposed $IS-DCC$ provides a general framework for modeling multi-state regime switching time-varying correlation and results of hedging exercises illustrate the importance of modeling this feature for optimal dynamic futures hedging.

Table I
Summary Statistics for Spot and Futures Returns (In Percentage) of Full Sample and Two Sub-Sample Periods

	Sample Period: 1991-2008				Sample Period: 1991-1999				Sample Period: 2000-2008			
	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures
	WHEAT		CORN		WHEAT		CORN		WHEAT		CORN	
Mean	0.094	0.090	0.048	0.060	-0.006	-0.014	-0.053	-0.027	0.194	0.193	0.149	0.147
Maximum	20.955	22.706	15.979	13.799	14.864	16.954	15.978	13.799	20.955	22.706	15.979	13.490
Minimum	-18.275	-18.332	-18.232	-16.895	-12.609	-18.332	-14.833	-15.310	-18.275	-17.742	-18.232	-16.895
Std. Dev.	3.762	3.983	3.809	3.624	3.163	3.378	3.183	3.027	4.278	4.508	4.345	4.136
Skewness	0.252	0.271	-0.299	-0.053	-0.050	0.017	-0.201	0.115	0.332	0.337	-0.357	-0.146
Kurtosis	6.432	5.522	5.789	5.138	5.346	6.109	6.233	5.660	6.068	4.781	5.071	4.454
Jarque-Bera	470.75***	260.36***	318.40***	179.33***	107.79***	188.95***	207.41***	139.31***	193.01***	71.01***	93.96***	43.09***
	COCOA		COFFEE		COCOA		COFFEE		COCOA		COFFEE	
Mean	0.114	0.090	0.037	0.026	0.003	-0.065	0.073	0.066	0.225	0.244	0.000	-0.015
Maximum	33.841	20.191	31.038	39.309	33.841	19.133	31.038	39.309	19.129	20.191	17.494	18.906
Minimum	-20.371	-14.129	-23.245	-25.093	-20.371	-10.182	-20.918	-22.398	-18.539	-14.129	-23.245	-25.093
Std. Dev.	4.055	4.248	5.052	5.614	3.763	3.701	5.764	6.445	4.328	4.731	4.229	4.645
Skewness	0.803	0.256	0.392	0.530	2.238	0.584	0.619	0.722	-0.165	0.054	-0.249	-0.078
Kurtosis	12.066	4.445	7.458	7.941	23.248	5.342	7.103	7.742	5.520	3.810	5.879	5.216
Jarque-Bera	3316.67***	91.99***	801.80***	999.20***	8402.77***	133.83***	359.01***	480.27***	126.46***	13.09***	167.22***	96.61***
	CRUDE OIL		NATURAL GAS		CRUDE OIL		NATURAL GAS		CRUDE OIL		NATURAL GAS	
Mean	0.055	0.055	0.001	0.001	-0.001	0.000	0.035	0.043	0.111	0.111	0.186	0.182
Maximum	30.305	23.244	0.944	0.398	15.036	16.032	94.446	31.573	30.305	23.244	54.714	39.805
Minimum	-29.214	-37.288	-0.728	-0.372	-29.214	-37.288	-72.789	-37.165	-23.263	-23.591	-33.694	-27.852
Std. Dev.	4.970	4.747	0.105	0.081	4.581	4.420	11.806	7.820	5.335	5.056	9.462	8.362
Skewness	-0.342	-0.697	0.498	0.198	-0.446	-0.998	0.535	0.120	-0.281	-0.496	0.441	0.237
Kurtosis	6.483	8.345	16.538	5.324	6.923	13.630	21.515	5.924	6.022	5.027	6.533	4.983
Jarque-Bera	492.89***	1193.66***	6004.46***	181.07***	316.20***	2286.20***	4471.58***	111.90***	185.00***	99.75***	259.65***	81.40***
	HEATING OIL		PLATINUM		HEATING OIL		PLATINUM		HEATING OIL		PLATINUM	
Mean	0.060	0.069	0.083	0.087	-0.014	-0.009	0.017	0.009	0.133	0.147	0.149	0.165
Maximum	41.700	17.246	13.248	17.728	26.489	17.246	8.900	10.127	41.700	15.963	13.248	17.728
Minimum	-28.471	-26.680	-19.299	-16.074	-24.547	-26.680	-11.315	-8.681	-28.471	-21.360	-19.299	-16.074
Std. Dev.	5.277	4.697	2.893	3.012	4.642	4.156	2.214	2.299	5.846	5.183	3.442	3.586
Skewness	0.218	-0.333	-0.755	-0.170	0.148	-0.511	-0.265	0.039	0.232	-0.249	-0.861	-0.250
Kurtosis	9.975	5.153	7.884	7.872	8.385	7.283	5.731	5.155	9.921	3.947	6.964	6.996
Jarque-Bera	1910.81***	198.76***	1022.53***	933.28***	568.38***	378.90***	151.21***	90.89***	942.16***	22.43***	365.88***	317.64***

Note: *** indicates significance at the 1% level and returns are calculated as the differences in the logarithm of price multiplied by 100.

Table II.
Estimates of Unknown Parameters of Alternative Models
Data period is from January 1991 to December 2007

	WHEAT			CORN				
	DCC	IS-DCC(2)[1]	IS-DCC(3)	DCC	IS-DCC(2)	IS-DCC(3)	IS-DCC(4)	IS-DCC(5)
	Mean Equation			Mean Equation				
μ_c	0.087	0.088	0.131	0.301	0.266	0.366	0.527	0.534
	(0.090)[2]	(0.123)	(0.127)	(0.102)***	(0.098)***	(0.087)***	(0.032)***	(0.028)***
μ_f	0.102	0.063	0.118	0.330	0.188	0.305	0.513	0.516
	(0.109)	(0.090)	(0.124)	(0.106)***	(0.111)**	(0.090)***	(0.034)***	(0.034)***
	Volatility Equation			Volatility Equation				
γ_c	2.704	3.178	3.060	0.955	0.974	1.021	0.860	0.944
	(0.590)***	(4.005	(0.757)***	(0.195)***	(0.194)***	(0.207)***	(0.216)***	(0.123)***
γ_f	7.902	6.979	6.809	0.905	0.905	1.123	0.920	1.035
	(2.118)***	(11.287)	(2.423)***	(0.244)***	(0.237)***	(0.288)***	(0.292)***	(0.183)***
α_c	0.192	0.171	0.165	0.148	0.143	0.108	0.104	0.105
	(0.037)***	(0.043)***	(0.035)***	(0.022)***	(0.021)***	(0.014)***	(0.009)***	(0.006)***
α_f	0.164	0.143	0.138	0.094	0.110	0.099	0.100	0.100
	(0.041)***	(0.091)*	(0.040)***	(0.018)***	(0.019)***	(0.014)***	(0.009)***	(0.007)***
β_c	0.575	0.547	0.563	0.777	0.786	0.802	0.817	0.809
	(0.066)***	(0.388)*	(0.081)***	(0.029)***	(0.026)***	(0.026)***	(0.029)***	(0.016)***
β_f	0.271	0.347	0.364	0.828	0.819	0.801	0.817	0.807
	(0.161)**	(0.905)	(0.194)**	(0.032)***	(0.030)***	(0.033)***	(0.035)***	(0.020)***
	Correlation Equation			Correlation Equation				
$\theta_1(1)$	0.115	0.021	0.119	0.182	0.342	0.266	0.000	0.079
	(0.037)***	(0.115)	(0.063)**	(0.040)***	(0.074)***	(0.066)***	(0.018)	(0.054)*
$\theta_1(2)$		0.467	0.533		0.048	0.988	0.986	0.000
		(0.056)***	(0.058)***		(0.030)*	(0.006)***	(0.008)***	(0.006)
$\theta_1(3)$			0.009			0.039	0.291	0.272
			(0.014)			(0.024)**	(0.046)***	(0.048)***
$\theta_1(4)$							0.093	0.987
							(0.070)*	(0.001)***
$\theta_1(5)$								0.956
								(0.030)***
$\theta_2(1)$	0.638	0.299	0.881	0.113	0.658	0.734	0.285	0.488
	(0.117)***	(13.974)	(0.090)***	(0.061)**	(0.080)***	(0.075)***	(2.342)	(0.362)*
$\theta_2(2)$		0.533	0.466		0.175	0.007	0.014	0.379
		(0.061)***	(0.060)***		(0.124)*	(0.007)	(0.008)**	(2.621)
$\theta_2(3)$			0.238			0.360	0.709	0.728
			(2.686)			(0.219))*	(0.031)***	(0.056)***
$\theta_2(4)$							0.384	0.013
							(0.337)	(0.001)***
$\theta_2(5)$								0.002
								(0.007)
LRT[3]		44.38	14.94		86.39	120.57	44.36	11.80

Note:
1. $IS-DCC(i)$ stands for the i-state independent switching $DCC-GARCH$ model.
2. Figures in parentheses are standard errors and *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.
3. LRT stands for the likelihood ratio test. The likelihood ratio test statistics is given by $LRT = -2(\ln(L(i-1)) - \ln(L(i)))$, where $L(i)$ is the likelihood value of $IS-DCC(i)$. The critical values at 1% for $i = 2,3,4,5$ are 13.28, 16.81, 20.09 and 23.21, respectively.

Table II. Continue
Estimates of Unknown Parameters of Alternative Models
Data period is from January 1991 to December 2007

	COCOA			COFFEE			
	DCC	IS-DCC(2)[1]	IS-DCC(3)	DCC	IS-DCC(2)	IS-DCC(3)	IS-DCC(4)
	Mean Equation			Mean Equation			
μ_c	0.119	0.111	0.115	-0.013	-0.042	-0.008	-0.030
	(0.099)[2]	(0.088)	(0.112)	(0.057)	(0.097)	(0.005)*	(0.023)*
μ_f	0.098	0.099	0.091	-0.018	-0.072	-0.038	-0.057
	(0.107)	(0.094)	(0.113)	(0.055)	(0.111)	(0.043)	(0.037)*
	Volatility Equation			Volatility Equation			
γ_c	0.752	0.605	0.694	1.684	2.581	0.608	1.921
	(0.240)***	(0.195)***	(0.222)***	(0.654)***	(0.883)***	(0.285)**	(0.733)***
γ_f	0.200	0.130	0.172	2.046	3.452	0.586	1.968
	(0.103)**	(0.077)**	(0.098)**	(0.794)***	(1.188)***	(0.357)*	(0.646)***
α_c	0.056	0.047	0.046	0.085	0.083	0.051	0.076
	(0.014)***	(0.011)***	(0.011)***	(0.017)***	(0.018)***	(0.015)***	(0.018)***
α_f	0.031	0.033	0.030	0.087	0.078	0.048	0.073
	(0.010)***	(0.008)***	(0.009)***	(0.017)***	(0.018)***	(0.015)***	(0.018)***
β_c	0.895	0.911	0.904	0.848	0.808	0.926	0.848
	(0.025)***	(0.021)***	(0.024)***	(0.038)***	(0.049)***	(0.023)***	(0.041)***
β_f	0.957	0.961	0.960	0.845	0.798	0.931	0.856
	(0.013)***	(0.010)***	(0.012)***	(0.037)***	(0.053)***	(0.024)***	(0.033)***
	Correlation Equation			Correlation Equation			
$\theta_1(1)$	0.087	0.371	0.370	0.063	0.113	0.810	0.111
	(0.025)***	(0.114)***	(0.127)***	(0.009)***	(0.026)***	(0.000)***	(0.033)***
$\theta_1(2)$		0.029	0.185		0.216	0.080	0.000
		(0.009)***	(0.116)**		(0.082)***	(0.032)***	(0.025)
$\theta_1(3)$			0.039			0.011	0.795
			(0.013)***			(0.091)	(0.002)***
$\theta_1(4)$							0.036
							(0.019)**
$\theta_2(1)$	0.896	0.627	0.628	0.924	0.887	0.190	0.889
	(0.029)***	(0.117)***	(0.130)***	(0.010)***	(0.029)***	(0.000)***	(0.032)***
$\theta_2(2)$		0.962	0.000		0.003	0.920	0.231
		(0.012)***	(0.024)		(0.010)***	(0.033)***	(1.184)
$\theta_2(3)$			0.961			0.156	0.205
			(0.013)***			(0.227)***	(0.002)***
$\theta_2(4)$							0.964
							(0.019)***
LRT[3]		58.81	12.43		80.14	17.04	15.63

Note: 1. $IS-DCC(i)$ stands for the i-state independent switching $DCC-GARCH$ model.

2. Figures in parentheses are standard errors and *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.

3. LRT stands for the likelihood ratio test. The likelihood ratio test statistics is given by $LRT = -2(\ln(L(i-1)) - \ln(L(i)))$, where $L(i)$ is the likelihood value of $IS-DCC(i)$. The critical values at 1% for $i = 2,3,4,5$ are 13.28, 16.81, 20.09 and 23.21, respectively.

Table II. Continue
Estimates of Unknown Parameters of Alternative Models
Data period is from January 1991 to December 2007

	CROUD OIL					NATURAL GAS			
	DCC	IS-DCC(2)[1]	IS-DCC(3)	IS-DCC(4)	IS-DCC(5)	DCC	IS-DCC(2)	IS-DCC(3)	IS-DCC(4)
	Mean Equation					Mean Equation			
μ_c	0.087 (0.178)[2]	0.046 (0.126)	0.159 (0.677)	0.184 (0.052)***	0.163 (0.119)*	0.168 (0.354)**	0.108 (0.248)*	0.069 (0.555)*	0.083 (0.217)*
μ_f	0.085 (0.173)	0.028 (0.124)	0.171 (0.677)	0.197 (0.051)***	0.167 (0.121)*	0.248 (0.315)**	0.130 (0.306)**	0.247 (0.367)*	0.230 (0.186)**
	Volatility Equation					Volatility Equation			
γ_c	1.652 (0.490)***	1.966 (0.522)***	2.648 (0.868)***	1.339 (0.165)***	2.719 (0.768)***	9.848 (2.125)***	11.597 (2.205)***	11.603 (2.195)***	12.156 (2.328)***
γ_f	1.441 (0.418)***	1.971 (0.516)***	2.706 (0.915)***	1.535 (0.192)***	2.904 (0.781)***	5.366 (1.485)***	7.826 (1.931)***	8.947 (2.174)***	9.556 (2.392)***
α_c	0.222 (0.030)***	0.167 (0.025)***	0.159 (0.027)***	0.080 (0.007)***	0.104 (0.015)***	0.310 (0.039)***	0.305 (0.041)***	0.289 (0.039)***	0.288 (0.038)***
α_f	0.218 (0.029)***	0.169 (0.024)***	0.146 (0.026)***	0.072 (0.007)***	0.100 (0.016)***	0.240 (0.036)***	0.227 (0.035)***	0.244 (0.035)***	0.245 (0.036)***
β_c	0.744 (0.035)***	0.759 (0.031)***	0.743 (0.046)***	0.871 (0.006)***	0.790 (0.039)***	0.643 (0.036)***	0.624 (0.037)***	0.637 (0.036)***	0.630 (0.037)***
β_f	0.751 (0.033)***	0.758 (0.031)***	0.750 (0.047)***	0.868 (0.007)***	0.785 (0.039)***	0.721 (0.034)***	0.701 (0.039)***	0.680 (0.043)***	0.668 (0.048)***
	Correlation Equation					Correlation Equation			
$\theta_1(1)$	0.156 (0.021)***	0.000 (0.025)	0.880 (0.025)***	0.670 (0.032)***	0.376 (0.107)***	0.068 (0.013)***	0.097 (0.051)***	0.141 (0.039)***	0.169 (0.111)***
$\theta_1(2)$		0.455 (0.034)***	0.000 (0.006)***	0.268 (0.034)***	0.997 (0.001)***		0.135 (0.035)***	0.670 (0.173)***	0.000 (0.004)
$\theta_1(3)$			0.318 (0.031)***	0.000 (0.003)	0.776 (0.038)***			0.081 (0.042)	0.744 (0.222)***
$\theta_1(4)$				0.998 (0.001)***	0.000 (0.007)				0.139 (0.037)***
$\theta_1(5)$					0.289 (0.034)***				
$\theta_2(1)$	0.599 (0.045)***	0.010 (0.109)	0.120 (0.026)***	0.330 (0.033)***	0.000 (0.062)	0.928 (0.013)***	0.000 (0.046)	0.852 (0.042)	0.000 (0.016)
$\theta_2(2)$		0.543 (0.035)***	0.115 (2.818)	0.731 (0.039)***	0.001 (0.001)		0.858 (0.037)***	0.330 (0.181)***	0.247 (2.790)
$\theta_2(3)$			0.681 (0.033)***	0.119 (0.476)	0.224 (0.038)***			0.000 (0.053)	0.253 (0.227)**
$\theta_2(4)$				0.001 (0.001)	0.000 (0.226)				0.855 (0.040)***
$\theta_2(5)$					0.710 (0.035)***				
LRT[3]		389.94	77.13	52.54	45.95[4]		75.37	17.59	1.5

Note:
1. $IS-DCC(i)$ stands for the i -state independent switching $DCC-GARCH$ model.
2. Figures in parentheses are standard errors and *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.
3. LRT stands for the likelihood ratio test. The likelihood ratio test statistics is given by $LRT = -2(\ln(L(i-1)) - \ln(L(i)))$, where $L(i)$ is the likelihood value of $IS-DCC(i)$. The critical values at 1% for $i = 2,3,4,5$ are 13.28, 16.81, 20.09 and 23.21, respectively.
4. The LRT of crude oil is still significant. However, we do not proceed to $IS-DCC(6)$ since there are fifty parameters to be estimated and empirically, increasing the number of states to five does not create significant gains compared to four states.

Table II. Continue
Estimates of Unknown Parameters of Alternative Models
Data period is from January 1991 to December 2007

	HEATING OIL				
	DCC	IS-DCC(2)[1]	IS-DCC(3)	IS-DCC(4)	IS-DCC(5)
	Mean Equation				
μ_c	0.004 (0.021)[2]	0.044 (0.069)	-0.018 (0.038)	-0.114 (0.063)**	-0.111 (0.064)**
μ_f	0.038 (0.037)	0.035 (0.069)	-0.024 (0.039)	-0.095 (0.060)*	-0.093 (0.062)*
	Volatility Equation				
γ_c	1.225 (0.343)***	1.961 (0.418)***	2.513 (0.502)***	2.874 (0.420)***	2.829 (0.441)***
γ_f	0.998 (0.311)***	1.885 (0.438)***	2.457 (0.568)***	2.522 (0.368)***	2.478 (0.384)***
α_c	0.271 (0.022)***	0.254 (0.026)***	0.251 (0.026)***	0.251 (0.029)***	0.246 (0.048)***
α_f	0.234 (0.024)***	0.251 (0.026)***	0.265 (0.027)***	0.255 (0.029)***	0.250 (0.048)***
β_c	0.729 (0.021)***	0.705 (0.025)***	0.690 (0.027)***	0.680 (0.022)***	0.684 (0.02)***
β_f	0.759 (0.024)***	0.704 (0.029)***	0.676 (0.032)***	0.685 (0.023)***	0.689 (0.027)***
	Correlation Equation				
$\theta_1(1)$	0.151 (0.020)***	0.006 (0.009)	0.741 (0.050)***	0.995 (0.002)***	0.657 (0.051)***
$\theta_1(2)$		0.444 (0.045)***	0.005 (0.009)	0.005 (0.009)	0.000 (0.057)
$\theta_1(3)$			0.210 (0.027)***	0.209 (0.026)***	0.211 (0.025)***
$\theta_1(4)$				0.648 (0.047)***	0.995 (0.003)***
$\theta_1(5)$					0.106 (0.273)
$\theta_2(1)$	0.720 (0.034)***	0.000 (0.026)	0.253 (0.052)***	0.004 (0.003)**	0.338 (0.052)***
$\theta_2(2)$		0.544 (0.046)***	0.000 (0.034)	0.000 (0.015)	0.380 (7.239)
$\theta_2(3)$			0.788 (0.029)***	0.791 (0.027)***	0.788 (0.026)***
$\theta_2(4)$				0.346 (0.047)***	0.005 (0.003)**
$\theta_2(5)$					0.000 (0.123)
LRT[3]		253.71	46.57	31.87	8.52

	PLATINUM			
	DCC	IS-DCC(2)	IS-DCC(3)	IS-DCC(4)
	Mean Equation			
μ_c	0.104 (0.069)*	0.089 (0.080)	0.108 (0.069)*	0.102 (0.290)
μ_f	0.109 (0.072)*	0.092 (0.084)	0.114 (0.069)*	0.108 (0.294)
	Volatility Equation			
γ_c	0.175 (0.052)***	0.159 (0.051)***	0.168 (0.049)***	0.174 (0.086)***
γ_f	0.180 (0.049)***	0.169 (0.049)***	0.183 (0.049)***	0.184 (0.075)***
α_c	0.093 (0.013)***	0.088 (0.013)***	0.084 (0.012)***	0.084 (0.012)***
α_f	0.079 (0.011)***	0.077 (0.011)***	0.074 (0.010)***	0.073 (0.011)***
β_c	0.886 (0.016)***	0.887 (0.016)***	0.893 (0.014)***	0.891 (0.018)***
β_f	0.900 (0.013)***	0.899 (0.013)***	0.900 (0.013)***	0.899 (0.014)***
	Correlation Equation			
$\theta_1(1)$	0.082 (0.016)***	0.273 (0.053)***	0.143 (0.049)***	0.670 (0.080)***
$\theta_1(2)$		0.132 (0.051)***	0.649 (0.076)***	0.056 (0.040)*
$\theta_1(3)$			0.080 (0.024)***	0.238 (0.087)***
$\theta_1(4)$				0.127 (0.133)
$\theta_1(5)$				
$\theta_2(1)$	0.911 (0.020)***	0.724 (0.056)***	0.000 (0.027)	0.327 (0.080)***
$\theta_2(2)$		0.000 (0.029)	0.348 (0.075)***	0.944 (0.044)***
$\theta_2(3)$			0.920 (0.026)***	0.762 (0.111)***
$\theta_2(4)$				0.000 (0.121)
$\theta_2(5)$				
LRT[3]		64.35	35.52	7.34

Note:
1. $IS-DCC(i)$ stands for the i -state independent switching $DCC-GARCH$ model.
2. Figures in parentheses are standard errors and *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.
3. LRT stands for the likelihood ratio test. The likelihood ratio test statistics is given by $LRT = -2(\ln(L(i-1)) - \ln(L(i)))$, where $L(i)$ is the likelihood value of $IS-DCC(i)$. The critical values at 1% for $i = 2,3,4,5$ are 13.28, 16.81, 20.09 and 23.21, respectively.

Table III
Out-of-Sample Hedging Effectiveness. Hedging period is from January 2008 to December 2008

	Variance of Hedged Portfolio Return	Percentage Variance Reduction[1]	Improvement of IS-DCC(3) over Other model[2]	Expected Weekly Utility[3]	Utility Gain of Dynamic Hedging Models over OLS[4]	Variance of Hedged Portfolio Return	Percentage Variance Reduction[1]	Improvement of IS-DCC(3) over Other model[2]	Expected Weekly Utility[3]	Utility Gain of Dynamic Hedging Models over OLS[4]
	WHEAT (RC=0.074*)[5]					CORN (RC=0.000***)				
Unhedged	59.674					52.052				
OLS	11.753	80.30%	6.22%	-47.177		2.514	95.17%	2.37%	-10.089	
DCC	9.414	84.22%	2.30%	-37.212	9.964	1.580	96.97%	0.58%	-6.268	3.821
IS-DCC(2)	8.707	85.41%	1.12%	-34.628	12.548	1.331	97.44%	0.10%	-5.286	4.803
IS-DCC(3)	**8.039**	**86.53%**		**-31.878**	**15.298**	**1.280**	**97.54%**		**-5.079**	**5.010**
IS-DCC(4)						**1.143**	97.80%	-0.26%	-4.519	5.570
IS-DCC(5)						1.152	97.79%	-0.25%	-4.546	5.543
	COCOA (RC=0.423)					COFFEE (RC=0.085*)				
Unhedged	26.518					11.435				
OLS	10.956	58.68%	2.63%	-43.731		4.196	63.31%	2.66%	-16.676	
DCC	11.022	58.44%	2.88%	-43.877	-0.146	4.081	64.31%	1.65%	-16.191	0.485
IS-DCC(2)	10.934	58.77%	2.55%	-43.596	0.135	4.102	64.13%	1.83%	-16.261	0.415
IS-DCC(3)	**10.258**	**61.32%**		**-40.928**	**2.803**	**3.892**	**65.96%**		**-15.403**	**1.274**
IS-DCC(4)						3.987	65.13%	0.83%	-15.755	0.921
	CRUDE OIL (RC=0.010***)					NATURAL GAS (RC=0.267)				
Unhedged	68.080					40.122				
OLS	2.537	96.27%	1.50%	-10.182		16.790	58.15%	0.33%	-67.256	
DCC	1.717	97.48%	0.30%	-6.958	3.224	16.917	57.84%	0.65%	-67.885	-0.630
IS-DCC(2)	1.611	97.63%	0.14%	-6.504	3.678	16.668	58.46%	0.03%	-66.913	0.342
IS-DCC(3)	**1.516**	**97.77%**		**-6.109**	**4.073**	**16.656**	**58.49%**		**-66.874**	**0.381**
IS-DCC(4)	1.581	97.68%	0.09%	-6.293	3.890	16.856	57.99%	0.50%	-67.670	-0.414
IS-DCC(5)	1.561	97.71%	0.07%	-6.263	3.919					
	HEATING OIL (RC=0.097*)					PLATINUM (RC=0.039**)				
Unhedged	43.578					41.813				
OLS	1.989	95.43%	0.74%	-8.070		3.794	90.93%	3.50%	-15.445	
DCC	2.110	95.16%	1.01%	-8.466	-0.397	2.469	94.10%	0.33%	-9.977	5.468
IS-DCC(2)	1.781	95.91%	0.26%	-7.128	0.942	2.407	94.24%	0.19%	-9.770	5.675
IS-DCC(3)	**1.669**	**96.17%**		**-6.694**	**1.376**	**2.329**	**94.43%**		**-9.404**	**6.041**
IS-DCC(4)	**1.667**	96.17%	-0.0029%	-6.699	1.371	2.334	94.42%	0.01%	-9.443	6.002
IS-DCC(5)	1.677	96.15%	0.02%	-6.739	1.331					

Note:
1. Percentage variance reductions are calculated as the differences of variance of unhedged position and estimated variance of alterative models over variance of unhedged position multiplied by 100.
2. Improvement of $IS-DCC(3)$ over other hedging strategies is defined as the difference of the percentage variance reduction of $IS-DCC(3)$ and the percentage variance reduction of alternative hedging strategies
3. Expected weekly utility is calculated based on equation (32)
4. Utility gains of dynamic hedging models over OLS are defined as the differences of the expected utilities of alternative dynamic models and the expected utility of OLS.
5. RC stands for the White's reality check p-value testing the null that no improvement of the best $IS-DCC$ over OLS. *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.

Table IV

Diebold-Mariano-West Test Statistics of No Superiority of Unrestricted IS-DCC model over Its Nested model. Full Sample Period (from January 2008 to December 2008).

	WHEAT	CORN	COCOA	COFFEE	CRUDE OIL	NATURAL GAS	HEATING OIL	PLATINUM
IS-DCC(2) vs. DCC	0.852	1.464*	0.284	-0.429	1.344*	0.533	1.122*	0.209
IS-DCC(3) vs. IS-DCC(2)	1.315*	0.598	1.116**	1.763**	1.235*	0.082	1.048	0.464
IS-DCC(4) vs. IS-DCC(3)	NA	2.740***	NA	-1.269*	-0.320	-1.259*	0.026	-0.316
IS-DCC(5) vs. IS-DCC(4)	NA	-0.143	NA	NA	0.188	NA	-0.889	NA
IS-DCC(3) vs. DCC	1.462**	1.304*	1.013*	1.164*	1.549**	0.501	1.114*	0.800

Note:
1. The DMW statistic is shown in equation (36) with the adjusted critical values for nested models tabulated in McCracken (2007).
2. The N/R ratio is 0.06 and the number of additional estimated parameters for $IS-DCC(i)$, i=2,3,4 and 5 are four, six, eight, and ten, respectively. The critical values are tabulated for $N/R=0$ and 0.1, and we construct the values for $N/R=0.06$ by interpolation.
3. *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.
4. NA stands for not available since the likelihood value does not increase significantly when the number of states is further increased.

Table V

Diebold-Mariano-West Test Statistics of No Superiority of IS-DCC(3) and IS-DCC(2) over MS-DCC Model

	WHEAT	CORN	COCOA	COFFEE	CRUDE OIL	NATURAL GAS	HEATING OIL	PLATINUM
Variance of IS-DCC(2) hedging	8.707	1.331	10.934	4.102	1.611	16.668	1.781	2.407
Variance of IS-DCC(3) hedging	8.039	1.280	10.258	3.892	68.871.516	16.656	1.669	2.329
Variance of MS-DCC hedging	12.043	1.464	10.195	3.917	1.832	20.171	2.265	2.352
DMW test of no superiority of IS-DCC(2) over MS-DCC	1.526*	0.884	-0.656	-0.569	0.833	1.962**	1.192	-0.166
DMW test of no superiority of IS-DCC(3) over MS-DCC	1.839**	1.000	0.032	0.173	0.954	1.939**	1.167	0.117

Note:
1. Since $MS-DCC$ is not nested within the $IS-DCC$ model, regular DMW critical values are applied.
2. For one-sided test, the null that $MS-DCC$ is not outperformed by $IS-DCC$ at the 5% level is rejected if $DM > 1.645$.
3. *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.

Table VI
Out-of-Sample Hedging Effectiveness, Post-2000

	Variance of Hedged Portfolio Return	Percentage Variance Reduction[1]	Improvement of Best IS-DCC over Other model[2]	Expected Weekly Utility[3]	Utility Gain of Dynamic Hedging Models over OLS[4]	Variance of Hedged Portfolio Return	Percentage Variance Reduction[1]	Improvement of Best IS-DCC over Other model[2]	Expected Weekly Utility[3]	Utility Gain of Dynamic Hedging Models over OLS[4]
	WHEAT (RC=0.905)[5]					CORN (RC=0.075*)				
Unhedged	59.674					52.052				
OLS	11.461	80.79%	1.82%	-46.000		1.937	96.28%	0.81%	-7.772	
DCC	17.803	70.17%	12.45%	-70.896	-24.896	1.548	97.03%	0.06%	-5.988	1.784
IS-DCC(2)	**10.376**	**82.61%**		**-41.223**	**4.777**	**1.516**	**97.09%**		**-5.871**	**1.901**
IS-DCC(3)	10.387	82.59%	0.02%	-41.260	4.741	1.551	97.02%	0.07%	-6.006	1.765
	COCOA (RC=0.423)					COFFEE (RC=0.975)				
Unhedged	26.518					11.435				
OLS	10.448	60.60%	0.36%	-41.782		3.832	66.48%	-0.65%	-15.259	
DCC	10.393	60.81%	0.15%	-41.532	0.251	3.949	65.46%	0.37%	-15.677	-0.418
IS-DCC(2)	**10.352**	**60.96%**		**-41.369**	**0.413**	**3.907**	**65.83%**		**-15.497**	**-0.238**
IS-DCC(3)	10.493	60.43%	0.53%	-41.921	-0.139	3.964	65.33%	0.50%	-15.741	-0.482
	CRUDE OIL (RC=0.004***)					NATURAL GAS (RC=0.005***)				
Unhedged	68.080					40.122				
OLS	2.490	96.34%	1.46%	-9.988		16.861	57.98%	0.17%	-67.524	
DCC	2.421	96.44%	1.36%	-9.753	0.234	16.993	57.65%	0.51%	-68.174	-0.650
IS-DCC(2)	1.672	97.54%	0.26%	-6.690	3.298	**16.790**	**58.15%**		**-67.407**	**0.117**
IS-DCC(3)	1.896	97.22%	0.59%	-7.538	2.450	17.000	57.63%	0.52%	-68.206	-0.683
IS-DCC(4)	**1.496**	**97.80%**		**-5.989**	**3.999**	17.109	57.36%	0.79%	-68.642	-1.118
	HEATING OIL (RC=0.646)					PLATINUM (RC=0.043**)				
Unhedged	43.578					41.813				
OLS	2.171	95.02%	0.08%	-8.769		4.627	88.93%	5.16%	-18.821	
DCC	2.827	93.51%	1.59%	-11.169	-2.400	2.659	93.64%	0.46%	-10.631	8.190
IS-DCC(2)	2.153	95.06%	0.04%	-8.541	0.228	2.495	94.03%	0.06%	-10.015	8.806
IS-DCC(3)	**2.136**	**95.10%**		**-8.471**	**0.297**	**2.468**	**94.10%**		**-9.924**	**8.897**
IS-DCC(4)						2.578	93.83%	0.26%	-10.349	8.472

Note:
1. Percentage variance reductions are calculated as the differences of variance of unhedged position and estimated variance of alterative models over variance of unhedged position multiplied by 100.
2. Improvement of Best $IS-DCC$ over other hedging strategies is defined as the difference of the percentage variance reduction of best $IS-DCC$ and the percentage variance reduction of alternative hedging strategies
3. Expected weekly utility is calculated based on equation (32)
4. Utility gains of dynamic hedging models over OLS are defined as the differences of the expected utilities of alternative dynamic models and the expected utility of OLS.
5. RC stands for the White's reality check p-value testing the null that no improvement of the best $IS-DCC$ over OLS. *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.

Table VII
Out-of-Sample Hedging Effectiveness, Pre-2000

	Variance of Hedged Portfolio Return	Percentage Variance Reduction[1]	Improvement of Best IS-DCC over Other model[2]	Expected Weekly Utility[3]	Utility Gain of Dynamic Hedging Models over OLS[4]	Variance of Hedged Portfolio Return	Percentage Variance Reduction[1]	Improvement of Best IS-DCC over Other model[2]	Expected Weekly Utility[3]	Utility Gain of Dynamic Hedging Models over OLS[4]
	WHEAT (RC=0.775)[5]					CORN (RC=0.029**)				
Unhedged	9.906					14.738				
OLS	3.511	64.55%	-0.65%	-14.250		3.373	77.11%	3.36%	-13.556	
DCC	3.761	62.03%	1.88%	-15.238	-0.988	2.967	79.87%	0.60%	-11.914	1.642
IS-DCC(2)	3.677	62.88%	1.03%	-14.881	-0.631	2.914	80.23%	0.24%	-11.682	1.874
IS-DCC(3)	**3.575**	**63.91%**		**-14.498**	**-0.248**	3.033	79.42%	1.05%	-12.157	1.399
IS-DCC(4)						**2.878**	**80.47%**		**-11.538**	**2.018**
	COCOA (RC=0.113)					COFFEE (RC=0.150)				
Unhedged	20.227					60.153				
OLS	15.077	25.46%	23.15%	-61.081		1.823	96.97%	0.78%	-7.315	
DCC	**10.395**	**48.61%**		**-41.912**	**19.170**	1.366	97.73%	0.02%	-5.498	1.818
IS-DCC(2)	10.751	46.85%	1.76%	-43.417	17.664	1.365	97.73%	0.02%	-5.507	1.808
IS-DCC(3)	10.638	47.41%	1.21%	-42.865	18.216	**1.353**	**97.75%**		**-5.420**	**1.896**
IS-DCC(4)						1.456	97.58%	0.17%	-5.848	1.468
	CRUDE OIL (RC=0.378)					NATURAL GAS (RC=0.196)				
Unhedged	23.444					56.666				
OLS	0.563	97.60%	0.04%	-2.164		15.482	72.68%	3.66%	-61.764	
DCC	0.621	97.35%	0.28%	-2.448	-0.284	15.118	73.32%	3.02%	-60.090	1.675
IS-DCC(2)	0.568	97.58%	0.06%	-2.217	-0.053	**13.408**	**76.34%**		**-53.302**	**8.463**
IS-DCC(3)	0.582	97.52%	0.12%	-2.270	-0.106	15.195	73.18%	3.15%	-60.409	1.356
IS-DCC(4)	**0.555**	**97.63%**		**-2.141**	**0.023**					
IS-DCC(5)	0.571	97.56%	0.07%	-2.255	-0.091					
	HEATING OIL (RC=0.932)					PLATINUM (RC=0.322)				
Unhedged	22.917					5.321				
OLS	1.333	94.18%	-0.40%	-5.215		1.521	71.41%	0.21%	-6.085	
DCC	1.652	92.79%	1.00%	-6.624	-1.410	1.522	71.40%	0.22%	-5.950	0.135
IS-DCC(2)	1.508	93.42%	0.37%	-6.013	-0.799	1.514	71.55%	0.07%	-5.913	0.172
IS-DCC(3)	**1.423**	**93.79%**		**-5.656**	**-0.441**	**1.510**	**71.62%**		**-6.041**	**0.044**
IS-DCC(4)	1.677	92.68%	1.11%	-6.732	-1.518					

Note:
1. Percentage variance reductions are calculated as the differences of variance of unhedged position and estimated variance of alterative models over variance of unhedged position multiplied by 100.
2. Improvement of Best $IS-DCC$ over other hedging strategies is defined as the difference of the percentage variance reduction of best $IS-DCC$ and the percentage variance reduction of alternative hedging strategies
3. Expected weekly utility is calculated based on equation (32)
4. Utility gains of dynamic hedging models over OLS are defined as the differences of the expected utilities of alternative dynamic models and the expected utility of OLS.
5. RC stands for the White's reality check p-value testing the null that no improvement of the best $IS-DCC$ over OLS. *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.

Table VIII

Diebold-Mariano-West Test Statistics of No Superiority of Unrestricted IS-DCC model over Its Nested models. Two Sub-sample Periods.

	WHEAT	CORN	COCOA	COFFEE	CRUDE OIL	NATURAL GAS	HEATING OIL	PLATINUM
				Post-2000				
IS-DCC(2) vs. DCC	1.471**	0.783	-0.060	0.727	1.393**	-0.987*	1.682**	1.793**
IS-DCC(3) vs. IS-DCC(2)	-0.461	-0.744	-1.037*	-1.095*	-0.575	0.506	0.947*	0.240
IS-DCC(4) vs. IS-DCC(3)	NA	NA	NA	NA	0.910*	-0.545	NA	-1.062*
BEST IS-DCC vs. DCC	1.471**	0.783	-0.060	0.727	1.445***	-0.987*	1.704**	1.406**
				Pre-2000				
IS-DCC(2) vs. DCC	2.698***	0.300	-0.854	-0.008	0.951	0.889	1.548***	0.994*
IS-DCC(3) vs. IS-DCC(2)	-0.599	-0.553	0.464	0.723	-0.818	-0.915*	2.563***	-0.240
IS-DCC(4) vs. IS-DCC(3)	NA	1.575**	NA	-1.262**	0.990*	NA	NA	NA
IS-DCC(5) vs. IS-DCC(4)	NA	NA	NA	NA	-0.243	NA	NA	NA
BEST IS-DCC vs. DCC	1.809*	0.343	-0.926*	0.338	0.803	0.889	1.933***	0.640

Note:
1. The DMW statistic is shown in equation (36) with the adjusted critical values for nested models tabulated in McCracken (2007).
2. The N/R ratio is 0.125 and the number of additional estimated parameters for $IS-DCC(i)$, i=2,3,4 and 5 are four, six, eight, and ten, respectively. The critical values are tabulated for $N/R=0.1$ and 0.2, and we construct the values for $N/R=0.125$ by interpolation.
3. *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.
4. NA stands for not available since the likelihood value does not increase significantly when the number of states is further increased.

Figures

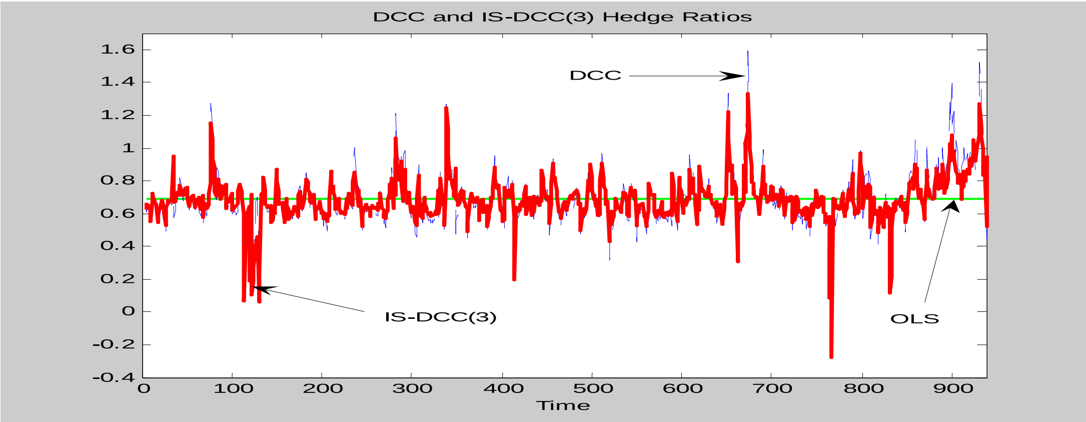


Figure 1 DCC and IS-DCC(3) Hedge Ratios for Wheat

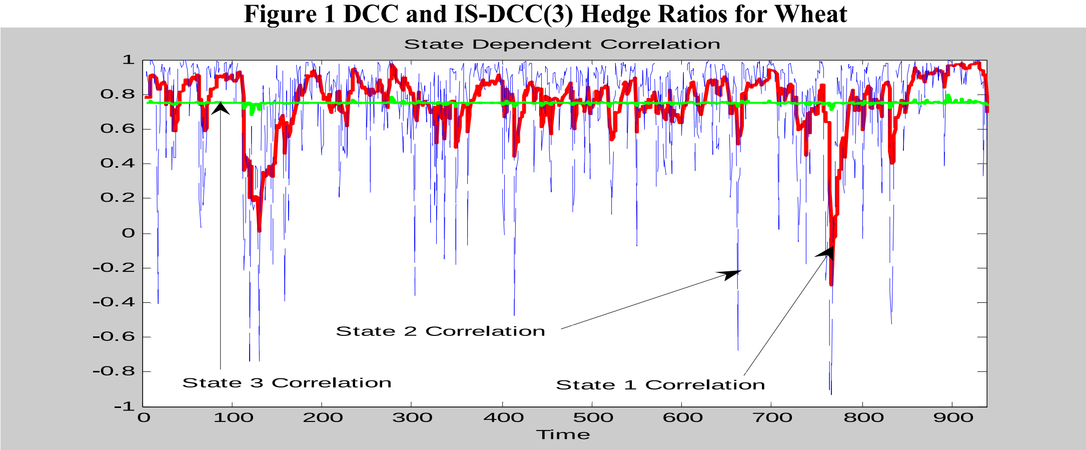


Figure 2 Correlations in Each Regime for Wheat

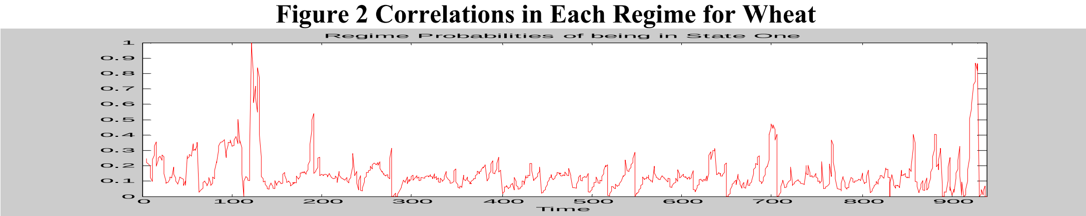


Figure 3 Regime Probability of being in State 1 for Wheat

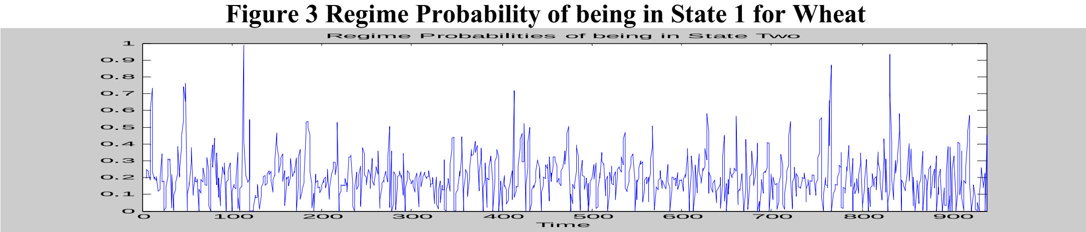


Figure 4 Regime Probability of being in State 2 for Wheat

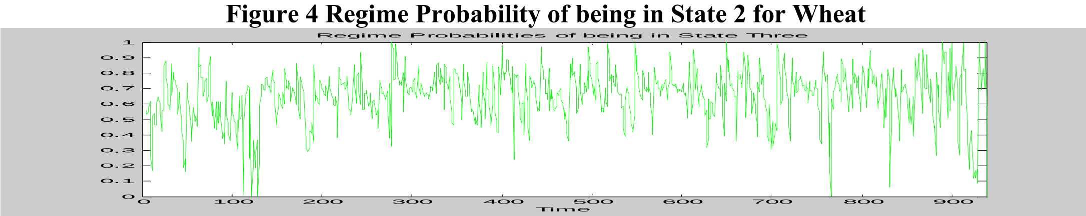


Figure 5 Regime Probability of being in State 3 for Wheat

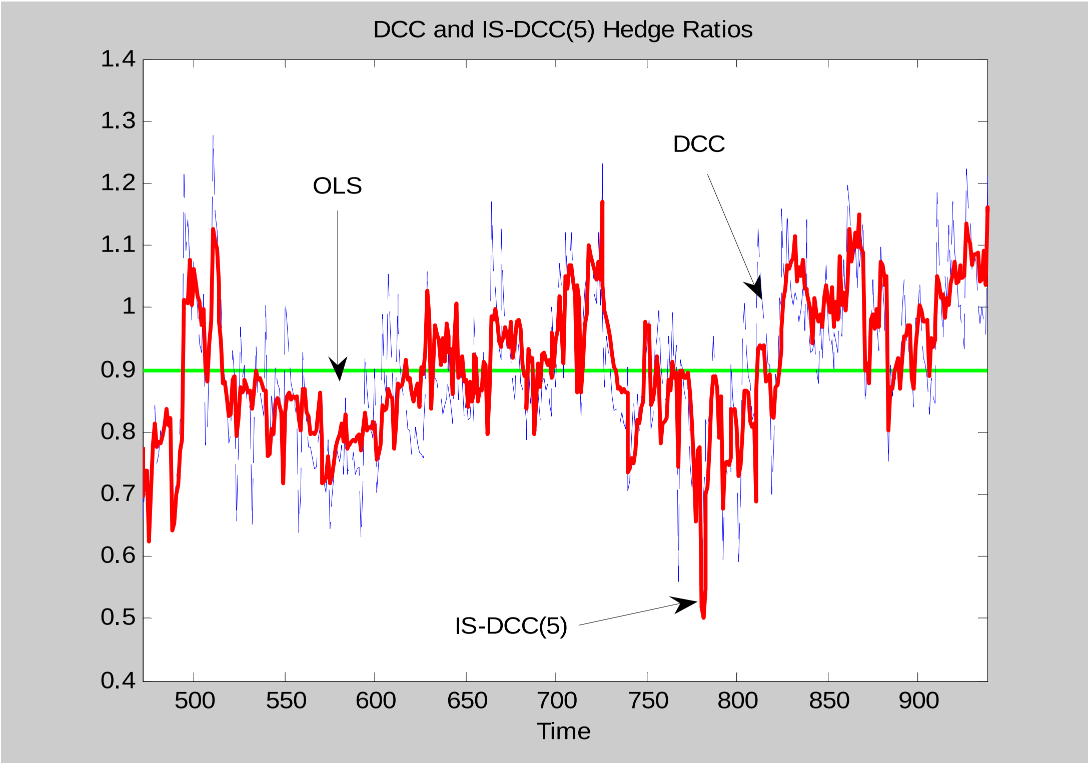


Figure 6 DCC and IS-DCC(5) Hedge Ratios for Corn

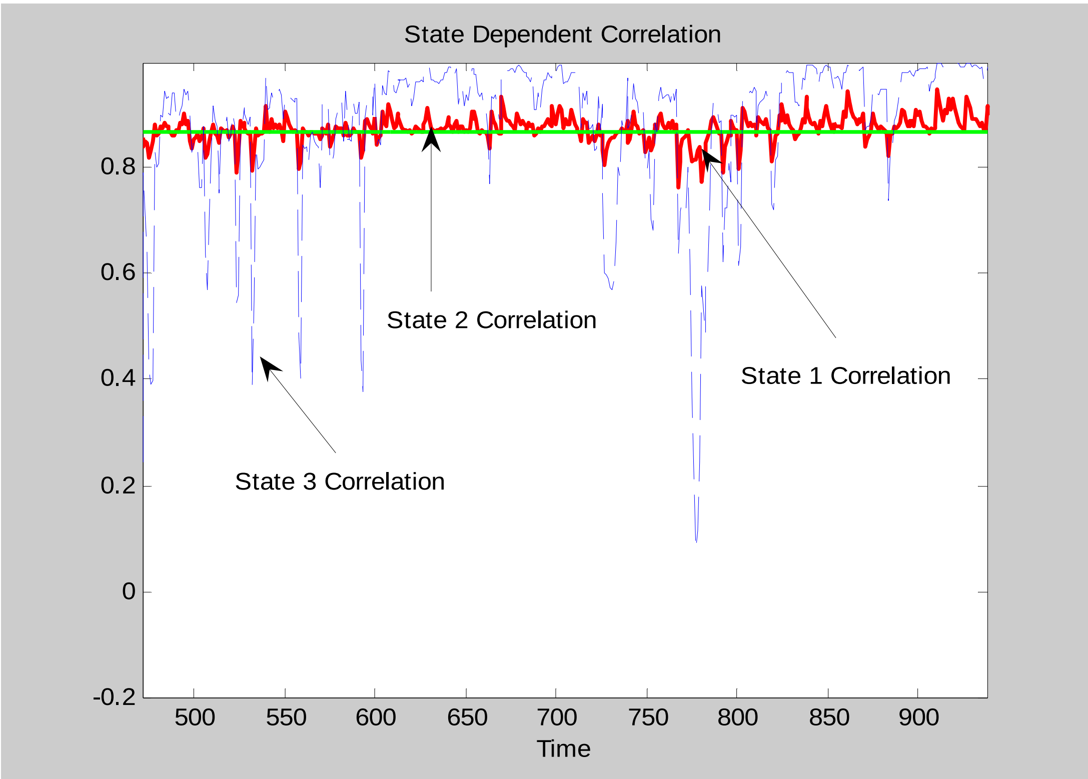


Figure 7 Correlations in state1, state2, and state 3 for Corn

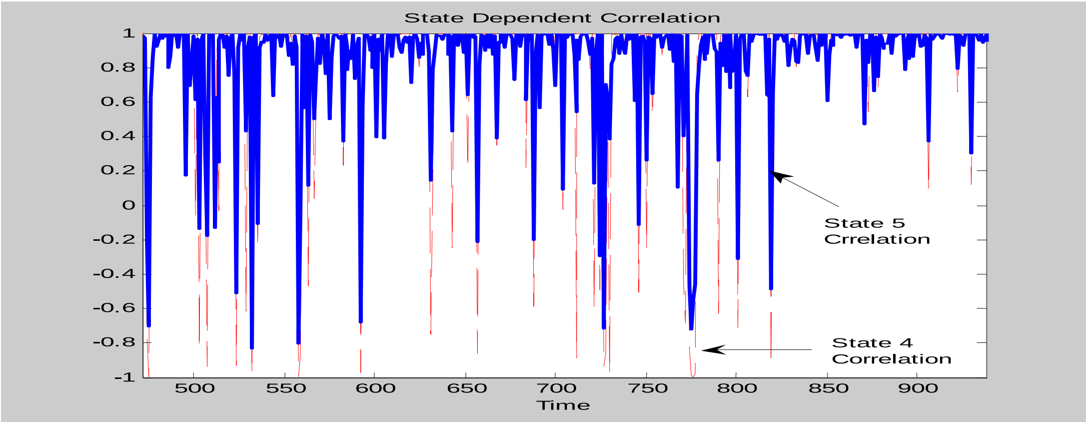


Figure 8 Correlations in state4 and state 5 for Corn

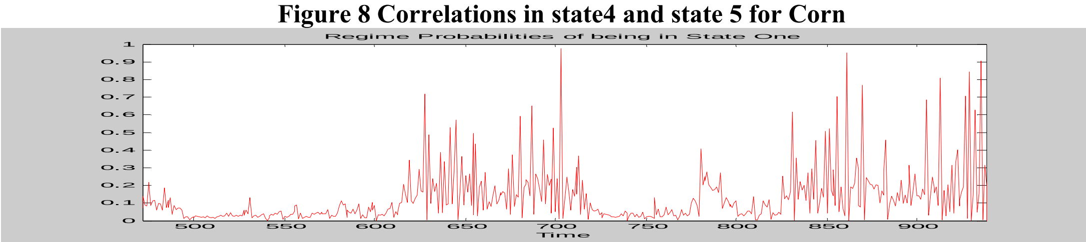


Figure 9 Regime Probability of being in State 1 for Corn

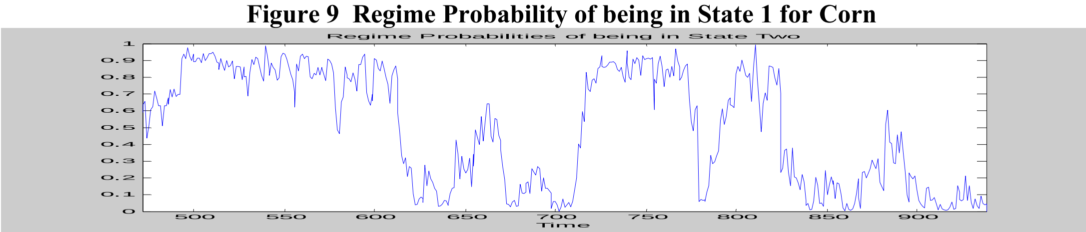


Figure 10 Regime Probability of being in State 2 for Corn

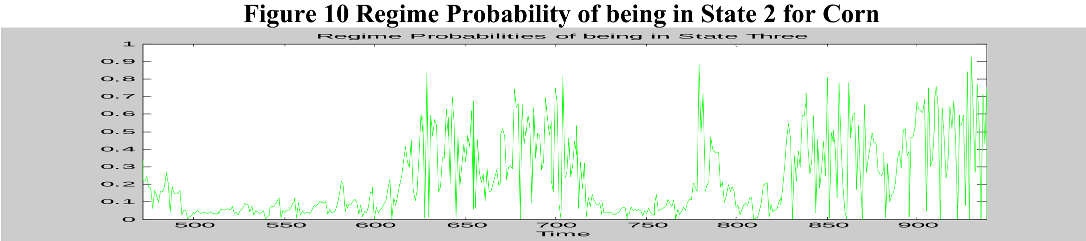


Figure 11 Regime Probability of being in State 3 for Corn

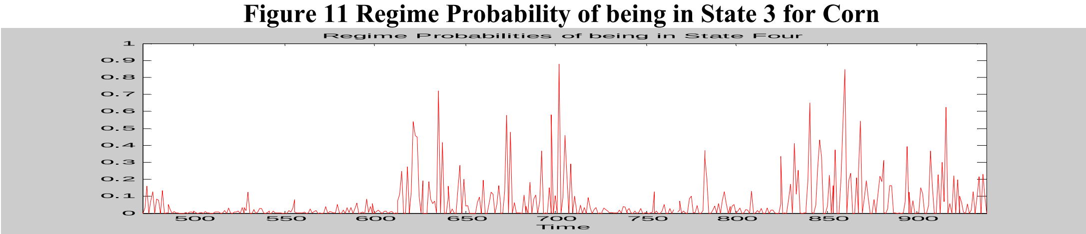


Figure 12 Regime Probability of being in State 4 for Corn

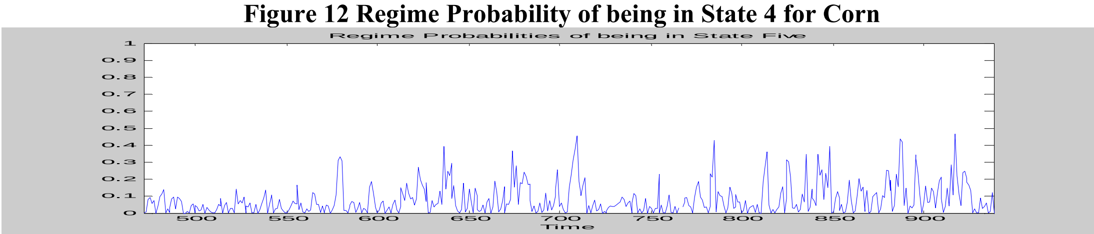
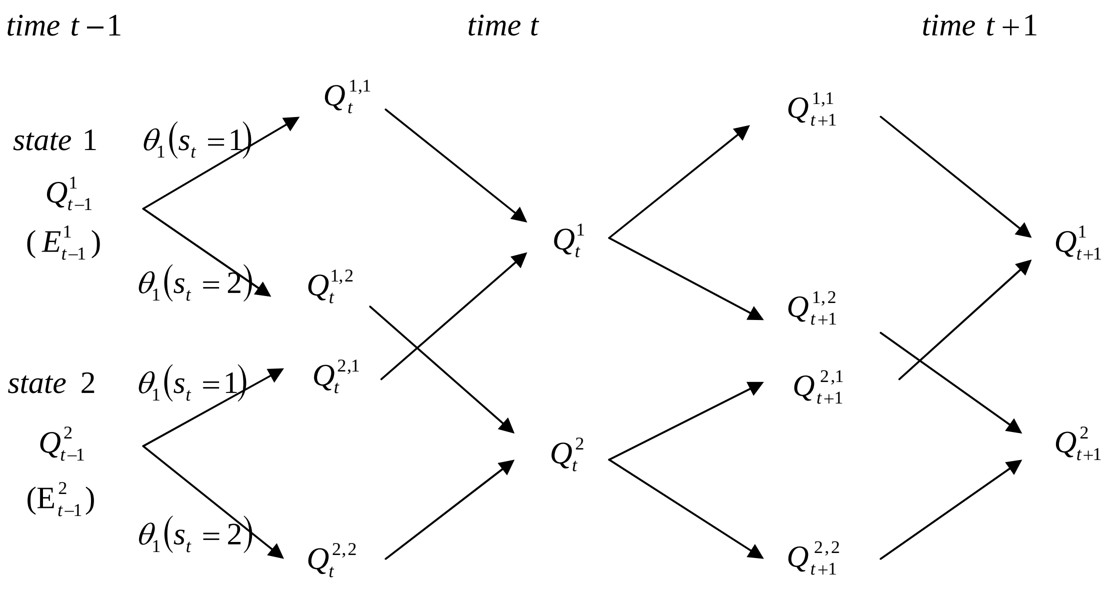


Figure 13 Regime Probability of being in State 5 for Corn

Appendix A. Problems of Path-dependency and Caporin and Billio's Recombining Method

In this appendix, the problem of path-dependency in covariance process and the problem of Caporin and Billio's recombining method will be addressed. Markov regime switching GARCH models are essentially intractable and impossible to estimate due to the dependence of the conditional covariance matrix on the entire past history of the data. This problem can be solved via Caporin and Billio's recombining method which is illustrated with the following figure



Without recombining, the tree will diverge and number of cases to be considered will be infinity and make the model intractable. With Caporin and Billio's recombining procedure, each conditional covariance depends only on the current regimes, not on the entire past history of the process and the estimation of the model will be feasible. Caporin and Billio's recombining method, however, is computational intensive due to the requirement of taking expectations in each time step. For instance, if the number of states is five, there will be $5^2 = \mathbf{Q}s$ in each time step and one has to collapse these 25 $\mathbf{Q}s$ into 5 $\mathbf{Q}s$ by taking expectations to make the model tractable. The correlation matrixes are collapsed by taking expectations based on equation (10). In addition to the problem of computational intensive, it is possible that the covariance of one regime will still be affected by shocks even if θ_1 in that regime is zero. To see this consider equation (12) rewritten below:

$$\mathbf{Q_t} = (1-\theta_1-\theta_2)\ (1-\theta_2)^{-1}\overline{\mathbf{Q}} + \theta_1 \sum_{i=1}^{\infty} \theta_2^{i-1} \mathbf{E_{t-\mathit{i}}}\ .$$

A state-dependent version of equation (12) is given by

$$\mathbf{Q_t}(s_t) = (1-\theta_1(s_t)-\theta_2(s_t))\ (1-\theta_2(s_t))^{-1}\overline{\mathbf{Q}} + \theta_1(s_t) \sum_{i=1}^{\infty} \theta_2^{i-1}(s_t)\ \mathbf{E_{t-\mathit{i}}}\ , \quad \text{(A1)}$$

where s_t is the state variable. Suppose that $\theta_1(s_t = 1) = 0$, this implies that all previous shocks $\mathbf{E_{t-\mathit{i}}}$, $i = 1,2,\cdots,\infty$ should have a zero impact on $\mathbf{Q_t}$ in state one. Although there is no effect of shocks at time $t-1$ ($\mathbf{E_{t-1}}$) on the covariance at time t at regime one, namely, Q_t^1, it could affects Q_t^2 through the channel of $Q_t^{1,2}$ and therefore affects Q_{t+i}^1, $i = 1,2,\cdots,\infty$ because of recombining. This contradicts to the fact that $\theta_1(s_t = 1) = 0$ implies a zero impact for all previous shocks in the regime one. The recombining procedure causes difficulty in interpreting the meaning of system parameters and Hass, et. al. (2004) refer this as the analytical intractability problem.

Bibliography

Alizadeh, A., Nomikos, N., 2004. A Markov Regime Switching Approach for Hedging Stock Indices. The Journal of Futures Markets 24, 649-674.

Alizadeh, A., Nomikos, N., 2008. A Markov regime switching approach for hedging energy commodities. Journal of Banking & Finance 32, 1970-1983.

Ang, A., Chen, J., 2002. Asymmetric correlations of equity portfolios. Journal of Financial Economics 63, 443-494.

Baillie, R. T., Myers, R. J., 1991. Bivariate GARCH Estimation of the Optimal Commodity Futures Hedge. Journal of Applied Econometrics 6, 109-124.

Bauwen, L., Laurent, S., Rombouts, J. V. K., 2006. Multivariate GARCH models: A Survey. Journal of Applied Econometrics 21, 79-109.

Brooks, C., Henry, O. T., Persand, G., 2002. The Effect of Asymmetries on Optimal Hedge Ratios. Journal of Business 75, 333-352.

Byström, H. N. E., 2003. The Hedging Performance of Electricity Futures on the Nordic Power Exchange. Applied Economics 35, 1-11.

Cai, J., 1994. A Markov Model of Switching-Regime ARCH. Journal of Business and Economic Statistics 12, 309-316.

Caporin, M., Billio, M., 2005. Multivariate Markov switching dynamic conditional correlation GARCH representations for contagion analysis, Statistical Methods & Applications 14, 145-161.

Diebold, F.X., Mariano, R.S., 1995. Comparing predictive accuracy. Journal of Business and Economic Statistics 13, 253–263.

Engle, R., 2002. Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models. Journal of Business & Economic Statistics 20, 339-350.

Gagnon, L., Lypny, G., 1998. Hedging Short-Term Interest Risk under Time-Varying Distribution. The Journal of Futures Markets 15, 767-783.

Gagnon, L., Lypny, G., McCurdy, T., 1998. Hedging foreign currency portfolios. Journal of Empirical Finance 5, 197–220.

Gray S. F., 1996. Modeling the Conditional Distribution of Interest Rates as a Regime-Switching Process. Journal of Financial Economics 42, 27-62

Hamilton, J. D., 1989. A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. Econometrica 57, 357-384.

Hamilton, J. D., 1994. Time Series Analysis. Princeton, NJ: Princeton University Press, 400-401.

Hamilton, J. D., Susmel, R., 1994. Autoregressive Conditional Heteroscedasticity and Changes in Regime. Journal of Econometrics 64, 307-333.

Hass, M., Mittnik, S., Paolella, M. S., 2004. A New Approach to Markov-Switching GARCH Methods. Journal of Financial Econometrics 2, 493-530.

Kim, C. J., 1994. Dynamic linear models with Markov-switching. Journal of Econometrics 60, 1-22.

Krishnan, C.N.V., Petkova, R., Ritchken, P., 2008. Correlation risk. Journal of Empirical Finance 16, 353-367.

Kroner, K. F., Sultan J., 1993. Time-Varying Distribution and Dynamic Hedging with Foreign Currency Futures. Journal of Financial and Quantitative Analysis 28, 535-551.

Lafuente, J., Novales, A., 2003. Optimal hedging under departures from the cost-of-carry valuation: Evidence from the Spanish stock index futures market. Journal of Banking & Finance 27, 1053–1078.

Ledoit, O., Santa-Clara, P., Wolf, M., 2003. Flexible Multivariate GARCH Modeling with an Application to International Stock Markets. Review of Economics & Statistics 85, 735-747.

Lee, H. T., Yoder, J. K., Mittelhammer, R. C., McCluskey, J. J., 2006. A Random Coefficient Autoregressive Markov Regime Switching Model for Dynamic Futures Hedging. The Journal of Futures Markets 26, 103-129.

Lee, H. T., Yoder, J. K., 2007a. Optimal Hedging with a Regime-Switching Time-Varying Correlation GARCH Model. The Journal of Futures Markets 27, 495-516.

Lee, H. T., Yoder, J. K., 2007b. A Bivariate Markov Regime Switching GARCH Approach to Estimate the Time Varying Minimum Variance Hedge Ratio. Applied Economics 39, 1253-1265.

Lee, H. T., 2009a. Optimal futures hedging under jump switching dynamics. Journal of Empirical Finance16, 446-456.

Lee, H. T., 2009b. A Copula-Based Markov Regime Switching GARCH model for optimal futures hedging. Journal of Futures Markets, Forthcoming.

McCracken, M., 2007. Asymptotics for Out-of-Sample Tests of Granger Causality. Journal of Econometrics 140, 719-752.

Park, T. H., Switzer, L. N., 1995. Bivariate GARCH Estimation of the Optimal Hedge Ratios for Stock Index Futures: A Note. The Journal of Futures Markets 15, 61-67.

Politis, D., Romano, J. P., 1994. The stationary bootstrap. Journal of the American Statistical Association 89, 1303-1313.

Sarno, L., Valente, G., 2000. The Cost of Carry Model and Regime Shifts in Stock Index Futures Markets: An Empirical Investigation. The Journal of Futures Markets 20, 603-624.

Sarno, L., Valente, G., 2005a. Empirical Exchange Rate Models and Currency Risk: Some Evidence from Density Forecasts. Journal of International Money and Finance 24, 363-385.

Sarno, L., Valente, G., 2005b. Modelling and Forecasting Stock Returns: Exploiting the Futures Market, Regime Shifts, and International Spillovers. Journal of Applied Econometrics 20, 345-376.

Sullivan, R., Timmermann, A., White, H., 1999. Data-snooping, technical trading rule performance, and the bootstrap. Journal of Finance 54, 1647-1692.

West, Kenneth D., 1996. Asymptotic Inference about Predictive Ability. Econometrica 64, 1067-1084

White, H., 2000. A reality check for data snooping. Econometrica 68, 1097-1126.